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STABILITY OF PREY-PREDATOR MODEL WITH HARVESTING ACTIVITY OF PREY

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Abstract: In this paper, we have proposed a prey-predator model with harvesting activity of prey proportional to their population size and studied the stability of the model using Holling's type II functional response and numerical response. The conditions for the equilibrium to be locally asymptotically stable have been obtained. The simulation has been carried out and it has been observed that when the harvesting activity of prey is taken into consideration then population size of predator decreases and the naturally stable equilibrium of model becomes unstable.

AMS Subject Classification: 97M60
Key Words: Holling's type II functional response and numerical response, Harvesting activity

1. Introduction

The Prey-Predator model is a topic of great interest for many ecologists and mathematicians. M. Danca et.al. [1] studied analytically and numerically a

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model of competition between populations of two species described by a two dimensional map and a rich dynamics has been analysed. J. Dhar [2] studied a prey-predator dynamics, where the predator species partially depends upon the prey species in a two patch habitat and obtained the conditions for asymptotic stability. In another piece of work J. Dhar [3] proposed a mathematical model to study the role of supplementary self-renewable resource on population in a two-patch habitat. The linear and non-linear asymptotic stability conditions of the model in a homogeneous habitat have been obtained. B. Dubey [4] proposed a prey-predator model and observed that the reserve zone has a stabilizing effect on prey-predator interactions. J.M. Jeschike et. al. [6] presented functional response model that incorporates handling and digesting prey. They found that maximum predation rate is determined not by the sum of time spent for handling and digesting prey but solely by larger of these two terms. N.P. Kumar et. al. [7] studied a mathematical model of commensalism between two species with limited resources. All the four equilibrium points of the model are identified and a criterion for stability has been discussed. K.L. Narayan et. al. [8] studied a prey-predator model in which the predator is provided with an alternative food in addition to the prey and both the prey and the predator harvested proportional to their population sizes. All the equilibrium points of the model were indentified and stability criterion has been discussed. B.R. Reddy et. al. [9] studied a model of two mutually interacting species with limited resources for first species and unlimited resources for second species. They have indentified two equilibrium points and described their stability criteria. In the present work we have proposed a prey-predator model with the harvesting activity of the prey proportional to their population size. The equilibrium of the model has been determined and the model has been subjected to Holling's type II functional response and numerical response. The conditions for equilibrium to be locally asymptotically stable has been obtained.

2. The Prey-Predator Model

Consider a habitat where prey and predator species are living together. Let u(t) is populations of the prey with natural growth rate 'a' and v(t) is the population of the predator with natural growth rate 'b'. Let λ is the rate of search for an individual prey by single predator, m is per capita predator mortality rate, α is the harvesting activity constant and h is handling time of single prey by a predator.

The proposed prey-predator model is

$$\begin{cases} \dfrac{du}{dt}=au-\dfrac{\lambda uv}{1+h\lambda u}-\alpha u \\ \dfrac{dv}{dt}=\dfrac{b\lambda uv}{1+h\lambda u}-mv \end{cases} \tag{1}$$

The equilibrium of model (1) has been given below:

$$u^{*}=\frac{m}{(b-mh)\lambda}$$

and

$$v^{*}=\frac{b(a-\alpha)}{(b-mh)\lambda},$$

where u^{*} and v^{*} are the populations of prey and predator respectively at equilibrium. It has been observed that the population of prey at equilibrium decreases with the increase of the growth rate of predator 'b' due to successful attack on prey. Interestingly, we found that the population of prey at equilibrium does not depend on prey growth rate 'a', whereas the population of predator at equilibrium depends on prey growth rate 'a'. The population of prey and predator at equilibrium depends on handling time 'h' of single prey by a predator. Moreover the population of predator at equilibrium decreases with increase of harvesting activity 'α' of prey.

3. Holling's Functional Response and Numerical Response

Holling's type II functional response [5] is given by

$$f(u)=\frac{\lambda u}{1+h\lambda u} \tag{2}$$

Numerical response [5] is given by

$$g(u)=\frac{b\lambda u}{1+h\lambda u} \tag{3}$$

The general representation of proposed prey-predator model (1) is

$$\begin{cases} \frac{du}{dt}=au-f(u)v-\alpha u \\ \frac{dv}{dt}=g(u)v-mv \end{cases} \tag{4}$$

The Jacobian matrix of model (4) at the equilibrium $(u^{\star}, v^{\star})$ is given by

$$\begin{pmatrix} a-\frac{\partial f(u^*)}{\partial u}v^{\star}-\alpha & -f\left(u^{\star}\right)-\frac{\partial f(u^*)}{\partial v}v^{\star} \\ \frac{\partial g(u^*)}{\partial u}v^{\star} & \frac{g(u^*)}{\partial v}v^{\star}+g\left(u^{\star}\right)-m \end{pmatrix}$$

