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A NEW ONE PARAMETER FAMILY OF ITERATIVE METHODS WITH EIGHTH-ORDER OF CONVERGENCE FOR SOLVING NONLINEAR EQUATIONS

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Abstract: In this paper, a new one parameter family of iterative methods with eighth-order of convergence for solving nonlinear equations is presented and analyzed. This new family of iterative methods is obtained by composing an iterative method proposed by Chun [3] with Newton's method and approximating the first-appeared derivative in the last step by a combination of already evaluated function values. The proposed family is optimal since its efficiency index is $8^{1/4} \approx 1.6818$. The convergence analysis of the new family is studied in this paper. Several numerical examples are presented to illustrate the efficiency and accuracy of the family.

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1. Introduction

In recent years, many researchers have been paid attention to develop several two-step, three-step and fourth-step iterative methods for solving nonlinear equations $f(x) = 0$, where $f : I \subseteq R \to R$ is a real and sufficiently smooth function in I, with I a real open interval with the assumption that f has a simple zero at α in I, i.e., $f(\alpha) = 0$ and $f'(\alpha) \neq 0$. These methods have been constructed using different techniques, see [1-20].

One of the powerful methods for solving nonlinear equation of the form

$$f(x) = 0, \tag{1}$$

is the Newton's method which given as comes next:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{2}$$

which has a quadratic convergence.

Chun in [3] proposed the following family of iterative methods:

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\begin{aligned} z_n &= x_n + (1+\beta)\frac{f(x_n)+f(y_n)}{f'(x_n)} - 2\frac{f^2(x_n)}{f'(x_n)(f(x_n)-f(y_n))} \\ &\quad -\beta\left(\frac{f(x_n)}{f'(x_n)} + \frac{f'(x_n)f(y_n)}{f^2(x_n)+f'^2(x_n)}\right), \end{aligned} \tag{3}$$

where β is any real number for solving equation (1). It has proven in [3] that the order of convergence of this family is four and the error equation is

$$e_{n+1} = (\beta c_2 + 3c_2^3 - c_2 c_3)e_n^4 + O(e_n^5). \tag{4}$$

Now by composing the above family with the classical Newton's method

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)},$$

we obtain the following family of three-step iterative methods

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\begin{aligned} z_n &= x_n + (1+\beta)\frac{f(x_n)+f(y_n)}{f'(x_n)} - 2\frac{f^2(x_n)}{f'(x_n)(f(x_n)-f(y_n))} \\ &\quad -\beta\left(\frac{f(x_n)}{f'(x_n)} + \frac{f'(x_n)f(y_n)}{f^2(x_n)+f'^2(x_n)}\right), \end{aligned}$$

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)}. \tag{5}$$

According to the following theorem, which can be found in [20].

Theorem 1. *Let $\phi_1(x)$ and $\phi_2(x)$ be two iterative methods with order of convergence p and q, respectively, then the order of convergence of the iterative method $\phi(x) = \phi_2(\phi_1(x))$ is pq.*

The order of convergence of the family (5) is eight. Per one iteration, any member in this family requires three evaluations of the function and two evaluations of its first derivative, so its efficiency index is $8^{1/5} \approx 1.5157$, where the efficiency index of a method is defined to $\rho^{1/\theta}$, with ρ is the order of convergence and θ is the total number of evaluations of the function and its derivative. The important issue here is "can we keep the order of convergence and the efficiency index increase as much as possible?". To improve the efficiency index, several estimations for the first-appeared derivative in the last step $f'(z_n)$ by a combination of already evaluated function values are proposed, see, for example [5,10].

In this paper, another estimation for $f'(z_n)$ is proposed by a combination of already evaluated function values. The derivation of the estimation and construction of the new family of iterative methods will be discussed in Section 2. The analysis convergence of the proposed family will be analyzed in Section 3. Several numerical examples are given and compared with other iterative methods of the same order in Section 4 to illustrate the efficiency and the accuracy of the proposed family of iterative methods. Finally, some conclusions are pointed in Section 5.

