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TWO-TEMPERATURE MAGNETO-THERMO-ELASTICITY RESPONSE IN A PERFECTLY CONDUCTING MEDIUM BASED ON GN III MODEL

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Abstract: This paper deals with magneto-thermo-elastic interaction due to thermal shock on the stress free boundary of the half-space of perfect conducting medium in the context of two-temperature generalized thermo-elasticity with energy dissipation (TEWED) (GN III model), see [1, 2, 3]. The governing equations are solved in the Laplace transformation domain using state-space approach. The inversion of the Laplace transformation is computed numerically using a method based on Fourier series expansion technique. Finally, the results obtained are presented graphically for the problem. Some comparisons have been shown in figures to estimate the effects of the two-temperature parameter, the damping coefficient and the applied magnetic field.

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Nomenclature

$\lambda,\ \mu$ — Lamè constants
t — time
ρ — density
C_E — specific heat at constant strain
$\mathbf{H}$ — magnetic field intensity vector
$\mathbf{E}$ — electric field intensity vector
H_0 — constant component of magnetic field
$\mathbf{J}$ — conduction current density vector
T — thermodynamic temperature
ϕ — conductive temperature
T_0 — reference temperature
α_T — coefficient of linear thermal expansion
σ_{ij} — component of stress tensor
e_{ij} — component of strain tensor
u_i — component of displacement vector
e — dilation
K — thermal conductivity
K^* — material constant
κ_0 — thermal diffusivity
μ_0 — magnetic permeability
$\delta_0 = \dfrac{\gamma T_0}{\rho C_0^2}$ — dimensionless constant for adjustment for the reference temperature
β_0 — the dimensionless temperature discrepancy
$\varepsilon = \dfrac{\delta_0 \gamma}{\rho C_E}$ — thermal coupling parameter
δ_{ij} — Kronecker's delta
$\gamma = (3\lambda + 2\mu)\alpha_T$
C_T — non-dimensional finite thermal wave speed
$c_1^2 = \dfrac{\lambda + 2\mu}{\rho}$ — speed of propagation of isothermal elastic waves
$c_3^2 = \dfrac{K^*}{\rho C_E}$ — finite thermal wave speed
$\alpha_0^2 = \dfrac{\mu_0 H_0^2}{\rho}$ — Alf'ven velocity

1. Introduction

The classical uncoupled theory of thermo-elasticity predicts two phenomena not compatible with physical observations. First the equation of heat conduction of this theory does not contain any elastic terms. Second, the heat equation is of a parabolic type, predicting infinite speeds of propagation for heat waves.

Biot [4] introduced the theory of coupled thermo-elasticity to overcome the first shortcoming. The governing equations for this theory are coupled, which eliminate the first paradox of the classical theory. However, both theories share second shortcoming since the heat equation for the coupled theory is also parabolic.

Two generalizations of the coupled theory were introduced. The extended thermo-elasticity theory proposed by Lord and Shulman [5] incorporates a flux-rate term into Fourier's law of heat conduction and formulates a generalized form that involves a hyperbolic type heat transport equation admitting finite speed of thermal signal.

Green and Lindsay [6] developed a temperature-rate dependent thermo-elasticity (TRDTE) theory by introducing relaxation time factors that do not violate the classical Fourier law of heat conduction, and this theory also predicts a finite speed for heat propagation. Because of the experimental evidence in support of the finiteness of the speed of propagation of a heat wave, generalized thermo-elasticity theories are more realistic than conventional thermo-elasticity theories in dealing with practical problems involving very short time intervals and high heat fluxes like those occurring in laser units, energy channels, nuclear reactors, etc.

The phenomenon of coupling between the thermo-mechanical behavior of materials and the electro-magnetic behavior of materials have been studied since the 19th century. By the middle of the 20th century, piezoelectric materials were finding their first applications in hydrophones. In the last two decades, the concept of electro-magnetic composite materials has arisen. Such composites can exhibit field coupling that is not present in any of the monolithic constituent materials. These so called "Smart" materials and composites have applications in ultrasonic imaging devices, sensors, actuators, transducers, and many other emerging components. Magneto-electro-elastic materials are used in various applications. Due to the ability of converting energy from one kind to another (among mechanical, electric, and magnetic energies), these materials have been used in high-tech areas such as lasers, supersonic devices, microwave, infrared applications, etc. Furthermore, magneto-electro-elastic materials exhibit coupling behavior among mechanical, electric, and magnetic fields and are inherently anisotropic. Problems related to the wave propagation in thermoelastic or magneto-thermo-elastic solids using these generalized theories have been studied by several authors. Among them, Paria [7] has presented some ideas about magneto-thermo-elastic plane waves. Neyfeh and Nemat-Nasser [8, 9] have studied thermo-elastic waves and electro-magneto-elastic waves in solids with a thermal relaxation time. Roychoudhuri and Chatterjee (Roy) [10] have introduced a coupled magneto-thermo-elastic problem in a perfectly conducting elastic half-space with thermal relaxation. Hsieh [11] has considered modeling of new electromagnetic materials. Ezzat [12] has studied the state space approaches to generalized magneto-thermo-elasticity with two relaxation times in a perfectly conducting medium. Ezzat et al. [13] have studied electro-magneto-thermo-elastic plane waves, with thermal relaxation in a medium of perfect conductivity. Problems related to magneto-thermo-elasticity with thermal relaxation have been investigated by Sherief and Youssef [14] and by Baksi and Bera [15].

Green and Naghdi [16] developed three models for generalized thermoelasticity of homogeneous isotropic materials which are labelled as models I, II, and III. The nature of these theories is such that when the respective theories are linearized, Model I reduces to the classical heat conduction theory (based on Fourier's law). The linearized versions of models II and III permit propagation of thermal waves at a finite speed. Model II, in particular, exhibits a feature that is not present in other established thermoelastic models as it does not sustain dissipation of thermal energy (Green and Naghdi [17]). In this model the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. Now the GN II model is employed to study the propagation of magneto-thermoelastic waves which do not undergo both attenuation and dispersion and which has been investigated by Roychoudhuri [18]). Green-Naghdi's third model admits the dissipation of energy. In this model the constitutive equations are derived by starting with the reduced energy equation, where the thermal displacement gradient in addition to the temperature gradient, are among the constitutive variables. Green and Naghdi [19] included the derivation of a complete set of governing equations of a linearized version of the theory for homogeneous and isotropic materials in terms of the displacement and temperature fields and a proof of the uniqueness of the solution for the corresponding initial boundary value problem. In the context of a linearized version of this theory (Green and Naghdi [17, 19]), a theorem on uniqueness of solutions has been established by Chandrasekhariah [20, 21]. Chandrashekhariah [22] have studied one-dimensional thermal wave propagation in a half-space based on the GN model due to the sudden exposure of the temperature to the boundary using the Laplace transform method. Chandrasekhariah and Srinath [23] have studied thermoelastic interactions caused by a continuous heat source in a homogeneous isotropic unbounded thermoelastic body by employing the linear theory of thermo-elasticity without energy dissipation (TEWOED).

Thermo-elastic interactions with energy dissipation in an infinite solid with distributed periodically varying heat sources have been studied by Banik et al. [24] and for functionally graded material without energy dissipation, have been studied by Mallik and Kanoria [25]. Das and Kanoria [26] have studied magneto -thermo-elastic interaction in a functionally graded isotropic unbounded medium due to the presence of periodically varying heat sources. Kar and Kanoria [27, 28] have analyzed thermoelastic interactions with energy dissipation in a transversely isotropic thin circular disc and in an unbounded body with a spherical hole. Mallik and Kanoria [29] also solved a two dimensional problem for a transversly isotropic generalized thick plate with spatially varying heat source. Das and Kanoria [30] described magneto-thermo-elastic wave propagation in an unbounded perfectly conducting elastic solid with energy dissipation. Generalized thermo-elastic problem of a spherical shell under thermal shock has been solved by Kar and Kanoria [31].

