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RECURRENCE RELATIONS FOR MOMENTS OF DOUBLY COMPOUND DISTRIBUTIONS

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Abstract: The paper contains recurrence formulae for moments of doubly compound distributions with the counting distributions satisfying some recursions.

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1. Introduction

Let N be a discrete random variable with a probability function $p(n)$. In this paper will be considered two cases:

R1. the probability function $p(n)$ satisfies recurrence relation of the first order

$$p(n) = \left(a + \frac{b}{n+c}\right) p(n-1)$$

for some constans a, b and c with $p(n) = 0$ for $n_0 < 0$ and $p(n_0) > 0$ for $n_0 \geq 0$ (cf. [3]),

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R2. the probability function $p(n)$ satisfies recurrence relation of k-th order

$$p(n) = \sum_{i=1}^{k} \left(a_i + \frac{b_i}{n} \right) p(n-i)$$

for some integer k and constants a_i and b_i $(i = 1, 2, ..., k)$ with $p(n) = 0$ for $n_0 < 0$ and $p(n_0) > 0$ for $n_0 \geq 0$ (cf. [7]).

The class of distributions satisfying both recursions contains important class of distributions studied by Panjer [4] and Sundt, Jewell [6] satisfying recursion:

$$p(n) = \left(a + \frac{b}{n} \right) p(n-1), \quad n = 1, 2, ..., \tag{1}$$

for some constants $a < 1$ and $a + b \geq 0$. The class of distributions satisfying relation R2 contains class of distributions considered by Schröter [5]

$$p(n) = \left(a + \frac{b}{n} \right) p(n-1) + \frac{c}{n} p(n-2), \quad n = 1, 2, ..., \tag{2}$$

for some constants $a < 1$, b and c.

Let $\{Y_n, n \geq 1\}$ be a sequence of independent identically distributed random variables independent of the random variable N and let

$$S_N(Y) := Y_1 + Y_2 + \cdots + Y_N.$$

The random sum $S(Y; N)$ is said to have a compound distribution. In collective risk theory Y_i corresponds to the amount of a claim, N corresponds to the number of claims in a certain period and a random sum $S_N(Y)$ represents the aggregate claims of portfolio.

Let $\{X_n, n \geq 1\}$ be a sequence of independent identically distributed random variables. Assume that $\{Y_n, n \geq 1\}$ are non-negative integer valued and independent of $\{X_n\}$. We defined doubly compound distribution as a distribution of the following random sum

$$S := S_{S_N(Y)}(X) = X_1 + X_2 + \cdots + X_{S_N(Y)}. \tag{3}$$

This random variable can be interpreted as the total claim of portfolio with the number of claims represented by a random sum $S(Y; N)$. Then the sum $S_N(Y)$ represents the aggregate number of claims and Y_n corresponds to the number of claims in the n-th period.

De Pril [1] derived relations for the moments of compound distribution $S_N(Y) = Y_1 + Y_2 + \cdots + Y_N$ with counting distribution satisfying recurrence relation (1) using moment generating function. Following his approach Murat and Szynal [3] derived relations for moments of compound distribution with counting distribution satisfying R1. In the case R2 such relations were establish by Murat and Szynal [2]. Different approach is presented by Sundt [7]. He derived recurrence relations for the moments of compound distribution with counting distribution satisfying R2 by a simple extension of result deduced by Panjer [4], Sundt and Jewell [6].

The presented contribution deals with recurrence relations for moments of doubly compound distribution $S(X; S_N(Y))$ when the counting distribution of N satisfying recursion R1 and R2. The method of moments generating function will be used to obtain this relations. In Section 2 will be considered the case R1 and in Section 3 the case R2.

