ON FUZZY ALMOST CONTRA $\gamma$-CONTINUOUS FUNCTIONS

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Abstract: Joseph and Kwack (see [11]) introduced the notion of $(\theta, s)$-continuous functions in order to investigate $S$-closed spaces. The aim of this paper is to introduce fuzzy almost contra $\gamma$-continuous functions related to $S$-closed spaces and to investigate some properties of such fuzzy functions.

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1. Introduction

Joseph and Kwack (see [11]) introduced $(\theta, s)$-continuous functions in order to investigate $S$-closed due to Thompson [23]. A function $f$ is called $(\theta, s)$-continuous if the inverse image of each regular open set is closed. Moreover,
Chang in [3] introduced fuzzy $S$-closed spaces in 1968. Fuzzy continuity is one of the main topics in fuzzy topology. Various authors introduce various types of fuzzy continuity. One of them is fuzzy $\gamma$-continuity. In 1999, Hanafy in [9] introduced the concept of fuzzy $\gamma$-continuity.

The purpose of this paper is to introduce fuzzy almost $\alpha$-continuous function and to investigate some of its properties. Using these properties of fuzzy almost contra continuous functions, properties of fuzzy almost contra $\alpha$-continuous functions, fuzzy almost contra precontinuous functions, fuzzy almost contra $\beta$-continuous and fuzzy almost contra semicontinuous functions are obtained.

2. Preliminaries

In the present paper, $X$ and $Y$ are always fuzzy topological spaces. The class of fuzzy sets on a universal set $X$ will be denoted by $I^X$ and fuzzy sets on $X$ will be denoted by Greek letters as $\mu, \rho, \eta$, etc. A family $\tau$ of fuzzy sets in $X$ is called a fuzzy topology for $X$ if

1. $0, 1 \in \tau$,
2. $\mu \wedge \rho \in \tau$, whenever $\mu, \rho \in \tau$ and
3. $\forall \{\mu_\alpha : \alpha \in I\} \in \tau$, whenever each $\mu_\alpha \in \tau (\alpha \in I)$.

Moreover, the pair $(X, \tau)$ is called a fuzzy topological space. Every member of $\tau$ is called a fuzzy open set. The complement of a fuzzy open set is fuzzy closed.

Let $\mu$ be a fuzzy set in $X$. We denote the complement, the interior and the closure of $\mu$ by $1 - \mu$ or $\mu^1$, $int(\mu)$ and $cl(\mu)$, respectively. A fuzzy set in $X$ is called a fuzzy point if and only if it takes the value 0 for all $y \in X$ except one, say, $x \in X$. If its value at $x$ is $\alpha (0 < \alpha \leq 1)$ we denote this fuzzy point by $x_\alpha$ where the point $x$ is called its support, see [16]. For any fuzzy point $x_\epsilon$ and any fuzzy set $\mu$, we write $x_\epsilon \in \mu$ if and only if $\epsilon \leq \mu(x)$. Two fuzzy sets $\lambda$ and $\beta$ are said to be quasi-coincident ($q$-coincident, shortly), denoted by $\lambda q \beta$, if there exists $x \in X$ such that $\lambda(x) + \beta(x) > 1$ (see [16]) and by $\overline{q}$ we denote “is not $q$-coincident”. It is known (see [16]) that $\lambda \leq \beta$ if and only if $\lambda \overline{q} (1 - \beta)$.

**Definition 2.1.** A fuzzy set $\mu$ in a space $X$ is called

1. fuzzy $\beta$-open [12] if $\mu \leq cl(int(cl(\mu)))$;
2. fuzzy semi-open [1] if $\mu \leq cl(int(\mu))$. 
(3) fuzzy α-open [2] if \( \mu \leq \text{int}(\text{cl}(\text{int}(\mu))) \);

(4) fuzzy preopen [1] if \( \mu \leq \text{int}(\text{cl}(\mu)) \);

(5) fuzzy γ-open [9] if \( \mu \leq \text{int}(\text{cl}(\mu)) \lor \text{cl}(\text{int}(\mu)) \).

The complements of the above mentioned open sets are called their respective closed sets.