The two temperature theory (2TT) of thermo-elasticity proposes that heat conduction in deformable media depends upon two distinct temperature, the conducting temperature Φ and the thermodynamic temperature Θ [32-34]. While under certain conditions these two temperature can be equal, in time-independent problems, however, in particular those involving wave propagation, Φ and Θ are generally distinct [34]. The key element that sets the 2TT apart from the classical theory of thermo-elasticity (CTE) is the (theory-specific) material parameter $a(\geq 0)$. Specifically if $a = 0$, then $\Phi = \Theta$ and the field equations of the 2TT reduces to these of CTE.

The two temperatures Θ and Φ and the strain are found to have representations in the form of a travelling wave plus a response, which occur instantaneously throughout the body (Boley and Tolins [35]). Youssef [1] studied the theory of two-temperature generalized thermo-elasticity. M. A. Ezzat and A. A. Bary [36] have solved one-dimensional problems for any set of boundary conditions using state-space approach of two temperature magneto-thermo-elasticity with thermal relaxation in a medium of prefect conductivity. The propagation of harmonic plane waves in media described by the two-temperature theory of thermo-elasticity (2TT) is investigated by Puri and Jordan [37].

Youssef and Al-Harby [38] studied state-space approach of two-temperature generalized thermo-elasticity of infinite body with a spherical cavity subjected to different types of thermal loading. The most important theoretical contributions to the subject are the proof of uniqueness theorems under different conditions by Ignaczak [39, 40] and by Sherief [41]. The state-space formulation for problems not containing heat sources was done by Anwar and Sherief [42] and the boundary element formulation was done by Anwar and Sherief [43]. Some concrete problems have also been solved. The fundamental solutions for the spherically symmetrical spaces were obtained by Sherief [44]. Sherief and Anwar [45, 46] have solved some two-dimensional problems, while Sherief and Hazma [47] have solved some two-dimensional problems and studied the wave propagation in this theory. El-Maghraby and Youssef [48] used the state-space approach to solve a thermo-mechanical shock problem. Youssef [49] constructed a model of dependence of the modulus of elasticity and the thermal conductivity on the reference temperature and solved a problem of an infinite material with spherical cavity. Kumar et al. [2] studied the variational and reciprocal principles in two-temperature generalized thermo-elasticity. Effects of thermal relaxation time on a plane wave propagation under two-temperature thermo-elasticity was investigated by Kumar and Mukhopadhyay [3].

Our main object in writing this paper is to present magneto-thermo-elastic interaction due to the thermal shock on a stress free boundary of a half-space in the context of two-temperature generalized thermo-elasticity with energy dissipation and without energy dissipation. The governing equations of the problem are solved in Laplace transform domain by using state-space approach. The inversion of the Laplace transform is computed numerically by using a method on Fourier expansion technique [50]. The effect of the applied magnetic field, the damping coefficient and the two-temperature parameter on the physical quantities are also studied.

2. Basic Equations

For perfectly conducting medium the constitutive equations are

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda\Delta - \gamma(T - T_0)]\delta_{ij}\ , \tag{2.1}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),\ \ \Delta = e_{ii}\ . \tag{2.2}$$

Stress equations of motion in the presence of body forces F_i are

$$\sigma_{ij,j} + F_i = \rho\ddot{u}_i\ . \tag{2.3}$$

The heat equation corresponding to generalized thermo-elasticity with energy dissipation in the absence of heat source is

$$\rho C_E \ddot{T} + \gamma T_0 \ddot{\Delta} = K\nabla^2 \dot{\phi} + K^* \nabla^2 \phi \, . \tag{2.4}$$

The relation between conductive temperature and thermodynamic temperature is,

$$\phi - T = a\phi_{,ii} \, , \tag{2.5}$$

where $a > 0$, is the temperature discrepancy.

3. Formulation of the Problem

We now consider a magneto- thermo-elastic medium of perfect conductivity in an initial magnetic field **H**. This produces an induced magnetic field **h** and induced electric field **E**, which satisfy the linearized equations of electromagnetism in a slowly moving continuum as

$$\text{curl}\, \mathbf{h} = \mathbf{J} \, , \tag{3.1}$$

$$\text{curl}\, \mathbf{E} = -\mu_{\mathbf{0}} \frac{\partial \mathbf{h}}{\partial \mathbf{t}} \, , \tag{3.2}$$

$$\mathbf{E} = -\mu_{\mathbf{0}} (\frac{\partial \mathbf{u}}{\partial \mathbf{t}} \times \mathbf{H}) \, , \tag{3.3}$$

$$\text{div}\, \mathbf{h} = \mathbf{0} \, . \tag{3.4}$$

Now, we shall consider a homogeneous isotropic thermo-elastic perfectly conducting solid occupying the half-space $x \geq 0$, which is initially quiescent and where all the state functions depend only on x and the time t.

The displacement vector has components

$$u_x = u(x,t), \qquad u_y = u_z = 0. \tag{3.5}$$

In the context of the linear theory of two-temperature generalized thermo-elasticity based on the Green-Naghdi model III, the equation of motion, heat equation and the constitutive equations can be written as

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} + F_x = \rho \frac{\partial^2 u}{\partial t^2} \, , \tag{3.6}$$

where

$$\mathbf{F} = \mu_0 (\mathbf{J} \times \mathbf{H}), \quad \mathbf{F} = (F_x, F_y, F_z) \, ,$$

$$\rho C_E \frac{\partial^2 T}{\partial t^2} + \gamma T_0 \frac{\partial^3 u}{\partial t^2 \partial x} = K \frac{\partial^2 \dot{\phi}}{\partial x^2} + K^* \frac{\partial^2 \phi}{\partial x^2} \ , \tag{3.7}$$

$$\sigma = \sigma_{xx} = (\lambda + 2\mu)e - \gamma(T - T_0) \ , \tag{3.8}$$

where

$$e = e_{xx} = \frac{\partial u}{\partial x} \ . \tag{3.9}$$

A constant magnetic field with components $(0, H_0, 0)$ is permeating the medium. The current density vector $\mathbf{J}$ will have one component in z-direction and the induced magnetic field $\mathbf{h}$ will have one component in the y-direction, while the induced electric field $\mathbf{E}$ will have one component in z-direction. i.e.

$$\mathbf{J} = (0, 0, J) \ ,$$
$$\mathbf{h} = (0, h, 0) \ ,$$
$$\mathbf{E} = (0, 0, E) \ .$$

Now equations (3.1)-(3.3) yield

$$J = \frac{\partial h}{\partial x} \ , \tag{3.10}$$

$$h = -H_0 e \ , \tag{3.11}$$

$$E = -\mu_0 H_0 \frac{\partial u}{\partial t} \ . \tag{3.12}$$

From equations (3.1) and (3.11)

$$(\mathbf{J} \times \mathbf{H})_x = H_0^2 \frac{\partial^2 u}{\partial x^2} \ . \tag{3.13}$$

Now equations (3.6) reduces to

$$[(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x}] + \mu_0 {H_0}^2 \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} \ . \tag{3.14}$$

Equation (3.14) can also be written as

$$c_1^2 (1 + R_H) \frac{\partial^2 u}{\partial x^2} - \frac{\gamma}{\rho} \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2} \ , \tag{3.15}$$

where $R_H = \frac{\mu_0 H_0^2}{\rho c_1^2} = \frac{\alpha_0^2}{c_1^2}$, $c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ and $\alpha_0 = \sqrt{\frac{\mu_0}{\rho}} H_0$ is the Alf'ven wave velocity of the medium. The coefficient R_H represents the effect of external magnetic field in the thermo-elastic process proceeding in the body.