2. Construction of a One Parameter Family of Iterative Methods with Eighth-Order of Convergence

To construct our one parameter family of iterative methods for solving nonlinear equation (1), consider the family of iterative methods (5). To derive the estimation of $f'(z_n)$, define the third degree polynomial

$$P_3(x) = f(x_n) + f[x_n, x_n](x - x_n) + f[x_n, x_n, y_n](x - x_n)^2$$

$$+f[x_n, x_n, y_n, z_n](x - x_n)^2(x - y_n), \tag{6}$$

where $f[x_n, x_n] = f'(x_n)$, $f[x_n, y_n, z_n] = \frac{f[y_n,z_n]-f[x_n,y_n]}{(z_n-x_n)}$ and $f[x_n, x_n, y_n, z_n] = \frac{f[x_n,y_n,z_n]-f[x_n,x_n,y_n]}{(z_n-x_n)}$. With a little simplification, the polynomial $P_3(x)$ can be written as:

$$\begin{aligned} P_3(x) &= f(x_n) + f'(x_n)(x - x_n) + f[x_n, x_n, y_n](x - x_n)^2[1 - \frac{(x - y_n)}{(z_n - x_n)}] \\ &\quad +f[x_n, y_n, z_n]\frac{(x - x_n)^2(x - y_n)}{(z_n - x_n)}. \end{aligned} \tag{7}$$

Now, it is easy to verify that $P_3(x_n) = f(x_n)$, $P_3(y_n) = f(y_n), P_3(z_n) = f(z_n)$ and $P_3'(x_n) = f'(x_n)$. Therefore, this polynomial interpolates f at x_n, y_n, z_n and also $f'(x_n) = P_3'(x_n)$. By differentiating $P_3(x)$ and substituting in z_n, we get:

$$\begin{aligned} P_3'(z_n) &= f'(x_n) + (f[x_n, y_n, z_n] + f[x_n, x_n, y_n])(z_n - x_n) \\ &+ 2(f[x_n, y_n, z_n] - f[x_n, x_n, y_n])(z_n - y_n). \end{aligned} \tag{8}$$

We approximate $f'(z_n)$ which will be denoted as $p_f(z_n)$ by the following formula

$$\begin{aligned} f'(z_n) &\approx f'(x_n) + (f[x_n, y_n, z_n] + f[x_n, x_n, y_n])(z_n - x_n) \\ &\quad +2(f[x_n, y_n, z_n] - f[x_n, x_n, y_n])(z_n - y_n) \\ &= p_f(z_n). \end{aligned} \tag{9}$$

Substitute (9) into (5) to obtain the new one parameter family of iterative methods.

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\begin{aligned} z_n &= x_n + (1 + \beta)\frac{f(x_n) + f(y_n)}{f'(x_n)} - 2\frac{f^2(x_n)}{f'(x_n)(f(x_n) - f(y_n))} \\ &\quad -\beta(\frac{f(x_n)}{f'(x_n)} + \frac{f'(x_n)f(y_n)}{f^2(x_n) + f'^2(x_n)}), \end{aligned}$$

$$x_{n+1} = z_n - \frac{f(z_n)}{p_f(z_n)}. \tag{10}$$

In the next section, we will prove that the family (10) has order of convergence eight for any $\beta \in R$. Per one iteration, any member of this family requires three evaluations of the function and one evaluation of its first derivative so its efficiency index is $8^{1/4} \approx 1.6818$, which implies that the efficiency index of the family of iterative methods (10) is optimal according to Kung and Trabu's conjecture [8].

3. Convergence Analysis

The convergence analysis of the family of three-step iterative methods (10) for solving equation (1) will be established in this section.

Theorem 2. *Let $\alpha \in I$ be a simple zero of a sufficiently differentiable function $f : I \subseteq R \to R$ for an open interval I. If x_0 is sufficiently close to α, then the family of iterative methods (10) has eighth-order of convergence.*

Proof. Let α be a simple zero of equation (1) and $x_n = \alpha + e_n$. By Taylor expansion, we have

$$f(x_n) = f'(\alpha)[e_n + c_2e_n^2 + c_3e_n^3 + c_4e_n^4 + c_5e_n^5 + c_6e_n^6 + c_7e_n^7 + c_8e_n^8 + \boldsymbol{O}(e_n^9)], \quad (11)$$

$$f'(x_n) = f'(\alpha)[1 + 2c_2e_n + 3c_3e_n^2 + 4c_4e_n^3 + 5c_5e_n^4 + \cdots + \boldsymbol{O}(e_n^8)], \quad (12)$$

where $c_k = \frac{f^{(k)}(\alpha)}{k!f'(\alpha)}, k = 2, 3, \ldots$.