Throughout this paper will be used following notations. $M_S(u)$, $M_{S_N(Y)}(u)$ and $M_X(u)$ denote moments generating function of S, $S_N(Y)$ and X, respectively. Moreover, $P_{S_N(Y)}(u)$, $P_N(u)$, $P_Y(u)$ denote probability generating function of $S_N(Y)$, N and Y, respectively. Using this notations we have

$$M_{S-\varrho}(u) = e^{-\varrho u} M_S(u),$$

with

$$M_S(u) = P_{S_N(Y)}(M_X(u)) = P_N(P_Y(M_X(u)) = P_N(M_{S_Y(X)}(u)),$$

where $S_Y(X) = X_1 + X_2 + \cdots + X_Y$ and Y has the same distribution as $\{Y_n\}$. Write $M(u) := M_{S_Y(X)}(u)$. Thus

$$M_{S-\varrho}(u) = e^{-\varrho u} P_N(M(u)).$$

Derivation gives

$$M'_{S-\varrho}(u) = -\varrho M_{S-\varrho}(u) + e^{-\varrho u} P'_N[M(u)]M'(u) \tag{4}$$

2. Relations for Moments of Doubly Compound Distribution with Counting Distribution Satisfying Relation R1

We will give some examples of probability functions satisfying relation R1 with $c \neq 0$ to begin with.

1. For $a = 0$, $b = \theta$ and $c = \lambda - 1$ we have *hyper-Poisson* distribution with probability function

$$p(n) = \frac{\Gamma(\lambda)\theta^n}{\Gamma(\lambda + n)M(1, \lambda, \theta)}, \ \lambda > 0,$$

where $\Gamma(\lambda) = \int\limits_0^\infty t^{\lambda-1}e^{-t}dt$ is a gamma function and $M(1, \lambda, \theta) = \sum\limits_{i=0}^{\infty} \frac{\theta^i}{\lambda_{(i)}}$ with $\lambda_{(i)} = \lambda(\lambda + 1) \ \ldots \ (\lambda + i - 1)$.

2. When $a = 0$, $b = \lambda$ and $c = \nu$ we obtain *displaced Poisson* distribution with probability function

$$p(n) = \frac{e^{-\lambda}\lambda^{n+\nu}}{I(\nu, \lambda)(n + \nu)!}, \ \theta = 0, 1, \ldots .$$

3. For $a = 1$, $b = -\nu - 1$ and $c = \nu + p$ one can get *Waring* distribution with probability function

$$p(n) = \frac{\nu\Gamma(p + n)\Gamma(\nu + p)}{\Gamma(p)\Gamma(\nu + p + n + 1)}, \ \nu, p > 0.$$

4. Putting $p = 1$ we get *Yule* distribution with probability function

$$p(n) = \frac{\nu n!}{(\nu + 1)_{(n+1)}}, \nu > 0.$$

Of course in this case $a = 1$, $b = -\nu - 1$ and $c = \nu + 1$.

5. For $a = \theta$, $b = -\theta$ and $c = 1$ we get *shifted logarithmic* distribution with probability function

$$p(n) = \frac{\theta^{n+1}}{-(n + 1)log(1 - \theta)}, \ 0 < \theta < 1.$$

6. For $a = q$, $b = q(m - 1)$ and $c = -m$ one can obtain *Pascal* distribution with probability function

$$p(n) = \binom{n-1}{m-1} p^m q^{n-m}, \ \ m = 1, 2, \ldots .$$

Let $a \neq 1$. The following Theorem gives recurrence relations for moments of the random sum $S = X_1 + X_2 + \ldots + X_{S_{N(Y)}}$.

Theorem 1. *Moments of S about a point ϱ with counting distribution belonging to class R1 with $a \neq 1$ satisfy the recurrence relation:*

$$(1-a)E[(S-\varrho)^{r+1}]$$

$$= \sum_{i=0}^{r} \binom{r}{i} \left[\left(\frac{r+1}{i+1} a + b + ac \right) E[S_Y(X)]^{j+1} + a\varrho E[S_Y(X)]^i \right] E[(S-\varrho)^{r-i}]$$

$$-\varrho E[(S-\varrho)^r] + p(n_0) \sum_{i=0}^{r} \binom{r}{i} (-\varrho)^{r-i} E\left[S_{S(Y;n_0)}(X))^{i+1} \right] \tag{5}$$

$$+ c \sum_{i=0}^{r} \sum_{j=0}^{i} \binom{r}{i} \binom{i}{j} E[S_Y(X)]^{j+1} G^{(i-j)}(0) \left(p(n_0)(-\varrho)^{r-i} - E[(S-\varrho)^{r-i}] \right)$$

for $r = 0, 1, 2, \ldots,$ where $G(0) = \frac{1}{M(u)}_{|u=0}$, $M(u) := M_{S_Y(X)}(u)$ and $S_{n_0}(Y) = Y_1 + Y_2 + \ldots + Y_{n_0}$.