**Remark 2.2.** (see [14])

- Fuzzy open set
- Fuzzy α-open
- Fuzzy preopen set
- Fuzzy beta-open set
- Fuzzy γ-open set
- Fuzzy semi-open set

None of the above implications is reversible.

**Definition 2.3.** [8] A space \( X \) is said to be fuzzy extremely disconnected if the closure of every fuzzy open set of \( X \) is fuzzy open in \( X \).

**Definition 2.4.** [1] Let \((X, \tau)\) be a fuzzy topological space. A fuzzy set \( \mu \) of \( X \) is called

(1) fuzzy regular open if \( \mu = \text{int}(\text{cl}(\mu)) \);

(2) fuzzy regular closed if \( \mu = \text{cl}(\text{int}(\mu)) \).

The complement of fuzzy regular open set is fuzzy regular closed.

The collection of all fuzzy regular closed sets of \( X \) is denoted by \( \text{FRC}(X) \).

**Definition 2.5.** [17] A subset \( \rho \) in a space \( X \) is said to be a fuzzy locally closed (briefly, a fuzzy LC) set if \( \rho = \alpha \land \beta \), where \( \alpha \) is a fuzzy open set and \( \beta \) is a fuzzy closed set.

**Theorem 2.6.** [20] Let \( X \) be a fuzzy extremely disconnected space and \( \mu \leq X \), the following properties are equivalent.

(1) \( \mu \) is a fuzzy open set.

(2) \( \mu \) is fuzzy α-open and a fuzzy LC set,
(3) $\mu$ is fuzzy preopen and a fuzzy LC set.

(4) $\mu$ is fuzzy semi-open and a fuzzy LC set.

(5) $\mu$ is fuzzy $\gamma$-open and a fuzzy LC set.

Let $\mu$ be a fuzzy set in a fuzzy topological space $X$. The fuzzy $\gamma$-closure and fuzzy $\gamma$-interior of $\mu$ are defined as $\bigwedge\{\rho : \mu \leq \rho, \rho$ is fuzzy $\gamma$-closed\}, $\bigvee\{\rho : \mu \geq \rho, \rho$ is fuzzy $\gamma$-open\} and denoted by $\gamma$-cl$(\mu)$ and $\gamma$-int$(\mu)$, respectively.

A fuzzy set $\mu$ is quasi-coincident with a fuzzy set $\nu$, denoted by $\mu \equiv \nu$, if there exists $x \in X$ such that $\mu(x) + \nu(x) > 1$. If $\mu$ is not quasi-coincident with $\nu$, then we write $\mu \not\equiv \nu$.

Lemma 2.7. [10] Let $A$ and $B$ be fuzzy sets in a fuzzy topological space $(X, \tau)$. Then

(1) if $A \cap B = 0_X$, then $A \not\equiv B$,

(2) $A \leq B$ iff $x_r q B$ for each $x_r q A$,

(3) $A \not\equiv B$ iff $A \leq B^1$.

(4) $x_r (\bigcup A_\alpha)(\alpha \in \Lambda)$ iff there is $\alpha_0 \in \Lambda$ such that $x_r q A_{\alpha_0}$.

Definition 2.8. [3] Let $f : (X, \tau) \rightarrow (Y, \rho)$ be a function. Let $A$ be a fuzzy subset in $X$ and $B$ be a fuzzy subset in $Y$. Then the Zadeh’s functions $f(A)$ and $f^{-1}(B)$ are defined by

(1) $f(A)$ is a fuzzy subset of $Y$ where

$$f(A) = \begin{cases} \sup_{z \in f^{-1}(y)} A(z), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

for each $y \in Y$.

(2) $f^{-1}(B)$ is a fuzzy subset of $X$ where $f^{-1}(B)(x) = B(f(x))$, for each $x \in X$.

Lemma 2.9. [3] Let $f : (X, \tau) \rightarrow (Y, \rho)$ be a function. For fuzzy sets $A$ and $B$ of $X$ and $Y$ respectively, the following statements hold:

(1) $ff^{-1}(B) \leq B$,

(2) $f^{-1}f(A) \geq A$;

(3) $f(A^1) \geq (f(A))^1$;

(4) $f^{-1}(B^1) = (f^{-1}(B))^1$;
(5) if $f$ is injective, then $f^{-1}(f(A)) = A$;
(6) if $f$ is surjective, then $f f^{-1}(B) = B$;
(7) if $f$ is bijective, then $f(A^1) = (f(A))^1$.