Dividing (11) by (12), gives us

$$\begin{aligned}\frac{f(x_n)}{f'(x_n)} = {} & e_n - c_2e_n^2 + (-2c_3 + 2c_2^2)e_n^3 + (-3c_4 + 7c_2c_3 - 4c_2^3)e_n^4 + (10c_2c_4 \\ & -4c_5 + 6c_3^2 - 20c_3c_2^2 + 8c_2^4)e_n^5 + \cdots + \boldsymbol{O}(e_n^9). \quad (13)\end{aligned}$$

Substituting the last equation into y_n in (10), we have:

$$\begin{aligned}y_n = {} & \alpha + c_2e_n^2 + (+2c_3 - 2c_2^2)e_n^3 + (+3c_4 - 7c_2c_3 + 4c_2^3)e_n^4 + (-10c_2c_4 \\ & +4c_5 + 6c_3^2 + 20c_3c_2^2 - 8c_2^4)e_n^5 + \cdots + \boldsymbol{O}(e_n^9). \quad (14)\end{aligned}$$

Expanding $f(y_n)$ about α to get

$$\begin{aligned}f(y_n) = {} & f'(\alpha)[c_2e_n^2 + (2c_3 - 2c_2^2)e_n^3 + (3c_4 - 7c_2c_3 + 5c_2^3)e_n^4 + (-10c_2c_4 \\ & +4c_5 - 6c_3^2 + 24c_3c_2^2 - 12c_2^4)e_n^5 + \cdots + \boldsymbol{O}(e_n^9)]. \quad (15)\end{aligned}$$

Substituting (11), (12) and (15) into z_n in (10), and simplifying to get

$$z_n = \alpha + (-c_2c_3 + 3c_2^3 + c_2\beta)e_n^4 + (-2c_2c_4 - 18c_2^4 - 2c_3^2 + 20c_3c_2^2 + 2\beta c_3$$

$$-6\beta c_2^2)e_n^5 + \cdots + \boldsymbol{O}(e_n^9). \tag{16}$$

Expanding $f(z_n)$ about α, to get

$$\begin{aligned} f(z_n) &= f'(\alpha)[(-c_2c_3 + 3c_2^3 + c_2\beta)e_n^4 + (-2c_2c_4 - 18c_2^4 - 2c_3^2 + 20c_3c_2^2 \\ &\quad +2\beta c_3 - 6c_2^2\beta)e_n^5 + \cdots + \boldsymbol{O}(e_n^9)]. \end{aligned} \tag{17}$$

The expressions $f[x_n, y_n, z_n]$ and $[f[x_n, x_n, y_n]$ can be written in terms of e_n as:

$$\begin{aligned} f[x_n, y_n, z_n] &= f'(\alpha)[c_2 + c_3e + (c_4 + c_2c_3)e_n^2 + (c_2c_4 + c_5 + 2c_3^2 - 2c_3c_2^2)e_n^3 \\ &\quad +(7c_3c_2^3 - 8c_2c_3^2 - c_4c_2^2 + c_2c_5 + c_2\beta c_3 + 5c_3c_4 + c_6)e_n^4 \\ &\quad + \cdots + \boldsymbol{O}(e_n^9])). \end{aligned} \tag{18}$$

$$\begin{aligned} f[x_n, x_n, y_n] &= f'(\alpha)[c_2 + 2c_3e + (3c_4 + c_2c_3)e_n^2 + (2c_2c_4 + 4c_5 + 2c_3^2 \\ &\quad -2c_3c_2^2)e_n^3 + (7c_3c_4 - 3c_4c_2^2 - 7c_2c_3^2 + 4c_3c_2^3 + 3c_2c_5 + 5c_6)e_n^4 \\ &\quad + \cdots + \boldsymbol{O}(e_n^9)]. \end{aligned} \tag{19}$$