Proof. Murat and Szynal [3] obtained

$$(1-au)P'_N(u) = n_0 u^{n_0+1} p(n_0) + \left(a + b + ac - \frac{c}{u} \right) P_N(u) + \frac{c}{u} p(n_0) \tag{6}$$

Inserting (6) to (4) we get after derivation

$$[1 - aM(u)]M'_{S-\varrho}(u) = -\varrho M_{S-\varrho}(u)[1 - aM(u)]$$

$$+ M_{S-\varrho}(u)(a+b+ac)M'(u)$$

$$+ c \left(p(n_0)e^{-\varrho u} - M_{S-\varrho}(u) \right) [log M(u)]' + p(n_0)e^{-\varrho u} \left([M(u)]^{n_0} \right)',$$

with $M(u) := M_{S_Y(X)}(u)$. According to Leibnitz formula taking the derivative of order r of both sides gives

$$M^{(r+1)}_{S-\varrho}(u) - a \sum_{i=0}^{r} \binom{r}{i} M^{(i)}(u) M^{(r-i+1)}_{S-\varrho}(u)$$

$$= \sum_{i=0}^{r} \binom{r}{i} \left[(a+b+ac)M^{(i+1)}(u) + a\varrho M^{(i)}(u) \right] M^{(r-i)}_{S-\varrho}(u) - \varrho M^{(r)}_{S-\varrho}(u)$$

$$+ c \sum_{i=1}^{r} \sum_{j=0}^{i} \binom{r}{i} \binom{i}{j} M^{(j+1)}(u) G^{(i-j)}(u) \left[p(n_0)(-\varrho)^{r-i} e^{-\varrho u} - M^{(r-i)}_{S-\varrho}(u) \right]$$

$$+p(n_0)\sum_{i=0}^{r}\binom{r}{i}(-\varrho)^{r-i}e^{-\varrho u}\Big([M(u)]^{n_0}\Big)^{(i+1)}.$$

We can put this, according easy computations, to the following form

$$M_{S-\varrho}^{(r+1)}(u)[1-aM(u)]$$

$$=\sum_{i=0}^{r}\binom{r}{i}\left[\left(\frac{r+1}{i+1}a+b+ac\right)M^{(i+1)}(u)+a\varrho M^{(i)}(u)\right]M_{S-\varrho}^{(r-i)}(u)$$

$$+c\sum_{i=0}^{r}\sum_{j=0}^{i}\binom{r}{i}\binom{i}{j}M^{(j+1)}(u)G^{(i-j)}(u)\left[p(n_0)e^{-\varrho u}(-\varrho)^{r-i}-M_{S-\varrho}^{(r-i)}(u)\right]$$

$$-\varrho M_{S-\varrho}^{(r)}(u)+p(n_0)\sum_{i=0}^{r}\binom{r}{i}(-\varrho)^{r-i}e^{-\varrho u}\Big([M(u)]^{n_0}\Big)^{(i+1)}.$$

Setting $u=0$ we obtain the recursion(5). □

Putting in (5) $\varrho=0$ we get the following recurrence relation for the ordinary moments of doubly compound distribution

Collorary 1. *The ordinary moments of S with counting distribution belonging to class R1 satisfy the recurrence relation:*

$$(1-a)E[S^{r+1}]=\sum_{i=0}^{r}\binom{r}{i}\left[\left(\frac{r+1}{i+1}a+b+ac\right)E[S_Y(X)]^{j+1}\right]E[S^{r-i}]$$

$$-cp(n_0)\sum_{i=0}^{r}\sum_{j=0}^{i}\binom{r}{i}\binom{i}{j}E[S_Y(X)]^{j+1}G^{(i-j)}(0)E[S^{r-i}]$$

$$+cp(n_0)\sum_{i=0}^{r}E[S_Y(X)]^{i+1}G^{(i-j)}(0)+p(n_0)E\Big[S_{S(Y;n_0)}(X))^{i+1}\Big],$$

for $r=0,1,2,\ldots$.