**Definition 2.10.** [3] Let $f : (X, \tau) \to (Y, \rho)$ be a function. Then $f$ is said to be
(1) fuzzy open if the image of every fuzzy open set of $X$ is fuzzy open in $Y$.
(2) fuzzy closed if the image of every fuzzy closed set of $X$ is fuzzy closed in $Y$.
(3) fuzzy continuous if the inverse image of every fuzzy open set of $Y$ is fuzzy open in $X$.

Let $f : X \to Y$ be a fuzzy function from a fuzzy topological space $X$ to a fuzzy topological space $Y$. Then the function $g : X \to X \times Y$ defined by $g(x_\epsilon) = (x_\epsilon, f(x_\epsilon))$ is called the fuzzy graph function of $f$, see [1].

Recall that for a fuzzy function $f : X \to Y$, the subset $\{(x_\epsilon, f(x_\epsilon)) : x_\epsilon \in X\} \subseteq X \times Y$ is called the fuzzy graph of $f$ and is denoted by $G(f)$.

### 3. Fuzzy Almost Contra $\gamma$-Continuous Functions

In this section, the notion of fuzzy almost contra $\gamma$-continuous functions is introduced.

**Definition 3.1.** Let $X$ and $Y$ be fuzzy topological spaces. A fuzzy function $f : X \to Y$ is said to be fuzzy almost contra $\gamma$-continuous if inverse image of each fuzzy regular open set in $Y$ is fuzzy $\gamma$-closed in $X$.

**Theorem 3.2.** For a fuzzy function $f : X \to Y$, the following statements are equivalent:

(1) $f$ is fuzzy almost contra $\gamma$-continuous,
(2) for every fuzzy regular closed set $\mu$ in $Y$, $f^{-1}(\mu)$ is fuzzy $\gamma$-open,
(3) for any fuzzy regular closed set $\mu \subseteq Y$ and for any $x_\epsilon X$ if $f(x_\epsilon) q \mu$, then $x_\epsilon q \gamma$-$\text{int}(f^{-1}(\mu))$,
(4) for any fuzzy regular closed set $\mu \subseteq Y$ and for any $x_\epsilon \in X$ if $f(x_\epsilon) q \mu$, then there exists a fuzzy $\gamma$-open set $\eta$ such that $x_\epsilon q \eta$ and $f(\eta) \leq \mu$,
(5) \( f^{-1}(\text{int}(\text{cl}(\mu))) \) is fuzzy \( \gamma \)-closed for every fuzzy open set \( \mu \),

(6) \( f^{-1}(\text{cl}(\text{int}(\rho))) \) is fuzzy \( \gamma \)-open for every fuzzy closed subset \( \rho \),

(7) for each fuzzy singleton \( x_\varepsilon \in X \) and each fuzzy regular closed set \( \eta \) in \( Y \)

containing \( f(x_\varepsilon) \), there exists a fuzzy \( \gamma \)-open set \( \mu \) in \( X \) containing \( x_\varepsilon \) such that \( f(\mu) \leq \eta \).

Proof. (1) \( \Leftrightarrow \) (2) : Let \( \rho \) be a fuzzy regular open set in \( Y \). Then, \( \rho^1 \) is fuzzy regular closed. By (2), \( f^{-1}(\rho^1) = (f^{-1}(\rho))^1 \) is fuzzy \( \gamma \)-open. Thus, \( f^{-1}(\rho) \) is fuzzy \( \gamma \)-closed.

Converse is similar.

(2) \( \Leftrightarrow \) (3) : Let \( \mu \leq Y \) be a fuzzy regular closed set and \( f(x_\varepsilon)q\mu \). Then \( x_\varepsilon qf^{-1}(\mu) \) and from (2), \( f^{-1}(\mu) = \gamma-\text{int}(f^{-1}(\mu)) \). Hence \( x_\varepsilon q\gamma-\text{int}(f^{-1}(\mu)) \). Thus, (3) holds.