Let us introduce the following dimensionless quantities:

$$x' = \frac{x}{l}, \ \ u' = \frac{\lambda + 2\mu}{\gamma T_0 l} u, \ \ t' = \frac{c_1 t}{l}, \ \ \theta' = \frac{T - T_0}{T_0} \ \ \phi' = \frac{\phi - \phi_0}{\phi_0},$$

$$\sigma' = \frac{\sigma}{\gamma T_0}, \ \ e' = e, \ \ R_M^2 = 1 + R_H, \ \ h' = \frac{h}{H_0}, \ \ E' = \frac{E}{\mu_0 H_0 c_1}.$$

where l =some standard length. Now omitting primes, equations (3.15), (3.7), (3.8), (3.9), (3.11), (3.12) and (2.5) can be re-written in dimensionless form as

$$R_M^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial \theta}{\partial x} = \frac{\partial^2 u}{\partial t^2} \ , \tag{3.16}$$

$$C_T^2 \frac{\partial^2 \phi}{\partial x^2} + \kappa_0 \frac{\partial^3 \phi}{\partial x^2 \partial t} = \frac{\partial^2 \theta}{\partial t^2} + \varepsilon \frac{\partial^3 u}{\partial x \partial t^2} \ , \tag{3.17}$$

$$\sigma = \frac{\partial u}{\partial x} - \theta \ , \tag{3.18}$$

$$e = \beta_1 \frac{\partial u}{\partial x} \ , \tag{3.19}$$

$$h = -e \ , \tag{3.20}$$

$$E = -\delta_0 \frac{\partial u}{\partial t} \ , \tag{3.21}$$

$$\phi - \theta = \beta_0 \frac{\partial^2 \phi}{\partial x^2} \ , \tag{3.22}$$

where $\varepsilon = \dfrac{\gamma^2 T_0}{(\lambda + 2\mu)\rho C_E}$, $C_T^2 = \dfrac{K^*}{\rho C_E c_1^2} = \dfrac{c_3^2}{c_1^2}$, $\kappa_0 = \dfrac{K}{\rho C_E c_1 l}$, $\beta_1 = \dfrac{\gamma T_0}{\lambda + 2\mu}$, $\beta_0 = \dfrac{a}{l^2}$.

Now the problem is to solve the equations (3.16) to (3.22) subject to the boundary conditions:

(i) Thermal Loading: A thermal shock is applied to the boundary plane $x = 0$ in the form

$$\phi(0, t) = \phi_0 U(t) \ , \tag{3.23}$$

where ϕ_0 is a constant and $U(t)$ is the Heaviside unit step function.

(ii) Mechanical Loading: The bounding plane $x = 0$ is taken to be traction-free, i.e.

$$\sigma(0, t) + T_{11}(0, t) - T_{11}^0(0, t) = 0, \tag{3.24}$$

where T_{11} and T_{11}^0 is the Maxwell stress tensor in a elastic medium and in a vacuum respectively.

Since the transverse components of the vectors **E** and **h** are continuous across the bounding plane, i.e. $E(0,t) = E^0(0,t)$ and $h(0,t) = h^0(0,t)$, $t > 0$, where E^0 and h^0 are the components of the induced electric and magnetic field in free space and the relative permeability is very nearly unity, it follows that $T_{11}(0,t) = T_{11}^0(0,t)$ and equation (3.24) reduces to :

$$\sigma(0,t) = 0 \ . \tag{3.25}$$

The initial and regularity conditions can be written as

$$\begin{gathered} u = \theta = \phi = 0 \ \text{ at } \ t = 0, \\ \frac{\partial u}{\partial t} = \frac{\partial \theta}{\partial t} = \frac{\partial \phi}{\partial t} = 0 \ \text{ at } \ t = 0 \ , \end{gathered} \tag{3.26}$$

and

$$u \to 0, \ \ \theta \to 0, \ \ \phi \to 0 \ \text{ as } \ x \to \infty \tag{3.27}$$

4. Method of Solution

Taking the Laplace transforms defined by the relation

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)\, dt = L\{f(t)\},$$

of both sides of equations (3.16)-(3.22), we obtain:

$$R_M^2 \frac{\partial^2 \bar{u}}{\partial x^2} - \frac{\partial \bar{\theta}}{\partial x} = s^2 \bar{u} \ , \tag{4.1}$$

$$C_T^2 \frac{\partial^2 \bar{\phi}}{\partial x^2} + \kappa_0 s \frac{\partial^2 \bar{\phi}}{\partial x^2} = s^2 \bar{\theta} + \varepsilon s^2 \frac{\partial \bar{u}}{\partial x} \ , \tag{4.2}$$

$$\bar{\sigma} = \frac{\partial \bar{u}}{\partial x} - \bar{\theta} \ , \tag{4.3}$$

$$\bar{e} = \beta_1 \frac{\partial \bar{u}}{\partial x} \ , \tag{4.4}$$

$$\bar{h} = -\bar{e} \ , \tag{4.5}$$

$$\bar{E} = -\delta_0 s \bar{u} \ , \tag{4.6}$$

$$\bar{\phi} - \bar{\theta} = \beta_0 \frac{\partial^2 \bar{\phi}}{\partial x^2} \ , \tag{4.7}$$

where all the initial state functions are equal to zero.

Eliminating $\bar{u}$ and $\bar{\theta}$ from equations (4.1)-(4.7), we obtain

$$\frac{\partial^2 \bar{\phi}}{\partial x^2} = L_1 \bar{\phi} + L_2 \bar{\sigma} \ , \tag{4.8}$$

where

$$L_1 = \frac{s^2(1+\varepsilon)}{(\kappa_0 s + C_T^2) + \beta_0 s^2 (1+\varepsilon)}, \quad L_2 = \frac{\varepsilon s^2}{(\kappa_0 s + C_T^2) + \beta_0 s^2 (1+\varepsilon)},$$

and

$$\frac{\partial^2 \bar{\sigma}}{\partial x^2} = M_1 \bar{\phi} + M_2 \bar{\sigma} \ , \tag{4.9}$$

where

$$M_1 = \frac{\{s^2 - L_1(R_M^2 - 1)\}(1 - \beta_0 L_1)}{\beta_0 L_2 + R_M^2 (1 - \beta_0 L_2)},$$

$$M_2 = \frac{s^2(1 - \beta_0 L_2) - (R_M^2 - 1) L_2 (1 - \beta_0 L_1)}{\beta_0 L_2 + R_M^2 (1 - \beta_0 L_2)} \ .$$

Choosing as state variables the conductive temperature $\bar{\phi}$ and the stress component $\bar{\sigma}$ in the x-direction, equations (4.8) and (4.9) can be written in the matrix form as:

$$\frac{d^2 \bar{v}(x,s)}{dx^2} = A(s) \bar{v}(x,s), \tag{4.10}$$

where

$$\bar{v}(x,s) = \begin{bmatrix} \bar{\phi}(x,s) \\ \bar{\sigma}(x,s) \end{bmatrix} \quad \text{and} \quad A(s) = \begin{bmatrix} L_1 & L_2 \\ M_1 & M_2 \end{bmatrix} .$$

The formal solution of system (4.10) can be written in the form

$$\bar{v}(x,s) = \exp[-\sqrt{A(s)} x] \bar{v}(0,s) \ , \tag{4.11}$$

where

$$\bar{v}(0,s) = \begin{bmatrix} \bar{\phi}(0,s) \\ \bar{\sigma}(0,s) \end{bmatrix} = \begin{bmatrix} \bar{\phi}_0 \\ \bar{\sigma}_0 \end{bmatrix} ,$$

where for bounded solution with large x, we have cancelled the part of exponential that has a positive power.

We shall use the spectral decomposition of the matrix $A(s)$ and the well-known Cayley- Hamiltonian theorem to find the matrix form of the of the expression $\exp[-\sqrt{A(s)}x]$. The characteristic equation of the matrix $A(s)$ can be written as follows:

$$k^2 - k(L_1 + M_2) + (L_1M_2 - L_2M_1) = 0 . \tag{4.12}$$

The roots of this equation, namely, k_1 and k_2, satisfy the following relations:

$$k_1 + k_2 = L_1 + M_2, \tag{4.13a}$$

$$k_1k_2 = L_1M_2 - L_2M_1 . \tag{4.13b}$$

The Taylor series expansion of the matrix exponential in equation (4.11) has the form

$$\exp[-\sqrt{A(s)}x] = \sum_{n=0}^{\infty} \frac{[-\sqrt{A(s)}x]^n}{n!} . \tag{4.14}$$

Using the Cayley-Hamilton theorem, we can express A^2 and higher powers of the matrix A in terms of I and A, where I is the unit matrix of second order.

Thus, the infinite series in equation (4.14) can be reduced to

$$\exp[-\sqrt{A(s)}x] = a_0(x, s)I + a_1(x, s)\sqrt{A(s)} , \tag{4.15}$$

where a_0 and a_1 are coefficients depending on x and s.