One can see that there are probability functions satisfying relation R1 with $a=1$ and $n_0=0$. Easy manipulations of above formulas give

Collorary 2. *The ordinary moments ES^r of S with counting distribution belonging to class R1 with $a = 1$ and $n_0 = 0$ satisfy the recurrence relation:*

$$(1+b)E[S_Y(X)]ES^r = \sum_{i=1}^{r}\binom{r}{i}\left[c\sum_{j=0}^{i}\binom{i}{j}E[S_Y(X)^{j+1}]G^{(i-j)}(0)\right.$$

$$\left.-\left(\frac{r+1}{i+1}+b+c\right)E[S_Y(X)^{i+1}]\right]ES^{r-i} - cp_0\sum_{j=1}^{r}\binom{r}{j}E[S_Y(X)^{j+1}]G^{(r-j)}(0),$$

for $r = 1, 2, \ldots$.

The central moments $E[(S-ES)^r]$ of S with counting distribution belonging to class R1 with $a = 1$ and $n_0 = 0$ satisfy the recurrence relation:

$$(1+b)E[S_Y(X)]E[(S-ES)^r] = \sum_{i=1}^{r}\binom{r}{i}\left[c\sum_{j=0}^{i}\binom{i}{j}E[S_Y(X)^{j+1}]G^{(i-j)}(0)\right.$$

$$\left.-\left(\frac{r+1}{i+1}+b+c\right)E[S_Y(X)^{i+1}] - ESE[S_Y(X)^{i}]\right]E[(S-ES)^{r-i}]$$

$$-cp_0\sum_{i=1}^{r}\sum_{j=0}^{i}\binom{r}{i}\binom{i}{j}E[S_Y(X)^{j+1}]G^{(i-j)}(0)\,[ES]^{r-i}$$

for $r = 1, 2, \ldots$.

One can check that for N satisfying relation (1) we get

Collorary 3. *Moments of S about a point ϱ with counting distribution (1) satisfy the recurrence relation:*

$$(1-a)E[S-\varrho]^{r+1} = \sum_{j=0}^{r}\binom{r}{j}\left[\left(\frac{r+1}{j+1}a+b\right)E[S_Y(X)]^{j+1}\right.$$

$$\left.+\varrho aE[S_Y(X)]^{j}\right]E[S-\varrho]^{r-j} - \varrho E[S-\varrho]^{r}$$

for $r = 0, 1, 2, \ldots$.

The moments of $S_Y(X)$ when Y satisfies R1 were establish by Murat and Szynal [3] and when Y satisfies R2 were given by Murat and Szynal [2].

3. Relations for Moments of Doubly Compound Distribution with Counting Distribution Satisfying Relation R2

We start with some examples of probability functions satisfying recurrence relation R2.

1. For $a_1 = \frac{1}{1+\alpha}$, $b_1 = \gamma + \frac{\beta-1}{1+\alpha}$, $a_2 = \frac{-\gamma}{1+\alpha}$ and $a_i = 0$, $i = 3, 4, \ldots, k$, $b_i = 0$, $i = 2, 3, \ldots, k$ we get *Delapote* distribution with parameters $\alpha > 0$, $\beta > 0$, $\gamma > 0$ which is defined as the convolution of a negative binomial distribution with parameters $(\beta, \frac{\alpha}{1+\alpha})$ and Poisson distribution with parameter γ considered by Schröter [5].

2. For $a_1 = \frac{-q^2}{1-q}$, $a_2 = \frac{q^2}{1-q}$, $b_1 = q\left(\frac{t+q}{1-q} + \alpha\right)$, $b_2 = \frac{q^2}{1-q}(\alpha - t - 2)$ and $a_i = 0$, $b_i = 0$, $i = 3, 4, \ldots, k$ we obtain the convolution of a binomial distribution with parameters (t, q) and a negative binomial distribution with parameters (α, q) considered by Sundt [7].

The following Theorem gives recurrence relations for moments of the random sum $S_{S_N(Y)}(X) = X_1 + X_2 + ... + X_{S_N(Y)}$.

Theorem 2. *Moments of S about a point ϱ with counting distribution belonging to class R2 satisfy the following recurrence relation:*

$$\left(1 - \sum_{i=1}^{k} a_i\right) E[(S-\varrho)^{r+1}] = \sum_{i=1}^{k}\sum_{j=0}^{r}\binom{r}{j}\Bigg\{\left(\frac{r+1}{j+1}a_i + \frac{b_i}{i}\right) E[S_{S_i(Y)}(X)^{j+1}]$$

$$+\varrho a_i E[S_{S_i(Y)}(X)^j]\Bigg\} E[(S-\varrho)^{r-j}] - \varrho E[(S-\varrho)^r], \qquad (7)$$

where $S_i(Y) = Y_1 + Y_2 + \cdots + Y_i$.