The reverse is obvious.

(3) \( \Rightarrow \) (4) : Let \( \mu \leq Y \) be any fuzzy regular closed set and for \( x_\varepsilon \in X \) let \( f(x_\varepsilon)q\mu \). Then \( x_\varepsilon q\gamma-\text{int}(f^{-1}(\mu)) \). Take \( \eta = \gamma-\text{int}(f^{-1}(\mu)) \), then \( f(\eta) = f(\gamma-\text{int}(f^{-1}(\mu))) \leq f(f^{-1}(\mu)) \leq \mu \), where \( \eta \) is fuzzy regular open in \( X \) and \( x_\varepsilon q\eta \).

(4) \( \Rightarrow \) (3) : Let \( \mu \leq Y \) be any fuzzy regular closed set and let \( f(x_\varepsilon)q\mu \). From (4), there exists fuzzy \( \gamma \)-open set \( \eta \) such that \( x_\varepsilon q\eta \) and \( f(\eta) \leq \mu \). Hence \( \eta \leq f^{-1}(\mu) \) and then \( x_\varepsilon q\gamma-\text{int}(f^{-1}(\mu)) \).

(1) \( \Leftrightarrow \) (5) : Let \( \mu \) be a fuzzy open set. Since \( \text{int}(\text{cl}(\mu)) \) is fuzzy regular open, by (1), it follows that \( f^{-1}(\text{int}(\text{cl}(\mu))) \) is fuzzy \( \gamma \)-closed.

The converse can be shown easily.

(2) \( \Leftrightarrow \) (6) : It can be obtained similar as (1) \( \Leftrightarrow \) (5).

(2) \( \Leftrightarrow \) (7) : Obvious.

**Theorem 3.3.** Let \( f : X \to Y \) be a fuzzy function and let \( g : X \to X \times Y \) be the fuzzy graph function of \( f \), defined by \( g(x_\varepsilon) = (x_\varepsilon, f(x_\varepsilon)) \) for every \( x_\varepsilon \in X \). If \( g \) is fuzzy almost contra \( \gamma \)-continuous, then \( f \) is fuzzy almost contra \( \gamma \)-continuous.

Proof. Let \( \eta \) be a fuzzy regular closed set in \( Y \), then \( X \times \eta \) is a fuzzy regular closed set in \( X \times Y \). Since \( g \) is fuzzy almost contra \( \gamma \)-continuous, then \( f^{-1}(\eta) = g^{-1}(X \times \eta) \) is fuzzy \( \gamma \)-open in \( X \). Thus, \( f \) is fuzzy almost contra \( \gamma \)-continuous.

**Definition 3.4.** [20] A fuzzy filter base \( \Lambda \) is said to be fuzzy \( \gamma \)-convergent to a fuzzy singleton \( x_\varepsilon \) in \( X \) if for any fuzzy \( \gamma \)-open set \( \eta \) in \( X \) containing \( x_\varepsilon \), there exists a fuzzy set \( \mu \in \Lambda \) such that \( \mu \leq \eta \).

**Definition 3.5.** [5] A fuzzy filter base \( \Lambda \) is said to be fuzzy \( rc \)-convergent to a fuzzy singleton \( x_\varepsilon \) in \( X \) if for any fuzzy regular closed set \( \eta \) in \( X \) containing
There exists a fuzzy set $\mu \in \Lambda$ such that $\mu \leq \eta$.

**Theorem 3.6.** If a fuzzy function $f : X \to Y$ is fuzzy almost contra $\gamma$-continuous, then for each fuzzy singleton $x_\epsilon \in X$ and each fuzzy filter base $\Lambda$ in $X\gamma$-converging to $x_\epsilon$, the fuzzy filter base $f(\Lambda)$ is fuzzy $rc$-convergent to $f(x_\epsilon)$.