Thus, the estimation of $f'(z_n) = p_f(z_n)$ described in (9) can be written in terms of e_n as:

$$\begin{aligned} p_f(z_n) &= f'(\alpha)[1 + (-2c_3c_2^2 + c_2c_4 + 6c_2^4 + 2\beta c_2^2)e^4 + (-4c_2c_3^2 + 40c_3c_2^3 \\ &\quad +4\beta c_2c_3 + 2c_3c_4 - 6c_4c_2^2 + 2c_2c_5 - 36c_2^5 - 12\beta c_2^3)e^5 + \ldots \\ &\quad +(-4c_5c_3^2 + 3706c_3^2c_2^4 + 2\beta c_2^2 + 7\beta c_2c_6 - 75c_5c_2^2\beta - 86\beta c_2^4 \\ &\quad -102c_3\beta c_2c_4 - 787c_3^3c_2^2 - 5\beta c_2c_4 + 60\beta c_3c_2^2 + 3c_2^2\beta^2c_3 + 380c_2^3c_4\beta \\ &\quad +464\beta c_3^2c_2^2 - 1278c_3\beta c_2^4 - 4\beta c_3c_5 + 5c_2c_8 - 455c_5c_2^4 + 14c_6c_4 \\ &\quad +96c_6c_2^3 - 15c_7c_2^2 - 10c_3c_4^2 + 184c_4^2c_2^2 + 1592c_4c_2^5 - 4125c_3c_2^6 \\ &\quad +1248c_2^8 + 8c_5^2 + 8c_7c_3 - 3\beta c_4^2 + 642\beta c_2^6 - 54c_5c_2c_4 + 309c_5c_3c_2^2 \\ &\quad -36c_6c_2c_3 + 243c_4c_2c_3^2 - 1908c_4c_3c_2^3)e^8 + \boldsymbol{O}(e_n^9)]. \end{aligned} \tag{20}$$

Substitute (16)-(20) into x_{n+1} in (10) to obtain that

$$\begin{aligned} x_{n+1} &= \alpha + (-c_4c_3c_2^2 + 9c_2^7 + 3c_4c_2^4 + c_3^2c_2^3 - 6c_3c_2^5 + 6\beta c_2^5 + c_2^3\beta^2 \\ &\quad -2\beta c_3c_2^3 + \beta c_4c_2^2)e_n^8 + \boldsymbol{O}(e_n^9). \end{aligned} \tag{21}$$

Therefore,

$$\begin{aligned} e_{n+1} &= (-c_4c_3c_2^2 + 9c_2^7 + 3c_4c_2^4 + c_3^2c_2^3 - 6c_3c_2^5 + 6\beta c_2^5 + c_2^3\beta^2 - 2\beta c_3c_2^3 \\ &\quad +\beta c_4c_2^2)e_n^8 + \boldsymbol{O}(e_n^9). \end{aligned} \tag{22}$$

This proves that the order of convergence of the family (10) is eight. This completes the proof.

4. Numerical Examples

In this section, we check for different numerical examples the effectiveness and performance of the family of iterative methods proposed in this paper. Specifically, taking $\beta = 0$ in (10), we have:

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = x_n + \frac{f(x_n) + f(y_n)}{f'(x_n)} - 2\frac{f^2(x_n)}{f'(x_n)(f(x_n) - f(y_n))},$$

$$x_{n+1} = z_n - \frac{f(z_n)}{p_f(z_n)}. \tag{23}$$

Taking $\beta = -1$, we have

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = x_n - 2\frac{f^2(x_n)}{f'(x_n)(f(x_n) - f(y_n))} + (\frac{f(x_n)}{f'(x_n)} + \frac{f'(x_n)f(y_n)}{f^2(x_n) + f'^2(x_n)}),$$

$$x_{n+1} = z_n - \frac{f(z_n)}{p_f(z_n)}. \tag{24}$$

We compare our methods (23) and (24) with the eighth-order iterative method of Mir et al. (MM) in [12] which is defined by:

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = y_n - \frac{h(y_n)}{1 - (h(y_n))^2}$$

$$x_{n+1} = z_n - (y_n - z_n)\frac{f(z_n)}{f(y_n) - 2f(z_n)}, \tag{25}$$

where

$$h(y_n) = \frac{2f(y_n)}{f'(y_n) + \sqrt{(f'(y_n))^2 + 4(f(y_n))^2}},$$

and the three eighth-order iterative methods of Wang and Liu (WLM) in [19] which are given as comes next:

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = x_n - \frac{f(x_n)}{f'(x_n)} \frac{f(x_n) - f(y_n)}{f(x_n) - 2f(y_n)},$$