By the Cayley-Hamilton theorem, the characteristic roots $\sqrt{k_1}$ and $\sqrt{k_2}$ of the matrix $\sqrt{A}$ must satisfy equation (4.15), thus

$$\exp[-\sqrt{k_1}x] = a_0 + a_1\sqrt{k_1} , \tag{4.16}$$

and

$$\exp[-\sqrt{k_2}x] = a_0 + a_1\sqrt{k_2} . \tag{4.17}$$

The solution of the above system is given by

$$a_0 = \frac{\sqrt{k_1}e^{-\sqrt{k_1}x} - \sqrt{k_2}e^{-\sqrt{k_2}x}}{\sqrt{k_1} - \sqrt{k_2}} ,$$

and

$$a_1 = \frac{e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x}}{\sqrt{k_1} - \sqrt{k_2}} .$$

Hence, we have

$$\exp[\sqrt{A(s)}x] = L_{ij}(x, s), \quad i, j = 1, 2$$

where

$$L_{11} = \frac{e^{-\sqrt{k_2}x}(k_1 - L_1) - e^{-\sqrt{k_1}x}(k_2 - L_1)}{k_1 - k_2}, \quad L_{12} = \frac{L_2(e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x})}{k_1 - k_2},$$
$$L_{22} = \frac{e^{-\sqrt{k_2}x}(k_1 - M_2) - e^{-\sqrt{k_1}x}(k_2 - M_2)}{k_1 - k_2}, \quad L_{21} = \frac{M_1(e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x})}{k_1 - k_2}. \tag{4.18}$$

The solution in equation (4.11) can be written in the form

$$\bar{v}(x,s) = L_{ij}\bar{v}(0,s)\ . \tag{4.19}$$

Hence, we obtain

$$\bar{\phi}(x,s) = \frac{(k_1\bar{\phi}_0 - L_1\bar{\phi}_0 - L_2\bar{\sigma}_0)e^{-\sqrt{k_2}x} - (k_2\bar{\phi}_0 - L_1\bar{\phi}_0 - L_2\bar{\sigma}_0)e^{-\sqrt{k_1}x}}{k_1 - k_2}, \tag{4.20}$$

$$\bar{\sigma}(x,s) = \frac{(k_1\bar{\sigma}_0 - M_1\bar{\phi}_0 - M_2\bar{\sigma}_0)e^{-\sqrt{k_2}x} - (k_2\bar{\sigma}_0 - M_1\bar{\phi}_0 - M_2\bar{\sigma}_0)e^{-\sqrt{k_1}x}}{k_1 - k_2}. \tag{4.21}$$

By using equations (4.20) and(4.21) with equation (4.7) we get

$$\bar{\theta}(x,s) = \frac{\begin{array}{c}(k_1\bar{\phi}_0 - L_1\bar{\phi}_0 - L_2\bar{\sigma}_0)(1-\beta_0 k_2)e^{-\sqrt{k_2}x} \\ -(k_2\bar{\phi}_0 - L_1\bar{\phi}_0 - L_2\bar{\sigma}_0)(1-\beta_0 k_1)e^{-\sqrt{k_1}x}\end{array}}{k_1 - k_2}. \tag{4.22}$$

It should be noted that the corresponding expressions for two-temperature generalized thermo-elasticity with one-relaxation time in the absence of magnetic field can be deduced by setting $R_M = 1$ and $\beta_0 = 0$ in equation (4.18).

Using Laplace transformation to the equations (3.23) and (3.25) we obtained

$$\bar{\phi}_0 = \frac{\phi_0}{s}, \tag{4.23}$$

$$\bar{\sigma}(0,t) = \bar{\sigma}_0 = 0\ . \tag{4.24}$$

Hence, we can use the conditions (4.23) and (4.24) into equations (4.20), (4.21) and (4.22) to get the exact solution in the Laplace transform domain as :

$$\bar{\phi}(x,s) = \frac{\phi_0[(k_1 - L_1)e^{-\sqrt{k_2}x} - (k_2 - L_1)e^{-\sqrt{k_1}x}]}{s(k_1 - k_2)}, \tag{4.25}$$

$$\bar{\sigma}(x,s) = \frac{\phi_0 M_1(e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x})}{s(k_1 - k_2)}, \tag{4.26}$$

$$\bar{\theta}(x,s) = \frac{\phi_0[Be^{-\sqrt{k_2}x} - Ae^{-\sqrt{k_1}x}]}{s(k_1-k_2)}\,, \tag{4.27}$$

$$\bar{e}(x,s) = \frac{\phi_0[(B-M_1)e^{-\sqrt{k_2}x} - (A-M_1)e^{-\sqrt{k_1}x}]}{s(k_1-k_2)}\,, \tag{4.28}$$

where $A = (k_2 - L_1)(1 - \beta_0 k_1)$, $B = (k_1 - L_1)(1 - \beta_0 k_2)$.

Using equations (4.1) and (4.3) the displacement can be written as:

$$\bar{u} = \frac{1}{s^2}[R_M^2 \frac{\partial\bar{\sigma}}{\partial x} + (R_M^2 - 1)\frac{\partial\bar{\theta}}{\partial x}]\,. \tag{4.29}$$

Substituting from equations (4.26) and (4.27) into (4.29) we get

$$\bar{u} = \frac{\phi_0}{s^3(k_1-k_2)}[\{(R_M^2-1)A - R_M^2 M_1\}\sqrt{k_1}e^{-\sqrt{k_1}x} - \{(R_M^2-1)B - R_M^2 M_1\}\sqrt{k_2}e^{-\sqrt{k_2}x}]. \tag{4.30}$$

Using equations (4.5) and (4.6) the expression for induced magnetic and electric field becomes

$$\bar{h}(x,s) = -\frac{\phi_0[(B-M_1)e^{-\sqrt{k_2}x} - (A-M_1)e^{-\sqrt{k_1}x}]}{s(k_1-k_2)}\,, \tag{4.31}$$

$$\bar{E}(x,s) = -\frac{\delta_0\phi_0}{s^2(k_1-k_2)}[\{(R_M^2-1)A - R_M^2 M_1\}\sqrt{k_1}e^{-\sqrt{k_1}x} - \{(R_M^2-1)B - R_M^2 M_1\}\sqrt{k_2}e^{-\sqrt{k_2}x}]\,. \tag{4.32}$$

5. Inversion of the Laplace Transform

In order to invert the Laplace transforms in the above equations we shall use a numerical technique based on Fourier expansions of functions.

Let $\bar{g}(s)$ be the Laplace transform of a given function $g(t)$. The inversion formula of Laplace transforms states that

$$g(t) = \frac{1}{2\pi i}\int_{d+i\infty}^{d-i\infty} e^{st}\bar{g}(s)\,ds,$$

where d is an arbitrary positive constant greater than all the real parts of the singularities of $\bar{g}(s)$. Taking $s = d + iy$, we get

$$g(t) = \frac{e^{dt}}{2\pi}\int_{\infty}^{-\infty} e^{ity}\bar{g}(d+iy)\,dy.$$

This integral can be approximated by

$$g(t) = \frac{e^{dt}}{2\pi} \sum_{k=-\infty}^{\infty} e^{ikt\Delta y} \bar{g}(d + ik\Delta y)\Delta y.$$

Taking $\Delta y = \dfrac{\pi}{t_1}$, we obtain

$$g(t) = \frac{e^{dt}}{t_1} \left[\frac{1}{2}\bar{g}(d) + Re\left(\sum_{k=1}^{\infty} e^{ik\pi t/t_1} \bar{g}(d + ik\pi/t_1) \right) \right].$$

For numerical purpose this is approximated by the function

$$g_N(t) = \frac{e^{dt}}{t_1} \left[\frac{1}{2}\bar{g}(d) + Re\left(\sum_{k=1}^{N} e^{ik\pi t/t_1} \bar{g}(d + ik\pi/t_1) \right) \right], \tag{5.1}$$

where N is a sufficiently large integer chosen such that

$$\frac{e^{dt}}{t_1} Re[e^{iN\pi t/t_1} \bar{g}(d + iN\pi/t_1)] < \eta$$

where η is a preselected small positive number that corresponds to the degree of accuracy to be achieved. Formula (4.28) is the numerical inversion formula valid for $0 \leq t \leq 2t_1$ (Sherief, 1986). In particular, we choose $t = t_1$, getting

$$g_N(t) = \frac{e^{dt}}{t} \left[\frac{1}{2}\bar{g}(d) + Re\left(\sum_{k=1}^{N} (-1)^k \bar{g}(d + ik\pi/t) \right) \right]. \tag{5.2}$$