The equilibrium is locally asymptotically stable if the sum of the two diagonal element of Jacobian matrix is negative and the determinant is positive. This leads to the following general condition.

$$a-\frac{\partial f\left(u^{\star}\right)}{\partial u}v^{\star}-\alpha+\frac{\partial g\left(u^{\star}\right)}{\partial v}v^{\star}+g(u^{\star})-m < 0, \tag{5}$$

$$\left(a-\frac{\partial f\left(u^{\star}\right)}{\partial u}v^{\star}-\alpha\right)\left(\frac{\partial g\left(u^{\star}\right)}{\partial v}v^{\star}+g(u^{\star})-m\right)+\left(f\left(u^{\star}\right)+\frac{\partial f\left(u^{\star}\right)}{\partial v}v^{\star}\right)\frac{\partial g(u^{\star})}{\partial u}v^{\star} > 0. \tag{6}$$

Since functional response and numerical response are independent of predator density, therefore $\frac{\partial f}{\partial v}= 0$ and $\frac{\partial g}{\partial v}= 0$ and also at equilibrium $g\left(u^{\star}\right) = m$.

Using the population predator equilibrium $v^{\star}=\frac{(a-\alpha)u^*}{f(u^*)}$ in (5) and (6), we get

$$\frac{df(u^{\star})}{du} > \frac{f(u^{\star})}{u^{\star}} \quad \text{and} \quad \frac{dg(u^{\star})}{du} > 0. \tag{7}$$

Equation (7) gives the general conditions for the equilibrium to be locally asymptotically stable. For the equilibrium to be locally asymptotically stable (i) the rate of change of functional response with respect to prey population should be greater than the ratio of functional response and population of prey at equilibrium and (ii) the rate change of numerical response with respect to prey population is always positive.

4. Simulation

In first stage simulation of model (1) has been carried out by taking values of parameters $a = 1$, $\lambda = 1, b = 0.2$, $m = 1, h = 0.02$. It has been observed that populations of predator as well as prey increases progressively with the passage of time in both cases when harvesting activity was not considered (i.e. $\alpha = 0$) (fig. 4.1a) and when the harvesting activity of prey was taken into consideration (i.e. $\alpha = 0.8$) (fig. 4.2a). One can clearly note that the time interval between two consecutive maxima increased for both prey and predator populations and less number of maxima appeared in the same time intervals in the case when harvesting activity was considered (fig. 4.2a) as compared to the case when harvesting activity was not considered (fig. 4.1a). Further, it has been observed that the population size of predator decreased when harvesting activity was considered (fig. 4.2b) as compared to the case when harvesting activity was not considered (fig. 4.1b).

In second stage simulation of model (1) has been carried out by keeping the parameters $\mathrm{a} = 1,\ \lambda = 1, \mathrm{b} = 0.2,\ \mathrm{m} = 1$ constant and changing the handling time 'h' from 0.02 to 0.04. It has been observed that the populations of predator as well as prey increases progressively with the passage of time and this increase is very high in both the cases when harvesting activity was not considered (i.e $\alpha = 0$) (fig. 4.3a) and when the harvesting activity of prey was taken into consideration (i.e. $\alpha = 0.8$) (fig. 4.4a) in second stage as compared to increase observed in first stage. In second stage also, it has been observed that the population size of predator decreased when harvesting activity was considered (fig. 4.4b) as compared to the case when harvesting activity was not considered (fig. 4.3b).

By critical analysis of the both the stages it has been observed that when harvesting activity is not considered (i.e. $\alpha = 0$) then the equilibrium of prey-predator model (1) is stable and when the harvesting activity of prey is taken into consideration (i.e $\alpha = 0.8$) then the population size of predator decreased and the naturally stable equilibrium of model (1) becomes unstable.

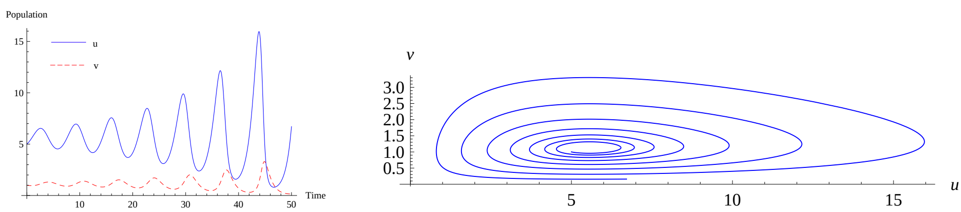

Figure 4.1: $\mathrm{a} = 1,\ \lambda = 1, \mathrm{b} = 0.2,\ \mathrm{m} = 1, \mathrm{h} = 0.02, \alpha = 0$

