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A GENERAL STRUCTURAL PROPERTY IN WAVELET PACKETS FOR DETECTING OSCILLATION AND NOISE COMPONENTS IN SIGNAL ANALYSIS

Paolo Mercorelli
Institute of Product and Process Innovation
Leuphana University of Lueneburg
Volgershall 1, D-21339 Lueneburg, GERMANY

Abstract: The paper presents a general structural mathematical property of the wavelet packets, which can be used in various industrial applications, such as the identification of signals, fault detection, harmonic detection, denoising and more in general oscillation detection. In particular, possible algorithms based on this mathematical property can find application and be implemented in wavelets-based outlier packages. Some of these algorithms are currently being integrated into the Inferential Model Platform of the Advanced Control and Simulation Solution Responsible Unit within ABB's Industry division.

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1. Introduction and Definitions

This paper combines and advances knowledge and methods already present in a technical form in [1] and [2] and derives a general mathematical property of the wavelet packets. Some definitions are given and after that the above mentioned property is mathematically derived. The proposed method find application in

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denoising techniques and oscillation detection. One of the pioneer methods, known as VisuShrink, was proposed by Donoho and Johnstone [3].

Another group of popular wavelet denoising methods is the Bayesian approach, often based on minimising the expected risk, with the expectation taken over a postulated prior distribution supposedly governing the underlying true signal [4], [5] and [6]. A different approach to wavelet denoising is based on the "minimum description length" (MDL) principle, as proposed in [7] and in [8]. This approach is based on comparisons of the "description lengths" of the data. The description length, which is an information theory criterion, is calculated for different subspaces of the basis. The method suggests choosing the noise variance and the subspace for which the description length of the data is minimal. In other words, noise is defined to be the part of the data in which the given model class cannot find any regular features. Ideally, this definition of noise does not include any assumptions of the noise distribution, even though a Gaussian noise model is usually assumed.

1.1. Some Definitions

Definition 1. Given the oscillating part of the sequence $y(t)$ as follows

$$y(t) - \sum_{n=0}^{2^{N_0-d}} s(d,n)\psi(d,n)(t) = \sum_{j=1}^{2^d-1}\sum_{n=0}^{2^{N_0-d}} w(d,j,n)\psi(d,j,n)(t), \tag{1}$$

where $s_{(d,n)} = w_{(d,0,n)}$, the time-frequency map of the peak values in the wavelet domain is a table of real values specified $\forall$ d $\in \mathsf{N}$ and by the triplet $(d,\hat{j},n)$ as

$$w_{p_{(d,\hat{j},n)}} = \max_{j}(\|w_{(d,j,n)}\psi_{(d,j,n)}(t)\|). \tag{2}$$

$\forall\ \hat{j} = 1 : 2^d - 1.$

Remark 1. The definition intends to select the maximum of the absolute value of the peak at every level of the packet tree and at every frequency cell to build a map of the peak values.

Definition 2. Let S be a sequence of length L consisting of N elements, function *sort* is the function that sorts the sequence in an increasing order. If *mean* is the mean value of a sequence and if sequence S consists of an even number of elements: $median = mean(sort(S))$. If sequence S consists of an odd number N of elements, then $sort(S) = S((N+1)/2)$. Then, $sort(S) = S((N+1)/2)$ =central element of the sequence.

Definition 3. Given an observed sequence

$$y(t) = x(t) + e(t).$$

Let $e(t)$ be defined as the incoherent part of the sequence $y(t)$ at every level d of the packet tree as follows:

$$e(t) = \sum_{n=0}^{2^{N_0-d}} W_{(d,\hat{j}^*,n)} \psi_{(d,\hat{j}^*,n)}(t),$$

where $\psi_{(d,\hat{j}^*,n)}(t)$ are the wavelet bases and $W_{(d,\hat{j}^*,n)}$ their coefficients. The selected wavelets are characterised by $(d, \hat{j}^*, n)$ indices such that:

$$\{(d, \hat{j}^*, n)\} = \arg\Big(\min_{\hat{j}} \left(median_n \{ w_{p_{(d,\hat{j},n)}} \}\right) : \\ \{0 < n \leq 2^{N_0-d}, 1 < \hat{j} \leq 2^d - 1, \forall d \in \mathsf{N}\}\Big) \quad (3)$$

where $median_n$ is the median calculated on the elements localised with index n (time localised). The coefficients $w_{p_{(d,\hat{j},n)}}$ are the wavelet table coefficients of the sequence $y(t)$ as given in Def. 1.