6. Numerical Results

The copper like material has been chosen for the purpose of numerical evaluations. The constants of the problem are taken as follows: (Ezzat [12])

$$K = 386 \ \text{N/Ks}, \quad \mu = 3.86 \times 10^{10} \ \text{N/m}^2, \quad T_0 = 293\text{K}, \ \varepsilon = 0.0168,$$

$$\alpha_T = 1.78 \times 10^{-5} \ \text{K}^{-1}, \ \lambda = 7.76 \times 10^{10} \ \text{N/m}^2, \ c_1 = 4.158 \times 10^3, \ \beta_0 = 0.1,$$

$$C_E = 383.1 \ \text{m}^2/\text{K}, \ \rho = 8954 \ \text{kg/m}^3, \ H_0 = 1.0, \ \mu_0 = 4 \times 10^{-7} \ \text{C}^2/\text{Nm}^2,$$

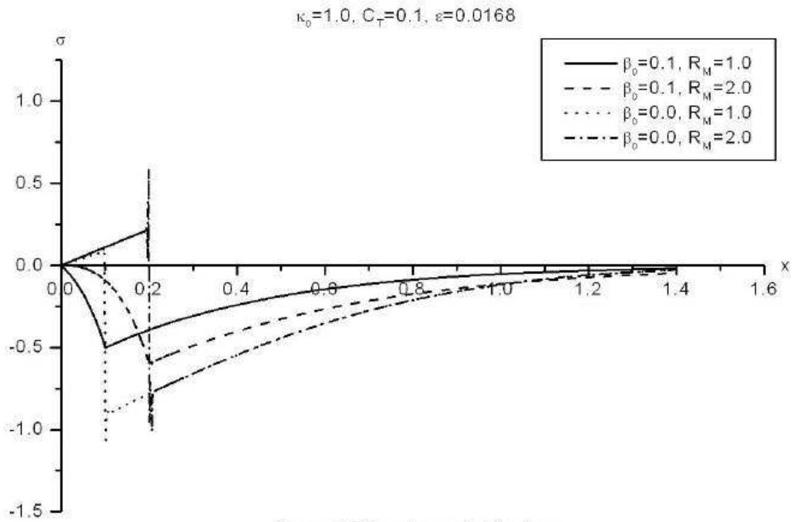

Figure 1. The stress distribution.

$$\beta = 0.1, \quad \delta_0 = 1.0$$

To get the solutions for stress, strain, temperature, induced magnetic field and induced electric field, we have applied the Laplace inversion formula to the equations (4.20), (4.21), (4.22), (4.23), (4.26) and (4.27). These have been done numerically using a method based on the Fourier series expansion technique. To get the roots of equation (4.12) in the complex domain, we have used Laguerre's method. The numerical code has been prepared using Fortran 77 programming language.

We now present our results in the form of graphs to compare stress, strain, temperatures, induced magnetic field and induced electric field by taking $t = 0.1$ and for the value of non-dimensional two-temperature parameters β_0 where $\beta_0 = 0.0$ indicates one type temperature $\beta_0 = 0.1$ indicates two type temperature.

Figure 1 depicts the space variation of stress for damping coefficient $\kappa_0 = 1.0$. In this figure a significant difference in the stress is noticed for different values of the non-dimensional two-temperature parameter β_0 in the presence ($R_M = 2.0$) and the absence ($R_M = 1.0$) of magnetic field. From this figure it is observed that in the case of one temperature ($\beta_0 = 0.0$) the discontinuity occurs in presence of magnetic field ($R_M = 2.0$) and also in absence of it ($R_M = 1.0$). But when there is two temperature ($\beta_0 = 0.1$) the curves show

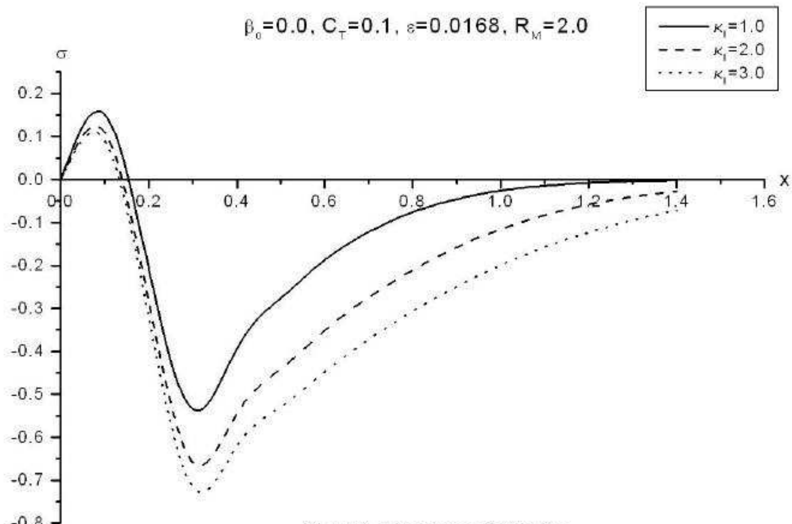

Figure 2. The stress distribution.

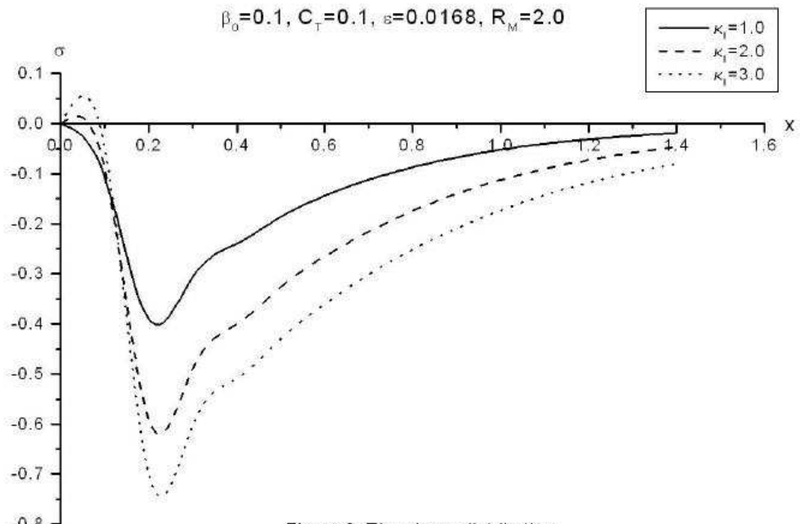

Figure 3. The stress distribution.

continuous behavior (for $R_M = 2.0$ and $R_M = 1.0$), which is quite plausible.

We also see that the magnitude of stress for $\beta_0 = 0.1$ is smaller than that of $\beta_0 = 0.0$. It is also observed from this figure that the magnetic field acts to decrease the magnitude of the stress component. This is mainly due to the fact of magnetic field corresponds to the term signifying positive force that tend to accelerate the metal particles.

Figure 2 depicts variation of stress versus distance for the damping coefficient $\kappa_0 = 1.0, 2.0$ and 3.0 in the presence of magnetic field ($R_M = 2.0$) and for the case of one type temperature ($\beta_0 = 0.0$). This figure shows positive values for stress in the range $0.0 \leq x \leq 1.9$ (for $\kappa_0 = 1.0, 2.0, 3.0$) and negative values for the stress in the range $0.2 \leq x \leq 1.2$ (for $\kappa_0 = 1.0$) and $0.2 \leq x \leq 1.4$ (for $\kappa_0 = 2.0, 3.0$) and finally diminishes. The stress distribution after assuming negative values goes on decreasing, attains maximum value in magnitude and then increases and finally vanishes.