**Proof.** Let $x_\epsilon \in X$ and $\Lambda$ be any fuzzy filter base in $X\gamma$-converging to $x_\epsilon$. To prove that the fuzzy filter base $f(\Lambda)$ is fuzzy $rc$-convergent to $f(x_\epsilon)$, let $\lambda$ be a fuzzy regular closed set in $Y$ containing $f(x_\epsilon)$. Since $f$ is almost fuzzy contra $\gamma$-continuous, there exists a fuzzy $\gamma$-open set $\mu$ in $X$ containing $x_\epsilon$ such that $f(\mu) \leq \lambda$. Since $\Lambda$ is fuzzy $\gamma$-converging to $x_\epsilon$, there exists a $\xi \in \Lambda$ such that $\xi \leq \mu$. This means that $f(\xi) \leq \lambda$ and therefore the fuzzy filter base $f(\Lambda)$ is fuzzy $rc$-convergent to $f(x_\epsilon)$.

**Definition 3.7.** [20] A space $X$ is called fuzzy $\gamma$-connected if $X$ cannot be expressed as $X = \mu_1 \lor \mu_2$ where

1. $\mu_1, \mu_2$ are fuzzy $\gamma$-open sets.
2. $\mu_1, \mu_2 \not= 0_X$.
3. $\mu_1 \bar{q} \mu_2$.

**Definition 3.8.** [18] A space $X$ is called fuzzy connected if $X$ cannot be expressed as $X = \mu_1 \lor \mu_2$ where

1. $\mu_1, \mu_2$ are fuzzy open sets.
2. $\mu_1, \mu_2 \not= 0_X$.
3. $\mu_1 \bar{q} \mu_2$.

**Theorem 3.9.** If $f : X \to Y$ is a fuzzy almost contra $\gamma$-continuous surjection and $X$ is fuzzy $\gamma$-connected, then $Y$ is fuzzy connected.

**Proof.** Suppose that $Y$ is not a fuzzy connected. Then there is a proper fuzzy clopen subset $\eta$ in $Y$. Therefore $\eta$ is fuzzy regular clopen in $Y$. Since $f$ is fuzzy almost contra $\gamma$-continuous surjection, $f^{-1}(\eta)$ is a proper fuzzy $\gamma$-clopen set in $X$. Thus $X$ is not fuzzy $\gamma$-connected and this is a contradiction. Hence $Y$ is fuzzy connected.

**Definition 3.10.** A fuzzy space $X$ is said to be fuzzy $\gamma$-normal if every pair of fuzzy closed sets $\mu$ and $\eta$ with $\mu \not= 0_X, \eta \not= 0_X$ and $\mu \bar{q} \eta$, can be separated by non-quasicoincident fuzzy $\gamma$-open sets.
Definition 3.11. [5] A fuzzy space $X$ is said to be fuzzy strongly normal if for every pair of fuzzy closed sets $\mu$ and $\eta$ with $\mu \bar{\eta} \rho$, there exist fuzzy open sets $\rho$ and $\xi$ such that $\mu \leq \rho, \eta \leq \xi$ and $\text{cl}(\rho) \bar{\eta} \text{cl}(\xi)$.

Theorem 3.12. If $Y$ is fuzzy strongly normal and $f : X \to Y$ is fuzzy almost contra $\gamma$-continuous closed injection, then $X$ is fuzzy $\gamma$-normal.

Proof. Let $\eta$ and $\rho$ be fuzzy closed sets of $X$ with $\eta, \rho \neq 0_X$ and $\eta \bar{\rho} \rho$. Since $f$ is injective and fuzzy closed, $f(\eta)$ and $f(\rho)$ are fuzzy closed sets with $f(\eta) \bar{\rho} f(\rho)$. Since $Y$ is fuzzy strongly normal, there exist fuzzy open sets $\mu$ and $\xi$ such that $f(\eta) \leq \mu$ and $f(\rho) \leq \xi$ and $\text{cl}(\mu) \bar{\eta} \text{cl}(\xi)$. Since $\mu$ and $\xi$ are fuzzy open in $X$, $\text{cl}(\mu)$ and $\text{cl}(\xi)$ are fuzzy regular closed in $X$. $f$ is fuzzy almost contra $\gamma$-continuous implies $f^{-1}(\text{cl}(\mu))$ and $f^{-1}(\text{cl}(\xi))$ are fuzzy $\gamma$-open sets in $X$, with $f^{-1}(\text{cl}(\mu)) \bar{\rho} f^{-1}(\text{cl}(\xi))$. Also $\eta \leq f^{-1}(\text{cl}(\mu)), \rho \leq f^{-1}(\text{cl}(\xi))$. Thus, $X$ is fuzzy $\gamma$-normal.