Proof. Using result given by Sundt [7] we have

$$P'_N(u) = \frac{\sum\limits_{i=1}^{k}(ia_i + b_i)u^{i-1}}{1 - \sum\limits_{i=1}^{k} a_i u^i} P_N(u). \qquad (8)$$

After insertion of (8) in (4) and developments we get

$$M'_{S-\varrho}(u) = \sum_{i=1}^{k} a_i M^i(u) M'_{S-\varrho}(u) + \sum_{i=1}^{k}(ia_i + b_i)M^{i-1}(u) M_{S-\varrho}(u) M'(u)$$

$$+\varrho\sum_{i=1}^{k}a_iM^i(u)M_{S-\varrho}(u)-\varrho M_{S-\varrho}(u)$$

$$=\sum_{i=1}^{k}a_iM^i(u)M'_{S-\varrho}(u)+\sum_{i=1}^{k}\left(a_i+\frac{b_i}{i}\right)[M^i(u)]'M_{S-\varrho}(u)$$

$$+\varrho\sum_{i=1}^{k}a_iM^i(u)M_{S-\varrho}(u)-\varrho M_{S-\varrho}(u)$$

with $M(u):=M_{S_Y(X)}(u)$. According to Lebnitz formula taking the derivative of order r of both sides gives

$$M_{S-\varrho}^{(r+1)}(u)=\sum_{i=1}^{k}a_i\sum_{j=0}^{r}\binom{r}{j}[M^i(u)]^{(j)}M_{S-\varrho}^{(r-j+1)}(u)$$

$$+\sum_{i=1}^{k}\left(a_i+\frac{b_i}{i}\right)\sum_{j=0}^{r}\binom{r}{j}[M^i(u)]^{(j+1)}M_{S-\varrho}^{(r-j)}(u)$$

$$+\varrho\sum_{i=1}^{k}a_i\sum_{j=0}^{r}\binom{r}{j}[M^i(u)]^{(j)}M_{S-\varrho}^{(r-j)}(u)-\varrho M_{S-\varrho}^{(r)}(u).$$

Hence we get

$$\left(1-\sum_{i=1}^{k}a_i\right)M_{S-\varrho}^{(r+1)}(u)=\sum_{i=1}^{k}\sum_{j=0}^{r}\binom{r}{j}\left[\left(\frac{r+1}{j+1}a_i+\frac{b_i}{i}\right)[M^i(u)]^{(j+1)}\right.$$

$$\left.+\varrho a_i[M^i(u)]^{(j)}\right]M_{S-\varrho}^{(r-j)}(u)-\varrho M_{S-\varrho}^{(r)}(u),$$

where $M^i(u)=M_{S_i}(u)$. Setting $u=0$ leads to the recursion (7). □

We have the following special case for $\varrho=0$

$$\left(1-\sum_{i=1}^{k}a_i\right)E(S^{r+1})$$

$$=\sum_{i=1}^{k}\sum_{j=0}^{r}\binom{r}{j}\left[\left(\frac{r+1}{j+1}a_i+\frac{b_i}{i}\right)E[S_{S_i(Y)}(X)^{j+1}]\right]E(S^{r-j}).$$

If we put in (7) $k=2$ we obtain the following statement.

Collorary 4. *The ordinary and the central moments of S under R2 with $k = 2$ satisfy the following recurrence relations:*

$$(1-a_1-a_2)E(S^{r+1}) = \sum_{j=0}^{r}\binom{r}{j}\left[\left(\frac{r+1}{j+1}a_1+b_1\right)E[S_i^{j+1}]\right.$$

$$\left.+\left(\frac{r+1}{j+1}a_2+\frac{b_2}{2}\right)\sum_{m=0}^{j+1}\binom{j+1}{m}E[S_i^m]E[S_i^{j+1-m}]\right]E[S^{r-j}];$$

$$(1-a_1-a_2)E[(S-ES)^{r+1}] = \sum_{j=0}^{r}\binom{r}{j}\left\{\left(\frac{r+1}{j+1}a_1+b_1\right)E[S_i^{j+1}]\right.$$