2. A General Mathematical Property in Wavelet Packets

It is easy to see how the intention of Def. 3 is to sort the basis, which can illuminate the difference between the coherent and incoherent part of the sequence, where *incoherent* is the part of the signal that has either no information or *contradictory* information. In fact, the procedure looks for the subspace characterised either by small components or by opposite components in the wavelet domain.

Proposition 1. *Let $y(t)$ be a sequence, and let $w_{p_{(d,j,n)}}$ be its corresponding peak values wavelet sequence on different levels of the tree. Because the wavelet functions are organised in packets, at every level d and at every frequency cell $\hat{j}$, the following relationship holds:*

$$\|w_{p_{(d,\hat{j},n)}}\| \leq \|w_{p_{(d+1,2\hat{j},n)}}\| + \|w_{p_{(d+1,2\hat{j}+1,n)}}\|. \quad (4)$$

Then,

$$\|median_n(w_{p_{(d,\hat{j},n)}})\| \leq \|median_n(w_{p_{(d+m,2\hat{j},n)}})\| + \|median_n(w_{p_{(d+m,2\hat{j}+1,n)}})\|. \quad (5)$$

$\forall\ m \geq 1.$

Proof. By observing that, from the orthogonality,

$$\sum_{(d,j,n)} w_{(d,j,n)}\psi_{(d,j,n)} = \sum_{(d,j,n)} w_{(d+1,2j,n)}\psi_{(d+1,2j,n)} + \sum_{(d,j,n)} w_{(d+1,2j+1,n)}\psi_{(d+1,2j+1,n)}. \quad (6)$$

Functions $\psi_{(d,j,n)}$ are organised in packets, and they are scaled functions, $\forall\ d$, j and $\forall\ m \geq 1$,

$$\|\max\big(\psi_{(d,j,n)}\big)\| \leq \|max\big(\psi_{(d+m,2j,n)}\big)\| + \|max\big(\psi_{(d+m,2j+1,n)}\big)\|. \quad (7)$$

Then, from (7) it follows:

$$\|\max_j\big(w_{(d,j,n)}\psi_{(d,j,n)}\big)\| \leq \|\max_j\big(w_{(d+1,2j,n)}\psi_{(d+1,2j,n)}\big)\|+ \|\max_j\big(w_{(d+1,2j+1,n)}\psi_{(d+1,2j+1,n)}\big)\|. \quad (8)$$

From Def. 1, $\forall\ d$ and $\forall\ n \geq 1$, it follows:

$$\|w_{p_{(d,\hat{j},n)}}\| \leq \|w_{p_{(d+1,2\hat{j},n)}}\| + \|w_{p_{(d+1,2\hat{j}+1,n)}}\|. \quad (9)$$

Considering that the "median" is a monotonic function:

$$\|median_n\big(w_{p_{(w,\hat{j},n)}}\big)\| \leq \|median_n\big(w_{p_{(d+1,2\hat{j},n)}}\big)\| + \|median_n\big(w_{p_{(d+1,2\hat{j}+1,n)}}\big)\|. \quad (10)$$

by proceeding $\forall\ m \geq 1$, the proposition is proven. □

Remark 2. In other words, the proposition says that if the minimum of the median at level d, according to Def. 1, is not more than the minimum of the median at level $d+1$ (one more in depth), then it is also not more than the minimum of the median at level $d+m$ (all the deeper levels), with integers d and m.

3. Conclusions

A general structural mathematical property of the wavelet packets, which can be applied to various industrial applications is presented in this paper. Some algorithms based on such kind of wavelet packets property are implemented in a wavelets-based outlier package and used within ABB's Industry division.

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