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(x_n)}[\frac{1}{2} + \frac{5(f(x_n))^2 + 8f(x_n)f(y_n) + 2(f(y_n))^2}{5(f(x_n))^2 - 12f(x_n)f(y_n)}(\frac{1}{2} + \frac{f(z_n)}{f(y_n)})], \quad (26)$$

and

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = x_n - \frac{f(x_n)}{f'(x_n)} \frac{f(x_n) - f(y_n)}{f(x_n) - 2f(y_n)},$$

$$\begin{aligned} x_{n+1} &= z_n - \frac{f(z_n)}{f'(x_n)}\{\frac{5(f(x_n))^2 - 2f(x_n)f(y_n) + (f(y_n))^2}{5(f(x_n))^2 - 12f(x_n)f(y_n)} \\ &+ [1 + \frac{4f(y_n)}{f(x_n)}]\frac{f(z_n)}{f(y_n)}\}, \end{aligned} \quad (27)$$

and

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = x_n - \frac{f(x_n)}{f'(x_n)} \frac{4f^2(x_n) - 5f(x_n)f(y_n) - f^2(y_n)}{4f^2(x_n) - 9f(x_n)f(y_n)},$$

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(x_n)}[1+4\frac{f(z_n)}{f(x_n)}][\frac{8(f(y_n))}{4f(x_n)-11f(y_n)}+1+\frac{f(z_n)}{f(y_n)}]. \quad (28)$$

The test functions and their roots, found up to the 32th decimal places, are as follows:

Example	the approximate zero α
$f_1(x) = \sin x - \frac{x}{2}$,	1.89549426703398094714403573809360 ,
$f_2(x) = x^3 - 10$,	2.15443469003188372175929356651935,
$f_3(x) = e^{-x} + \cos x$,	1.74613953040801241765070308895380 ,
$f_4(x) = \sin^2 x - x^2 + 1$,	1.40449164821534122603508681778695,
$f_5(x) = xe^{-x} - 0.1$,	.111832559158962964833569456820273.

All computations were done using MATLAB 7.6 with 400 digit floating arithmetic (VPA=400). The following criteria

$$|x_n - x_{n-1}| < \varepsilon \quad and \quad |f(x_n)| < \varepsilon,$$

	MM (25)	WLM (26)	WLM (27)	WLM (28)	method (23)	Method (24)
$f_1(x)$, initial guess $x_0 = 2.0$						
n	3	3	3	3	3	3
$\|f(x_n)\|$	5.3591e-327	4.0000e-400	4.0000e-400	4.0000e-400	4.0000e-400	4.0000e-400
δ	6.5433e-327	1.0000e-399	1.0000e-399	0	1.0000e-399	0
$f_2(x)$, initial guess $x_0 = 2$						
n	3	3	3	3	3	3
$\|f(x_n)\|$	3.0000e-399	3.0000e-399	3.0000e-399	3.0000e-399	1.0000e-398	3.0000e-399
δ	0	0	0	0	1.0000e-399	0
$f_3(x)$, initial guess $x_0 = 2$						
n	3	3	3	3	3	3
$\|f(x_n)\|$	6.2529e-337	5.0000e-400	5.0000e-400	5.0000e-400	5.0000e-400	5.0000e-400
δ	5.3945e-336	0	0	0	0	0
$f_4(x)$, initial guess $x_0 = 1.3$						
n	3	3	3	3	3	3
$\|f(x_n)\|$	5.5002e-392	1.0000e-399	1.0000e-399	1.7000e-399	1.0000e-399	1.0000e-399
δ	2.2156e-392	0	0	1.0000e-399	0	0
$f_5(x)$, initial guess $x_0 = 0.2$						
n	3	3	3	3	3	3
$\|f(x_n)\|$	0	0	0	0	0	0
δ	0	0	0	0	0	0

Table 1: Comparison of various iterative methods of the same order of convergence under the same stopping criteria

are used for stopping computer programmes. Displayed in Table 1 are the number of iterations n, such that the stopping criteria are satisfied, where ε is taken to be 10^{-300}, the value of $|f(x_n)|$ after the required iterations. Moreover, displayed is the distance of two consecutive approximations $\delta = |(x_n - x_{n-1})|$.

5. Conclusions

In this paper, a new one parameter family of iterative methods with eighth-order of convergence for solving nonlinear equations is presented and analyzed. This new proposed family is optimal since its efficiency index is $8^{1/4} \approx 1.6818$. The convergence analysis of the new family is also considered. Several numerical examples are presented to illustrate the efficiency and accuracy of our family. From Table (1), we observe that our family of iterative methods is comparable with all the methods cited in the table.

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