Figure 3 depicts variation of stress versus distance for the damping coefficient $\kappa_0 = 1.0, 2.0$ and 3.0 in the presence of magnetic field ($R_M = 2.0$) for the case of two type temperature ($\beta_0 = 0.1$). This figure shows negative values for stress in the range $0.0 < x \leq 0.2$ (for $\kappa_0 = 1.0$) and $0.02 \leq x \leq 0.2$ (for $\kappa_0 = 2.0$) and $0.07 \leq x \leq 0.2$ (for $\kappa_0 = 3.0$) and then finally diminishes. In this figure stress also assume positive values in the range $0.0 \leq x \leq 0.01$ (for $\kappa_0 = 2.0$) and $0.0 \leq x \leq 0.06$ (for $\kappa_0 = 3.0$). From Figure 2. and Figure 3. it is observed that the rate of decay is slower as the damping coefficient increases.

Figure 4 gives the variation of strain versus distance x for $\beta_0 = 0.0$ and $\beta_0 = 0.1$ in the presence of magnetic field ($R_M = 2.0$) and also in the absence of magnetic field ($R_M = 1.0$). The strain takes positive value in the range $0.0 \leq x < 0.1$ ($\beta_0 = 0.1, R_M = 1.0$), $0.0 \leq x < 0.21$ ($\beta_0 = 0.1, R_M = 2.0$), $0.0 \leq x < 0.2$ ($\beta_0 = 0.0, R_M = 1.0$), $0.0 \leq x < 0.3$ ($\beta_0 = 0.0, R_M = 2.0$), and then negative value and finally diminishes to zero. this is also in conformity with the fact that strain should decrease with increasing distance x. It is also observed from this figure that magnitude of strain in the case of two temperature ($\beta_0 = 0.1$) is smaller than the case of one-temperature ($\beta_0 = 0.0$).

Figure 5 and Figure 6 gives the variation of strain against distance x for various values of damping coefficient in the case of $\beta_0 = 0.0$ and $\beta_0 = 0.1$ in the presence of magnetic field ($R_M = 2.0$). It is observed from the figures that strain decreases with distance first and then takes negative values and finally goes to zero for all of the cases. We can also observe from this figure that for various values of strain the nature of strain is almost same.

Figure 7 depicts the variation of conductive temperature versus distance for the case of one-type temperature as well as two-type temperature in the presence of magnetic field ($R_M = 2.0$). For both of the cases it is observed

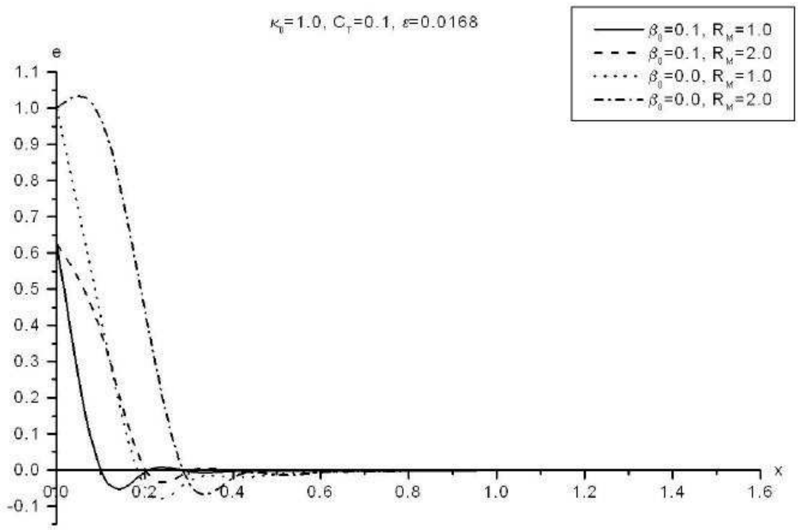

Figure 4. The strain distribution.

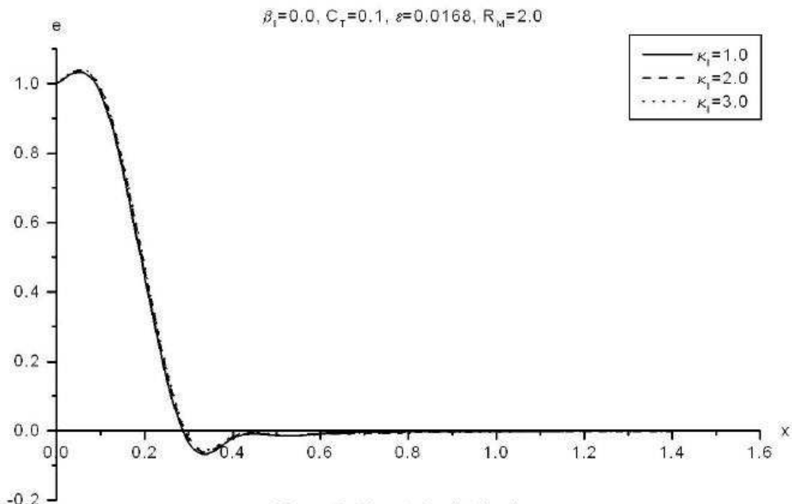

Figure 5. The strain distribution.

that conductive temperature decreases with the increase of distance and finally

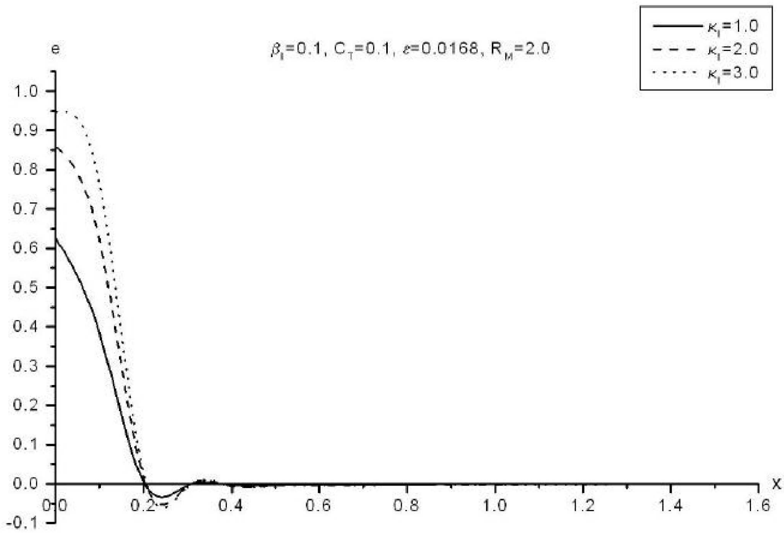

Figure 6. The strain distribution.

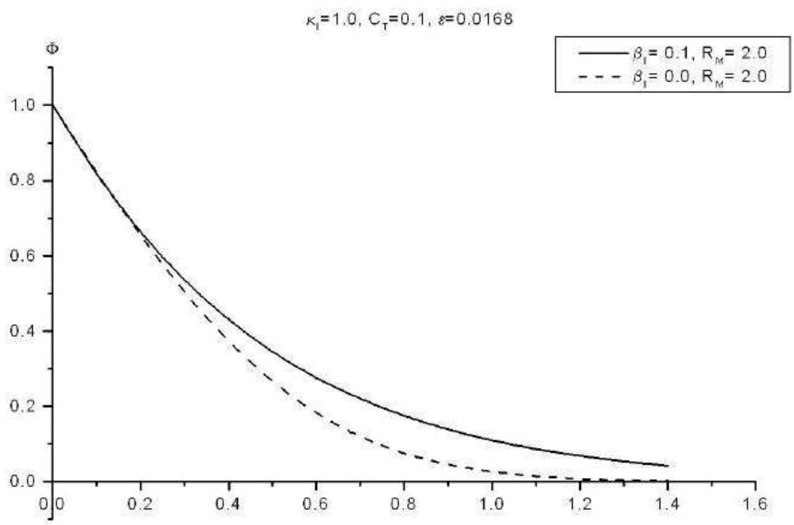

Figure 7. The conductive temperature distribution.

goes to zero. From this figure it can be observed that $\beta_0 = 0.1$ corresponds a slower rate of decay than the case when $\beta_0 = 0.0$. Figure 8 and Figure 9 are plotted to show the variation of conductive temperature versus distance when

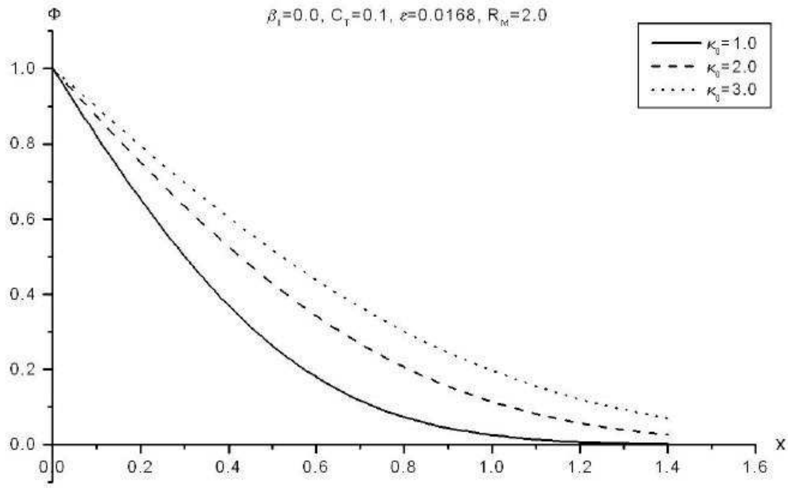

Figure 8. The conductive temperature distribution.