Definition 3.13. [5] A space $X$ is said to be fuzzy weakly $T_2$ if each element of $X$ is an intersection of fuzzy regular closed sets.

Definition 3.14. [20] A space $X$ is said to be fuzzy $\gamma$-$T_2$ if for each pair of distinct points $x_\epsilon$ and $y_\nu$ in $X$, there exist fuzzy $\gamma$-open sets $\mu$ and $\eta$ containing $x_\epsilon$ and $y_\nu$, respectively such that $\mu \bar{\eta} \rho$.

Definition 3.15. [20] A space $X$ is said to be fuzzy $\gamma$-$T_1$ if for each pair of distinct fuzzy singletons $x_\epsilon$ and $y_\nu$ in $X$, there exist fuzzy $\gamma$-open sets $\mu$ and $\eta$ containing $x_\epsilon$ and $y_\nu$, respectively, such that $y_\nu \notin \mu$ and $x_\epsilon \notin \eta$.

Theorem 3.16. If $f : X \to Y$ is a fuzzy almost contra $\gamma$-continuous injection and $Y$ is fuzzy Urysohn, then $X$ is fuzzy $\gamma$-$T_2$.

Proof. Let $x_\epsilon$ and $t_\gamma$ be any two distinct fuzzy singletons in $X$. Since $f$ is injective, $f(x_\epsilon) \neq f(t_\gamma)$ in $Y$. By assumption $Y$ is fuzzy Urysohn and therefore there exist fuzzy open sets $\eta$ and $\rho$ in $Y$ such that $f(x_\epsilon) \in \eta$ and $f(t_\gamma) \in \rho$ and $\text{cl}(\eta) \bar{\rho} \text{cl}(\rho)$. Since $\eta$ and $\rho$ are fuzzy open, $\text{cl}(\eta)$ and $\text{cl}(\rho)$ are fuzzy regular closed in $Y$. $f$ is fuzzy almost contra $\gamma$-continuous implies that there exists fuzzy $\gamma$-open sets $\mu$ and $\xi$ in $X$ containing $x_\epsilon$ and $t_\gamma$, respectively, such that $f(\mu) \leq \text{cl}(\eta)$ and $f(\xi) \leq \text{cl}(\rho)$. Since $\text{cl}(\eta) \bar{\rho} \text{cl}(\rho)$, we have $f(\mu) \bar{\rho} f(\xi)$ and hence $\mu \bar{\eta} \xi$. This shows that $X$ is fuzzy $\gamma$-$T_2$.

Theorem 3.17. If $f : X \to Y$ is a fuzzy almost contra $\gamma$-continuous injection and $Y$ is fuzzy weakly $T_2$, then $X$ is fuzzy $\gamma$-$T_1$.

Proof. Let $x_\epsilon$ and $t_\gamma$ be any two distinct fuzzy points in $X$. Since $f$ is injective, $f(x_\epsilon)$ and $f(t_\gamma)$ are distinct fuzzy points in $Y$. $Y$ is fuzzy weakly $T_2$ implies that there exist fuzzy regular closed sets $\eta$ and $\rho$ in $Y$ such that
continuous and open function, \( \prod f \) is fuzzy almost contra-\( \gamma \)-continuous, by Theorem 3.2, \( f^{-1}(\eta) \) and \( f^{-1}(\rho) \) are fuzzy \( \gamma \)-open sets in \( X \) such that \( x_\epsilon \in f^{-1}(\eta), t_\gamma \notin f^{-1}(\eta), x_\epsilon \notin f^{-1}(\rho) \) and \( t_\gamma \in f^{-1}(\rho) \). This shows that \( X \) is fuzzy \( \gamma \)-\( T_1 \).