$$\left.+Aa_1E[S_i]E[S_i^j]\right\}E[(S-ES)^{r-j}]$$

$$+\sum_{j=0}^{r}\binom{r}{j}\left\{\left(\frac{r+1}{j+1}a_2+\frac{b_2}{2}\right)\sum_{m=0}^{j+1}\binom{j+1}{m}E[S_i^m]E[S_i^{j+1-m}\right.$$

$$\left.+AE[S_i]\sum_{m=0}^{j}\binom{j}{m}E[S_i^m]E[S_i^{j-m}]\right\}E[(S-ES)^{r-j}]$$

$$-AE[S_i]E[(S-ES)^r], \qquad r = 0,1,2,\dots\ ,$$

respectively, where $A = \frac{a_1+b_1+2a_2+b_2}{1-a_1-a_2}$ *and* $S_i = S_{S_i(Y)}(X)$.

We end this Section with some examples.

1. When N has Delaport distribution from Corollary 4 we have

$$E(S^{r+1}) = \frac{1}{\alpha}\sum_{j=0}^{r}\binom{r}{j}\left[\left(\frac{r+1}{j+1}+\gamma(1+\alpha)+\beta-1\right)E[S_i^{j+1}]\right.$$

$$\left.-\frac{\gamma}{2}\sum_{m=0}^{j+1}\binom{j+1}{m}E[S_i^m]E[S_i^{j+1-m}]\right]E[S^{r-j}];$$

$$E[(S-ES)^{r+1}] = \frac{1}{\alpha}\sum_{j=0}^{r}\binom{r}{j}\left[\left(\frac{r+1}{j+1}+\gamma(1+\alpha)+\beta-1\right)E[S_i^{j+1}]\right.$$

$$\left.-\frac{\gamma}{2}\sum_{m=0}^{j+1}\binom{j+1}{m}E[S_i^m]E[S_i^{j+1-m}]+\frac{\beta+\gamma\alpha}{\alpha}E[S_i^j]\right]E[(S-ES)^{r-j}]$$

$$-\frac{(\beta+\gamma\alpha)(1+\alpha)}{\alpha^2}E[S_i]E[(S-ES)^r].$$

2. When N has distribution which is a convolution of a binomial and a negative binomial distribution from Corollary 4 we obtain

$$E(S^{r+1}) = \frac{q}{1-q}\sum_{j=0}^{r}\binom{r}{j}\left[\left(-q\frac{r+1}{j+1}+t+q+\alpha(1-q)\right)E[S_i^{j+1}]\right.$$
$$\left.+\left(q\frac{r+1}{j+1}+\frac{t+\alpha-2}{2}\right)\sum_{m=0}^{j+1}\binom{j+1}{m}E[S_i^m]E[S_i^{j+1-m}]\right]E[S^{r-j}];$$

$$E[(S-ES)^{r+1}]$$
$$=\frac{q}{1-q}\left\{\sum_{j=0}^{r}\binom{r}{j}\left[\left(-q\frac{r+1}{j+1}+t+q+\alpha(1-q)\right)E[S_i^{j+1}]\right.\right.$$
$$\left.-\frac{q^2(t(1-q)+\alpha)}{1-q}E[S_i]E[S_i^j]\right]E[(S-ES)^{r-j}]$$
$$+q\sum_{j=0}^{r}\binom{r}{j}\left[\left(\frac{r+1}{j+1}+\frac{\alpha-t-2}{2}\right)\sum_{m=0}^{j+1}\binom{j+1}{m}E[S_i^m]E[S_i^{j+1-m}]\right.$$
$$\left.+\frac{q(t(1-q)+\alpha)}{1-q}E[S_i]\sum_{m=0}^{j}\binom{j}{m}E[S_i^m]E[S_i^{j-m}]\right]E[(S-ES)^{r-j}]$$
$$\left.-(t(1-q)+\alpha)E[S_i]E[(S-ES)^r]\right\}.$$

4. Concluding Remarks

The purpose of this paper was to give general relations for moments of any order of doubly compound distributions. From those relations one can get other characteristics of doubly compound distribution such as the coefficient of skewness and kurtosis. Moreover, one can also get well known formulas for moments of compound distributions.

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