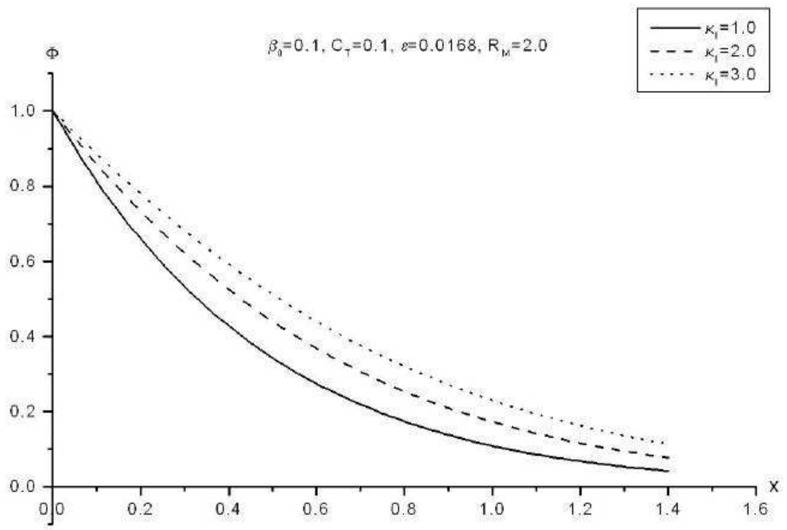

Figure 9. The conductive temperature distribution.

$R_M = 2.0$ in the case of $\beta_0 = 0.0$ and $\beta_0 = 0.1$ respectively. Here it can be

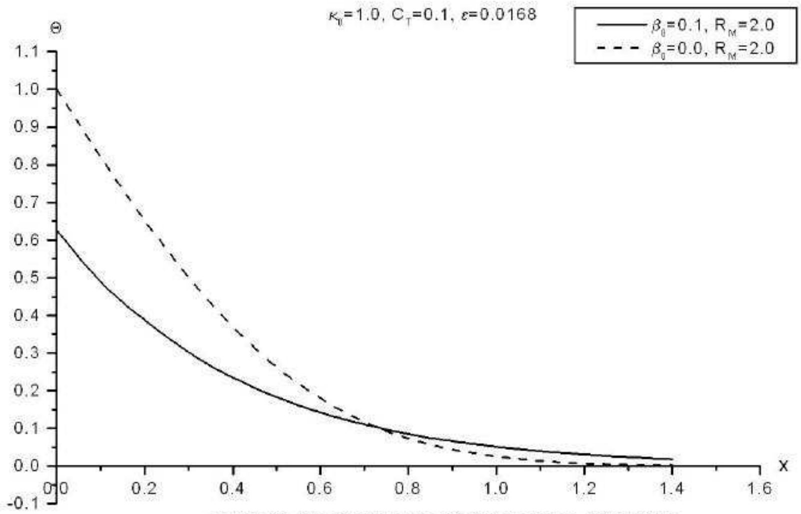

Figure 10. The thermodynamic temperature distribution.

observed that conductive temperature decreases with distance for $\kappa_0 = 1.0, 2.0$ and 3.0 and finally goes to zero but as damping coefficient increases, the rate of decay increases.

Figure 10 exhibits the space variation of thermodynamic temperature in which we observe that a significant difference in the thermodynamic temperature for the value of the non-dimensional two-temperature parameter β_0 where the case of $\beta_0 = 0.0$ indicates one -type temperature and the case of $\beta_0 = 0.1$ indicates two-type temperature. In both of the cases thermodynamic temperature decreases with distance and finally goes to zero but rate of decay for $\beta_0 = 0.1$ is slower than the rate of decay for $\beta_0 = 0.0$. Figure 11 and Figure 12 exhibit that space variation of thermodynamic temperature in the context of the one-type temperature generalized thermo-elasticity ($\beta_0 = 0.0$) and two-type temperature generalized thermo-elasticity ($\beta_0 = 0.1$) respectively in the presence of magnetic field ($R_M = 2.0$) for the different values of damping coefficient κ_0 and we have noticed thermodynamic temperature increases when the damping coefficient increases.

Figure 13 depicts to show the variation of induced magnetic field versus distance x for the value of the non-dimensional two-temperature parameter $\beta_0 = 0.1$ and $\beta_0 = 0.1$. In this figure we also observe the effect of magnetic field for both of the cases $\beta_0 = 0.0$ and $\beta_0 = 0.1$. It is seen form the figure that by

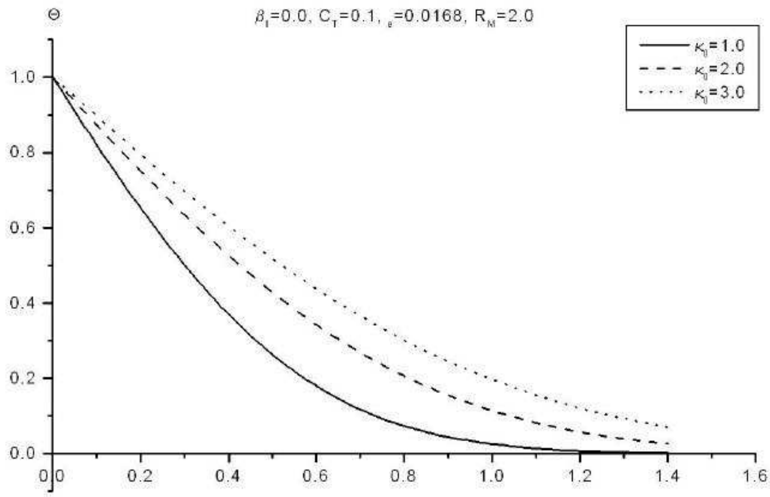

Figure 11. The thermodynamic temperature distribution.

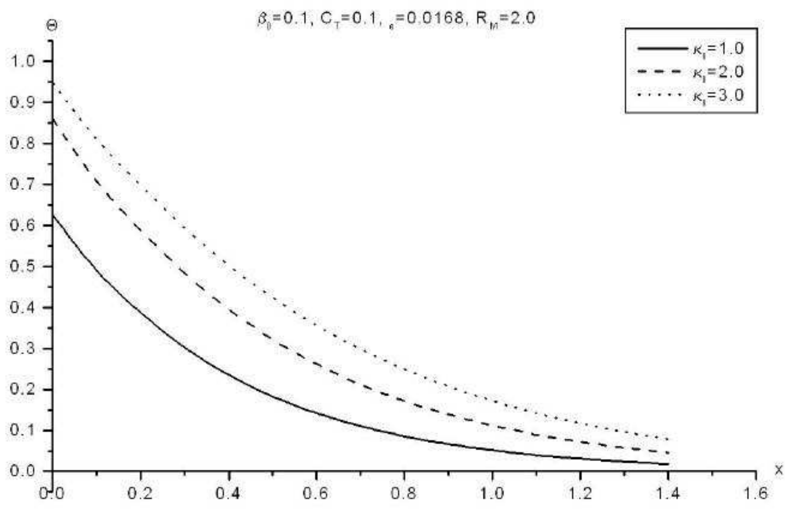

Figure 12. The thermodynamic temperature distribution.