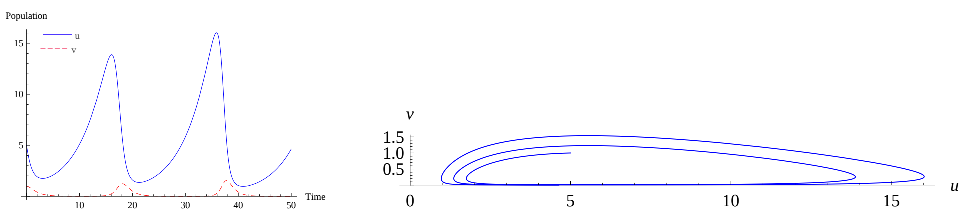

Figure 4.2: $\mathrm{a} = 1,\ \lambda = 1, \mathrm{b} = 0.2,\ \mathrm{m} = 1, \mathrm{h} = 0.02, \alpha = 0.8$

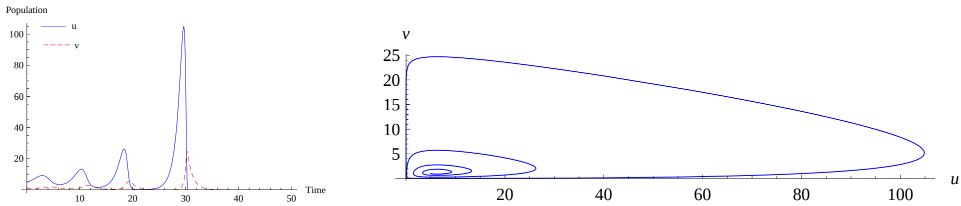

Figure 4.3: a $=1$, $\lambda=1$, b $=0.2$, m $=1$, h $=0.04$, $\alpha=0$

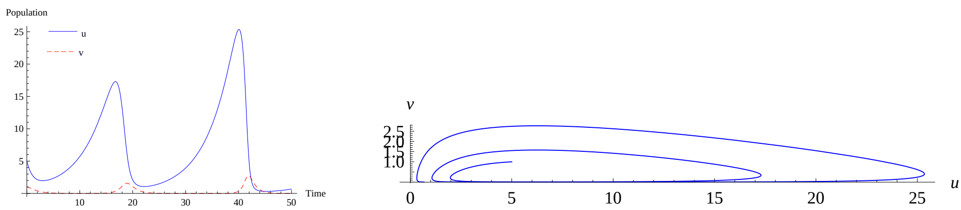

Figure 4.4: a $=1$, $\lambda=1$, b $=0.2$, m $=1$, h $=0.04$, $\alpha=0.8$

5. Conclusions

The stability of model has been studied with Holling's type II functional response and numerical response. For the equilibrium of model (1) to be locally asymptotically stable the conditions obtained are:

(i) the rate of change of functional response with respect to prey population should be greater than the ratio of functional response and prey equilibrium and

(ii) the rate change of numerical response with respect to prey population is always positive.

It has also been observed that when harvesting activity is not considered then the equilibrium of prey-predator model (1) is stable and when the harvesting activity of prey is taken into consideration then the population size of predator decreased and the naturally stable equilibrium of model (1) become unstable.

References

[1] M. Danca, S. Codreanu, B. BakzO, Detailed analysis of a nonlinear prey-predator model, *Journal of Biological Physics*, **23** (1997), 11-20.

[2] J. Dhar, A prey-predator model with diffusion and a supplementary resource for the prey in a two-patch environment, *Mathematical Modeling and Analysis*, **9** (2004), 9-24.

[3] J. Dhar, Population model with diffusion and supplementary forest resource in a two-patch habitat, *Applied Mathematical Modelling*, **32** (2008), 1219-1235.

[4] B. Dubey, A prey-predator model with a reserved area, *Nonlinear Analysis: Modelling and Control*, **12** (2007), 479-494.

[5] C.S. Holling, The functional response of invertebrate predators to prey density, *Memoris of the Entomological Society of Canada*, **48** (1966), 5-88.

[6] J.M. Jeschke, M. Kopp, R. Tollrian, Predator functional responses: Discriminating between handling and digesting prey, *Ecological Monographs*, **72**, No. 1 (2002), 95-112.

[7] N.P. Kumar, B.R. Reddy, Ramacharyulu, A two species commensalism model with limited recourses – A numerical approach, *ARPN Journal of Engineering and Applied Sciences*, **6**, No. 2 (2011).

[8] K.L. Narayan, N.C.P. Ramacharyulu, A prey-predator model with an alternative food for the predator, harvesting of both the species and with a gestation period for interaction, *Int. J. Opne Problems Compt. Math.*, **1**, No. 1 (2008).

[9] B.R. Reddy, N.P. Kumar, Pattabhiramacharyulu, A model mutually interacting species with limited resources of first species and unlimited for second species, *ARPN Journal of Engineering and Applied Sciences*, **6**, No. 1 (2011).