increasing x induced magnetic field is also increasing remaining negetive first

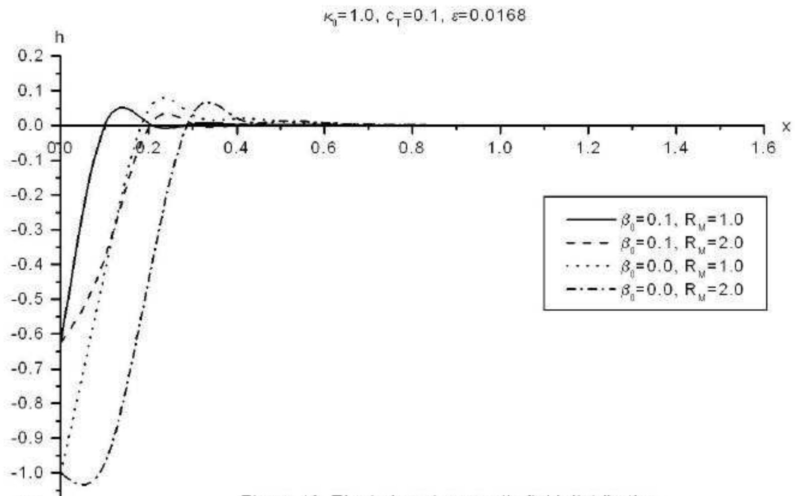

Figure 13. The induced magnetic field distribution.

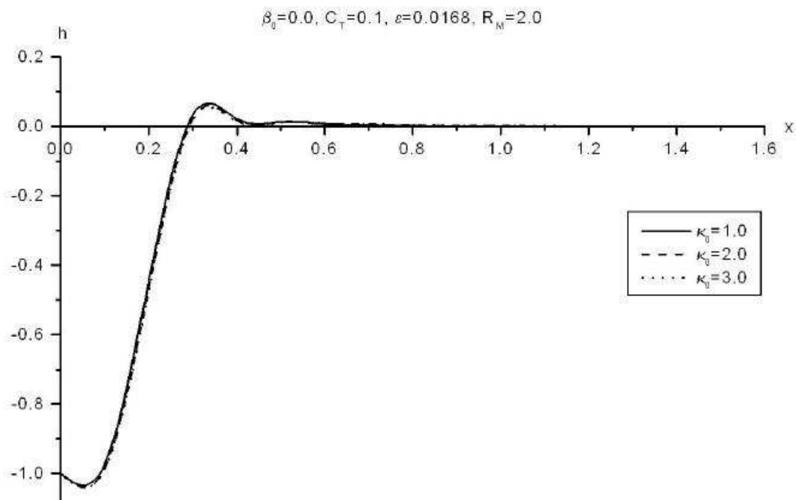

Figure 14. The induced magnetic field distribution.

and then it is positive in the range $x \geq 0.1$ for $(\beta_0 = 0.1, R_M = 1.0)$, $x > 0.2$

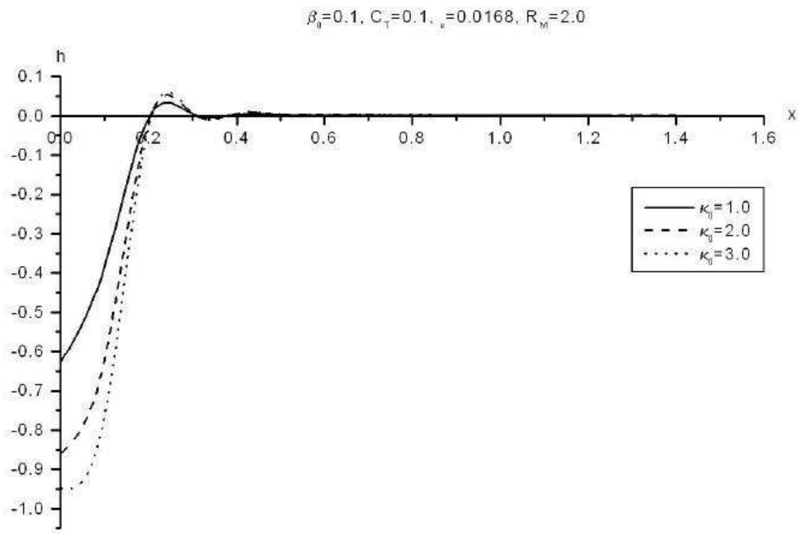

Figure 15. The induced magnetic field distribution.

for $(\beta_0 = 0.1, R_M = 2.0)$, $x > 0.1$ for $(\beta_0 = 0.0, R_M = 1.0)$ and $x > 0.2$ for $(\beta_0 = 0.0, R_M = 2.0)$ and finally diminishes.

Figure 14 is for the variation of the induced magnetic field against distance x for various value of the damping coefficient ($\kappa_0 = 1.0, 2.0$ and 3.0) for the case of one type temperature where $R_M = 2.0$. It is observed from these figure that as distance increases induced magnetic field tend to zero for all of the cases.

Figure 15. is plotted to show the variation of induced magnetic field with respect to that distance for the value of damping coefficient $\kappa_0 = 1.0, 2.0$ and 3.0 in case of two-type temperature ($\beta_0 = 0.1$) in the presence of magnetic field. For all of the cases induced magnetic field increases with distance remaining negative in the range $0.0 \leq x < 0.2$ and then positive in the range $0.2 < x < 0.3$ and finally goes to zero but as damping coefficient increases magnitude of induced magnetic field increases.

Figure 16. depicts to show the variation of induced electric field versus distance x. Here we can observe the effect of magnetic field with respect to one-type temperature and two-type temperature.

Figure 17. and Figure 18. are plotted to show the variation of induced electric field with respect to the distance for the various value of damping coefficient in the case of one-type temperature ($\beta_0 = 0.0$) and two-type temperature ($\beta_0 = 0.1$) respectively.

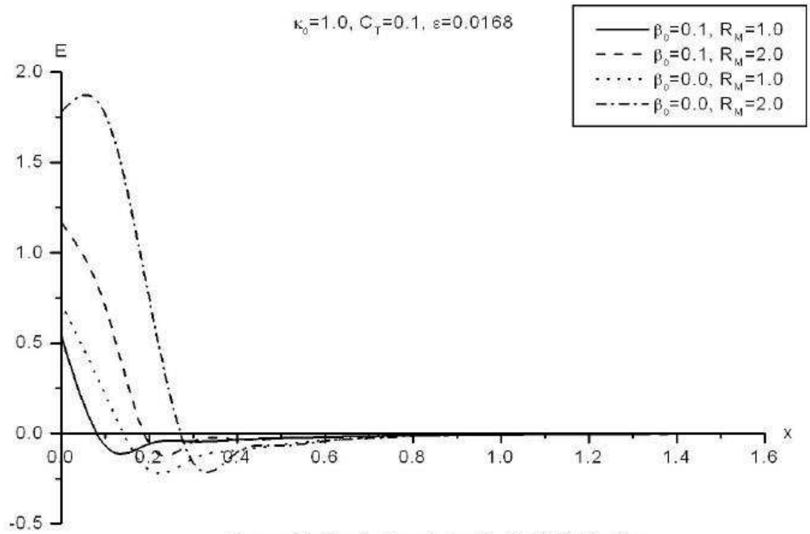

Figure 16. The induced electric field distribution.

7. Conclusion

This paper studies the magneto-thermo-elastic interaction in a stress free boundary of the half-space due to thermal shock in the context of two-temperature generalized thermo-elasticity with energy dissipation having finite conductivity. The state-space approach is used to solve the governing equations in the Laplace transformation domain. The inversion of the Laplace transform is carried out using a numerical approach. The analysis of the results permits some concluding remarks:

1. the discontinuity of the stress distribution was a critical situation and no one can explain the reason physically while in the context of the theory of two-temperature, the stress function is continuous.

2. The two-temperature generalized theory of magneto-thermo-elasticity describes the behavior of the particles of an elastic body is more realistic than the one temperature theory of generalized magneto-thermo-elasticity.

3. the results obtained in this paper are almost same in nature as they are obtained by Ezzat and Bary (2007) for L-S model.

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