**Theorem 3.18.** Let \( (X_i, \tau_i) \) be fuzzy topological spaces \( \forall i \in I \) and \( I \) be finite. Let \( f : (X, \tau) \to (\prod_{i \in I} X_i, \sigma) \) be a fuzzy function where \( (\prod_{i \in I} X_i, \sigma) \) is the product space. If \( f \) is fuzzy almost contra-\( \gamma \)-continuous, then \( \rho_i \) of is fuzzy almost contra-\( \gamma \)-continuous where \( \rho_i \) is projection function for each \( i \in I \).

Proof. Let \( \rho_i \) be a fuzzy regular closed set in \( (X_i, \tau_i) \). Since \( \rho_i \) is fuzzy continuous and open function, \( pr_i^{-1}(\rho_i) = X_1 \times X_2 \times \cdots X_i \times \rho_i \times X_{i+1} \times X_{i+2} \times \cdots \times X_n \) where \( I = \{1, 2, \cdots , n\} \) is finite, is a fuzzy regular closed set in \( (\prod_{i \in I} X_i, \sigma) \). By assumption \( f \) is fuzzy almost contra-\( \gamma \)-continuous

\[
f^{-1}(pr_i^{-1}(\rho_i)) = (pr_i \circ f)^{-1}(\rho_i)
\]

is fuzzy \( \gamma \)-open in \( X \). Hence \( pr_i \circ f \) is fuzzy almost contra-\( \gamma \)-continuous for each \( i \in I \).

**Definition 3.19.** The fuzzy graph \( G(f) \) of a fuzzy function \( f : X \to Y \) is said to be fuzzy strongly contra-\( \gamma \)-closed if for each \( (x_\epsilon, y_\nu) \in (X \times Y) \setminus G(f) \), there exist a fuzzy \( \gamma \)-open set \( \mu \) in \( X \) containing \( x_\epsilon \) and a fuzzy regular closed set \( \eta \) in \( Y \) containing \( y_\nu \) such that \( (\mu \times \eta) \notin G(f) \).

**Lemma 3.20.** The following properties are equivalent for the fuzzy graph \( G(f) \) of a fuzzy function \( f : X \to Y \):

1. \( G(f) \) is fuzzy strongly contra-\( \gamma \)-closed;

2. for each \( (x_\epsilon, y_\nu) \in (X \times Y) \setminus G(f) \), there exist a fuzzy \( \gamma \)-open set \( \mu \) in \( X \) containing \( x_\epsilon \) and a fuzzy regular closed set \( \eta \) containing \( y_\nu \) such that \( f(\mu) \notin \gamma \eta \).

**Theorem 3.21.** If \( f : X \to Y \) is fuzzy almost contra-\( \gamma \)-continuous and \( Y \) is fuzzy Urysohn, \( G(f) \) is fuzzy strongly contra-\( \gamma \)-closed in \( X \times Y \).

Proof. Suppose that \( Y \) is fuzzy Urysohn. Let \( (x_\epsilon, y_\nu) \in (X \times Y) \setminus G(f) \). It follows that \( f(x_\epsilon) \neq y_\nu \). Since \( Y \) is fuzzy Urysohn, there exist fuzzy open sets \( \eta \) and \( \rho \) such that \( f(x_\epsilon) \in \eta, y_\nu \in \rho \) and \( cl(\eta) \notin cl(\rho) \). \( \eta \) and \( \rho \) are fuzzy open in \( Y \) implies \( cl(\eta) \) and \( cl(\rho) \) are fuzzy regular closed in \( Y \). Since \( f \) is fuzzy almost contra-\( \gamma \)-continuous, there exists a fuzzy \( \gamma \)-open set \( \mu \) in \( X \) containing \( x_\epsilon \) such that \( f(\mu) \leq cl(\eta) \). Therefore, \( f(\mu) \notin \gamma \eta \) and thus \( G(f) \) is fuzzy strongly contra-\( \gamma \)-closed in \( X \times Y \).
Theorem 3.22. Let \( f : X \rightarrow Y \) have a fuzzy strongly contra \( \gamma \)-closed graph. If \( f \) is injective and fuzzy almost contra \( \gamma \)-continuous, then \( X \) is fuzzy \( \gamma \)-\( T_1 \).

Proof. Let \( x_\epsilon \) and \( y_\nu \) be any two distinct points of \( X \). Since \( f \) is injective, \( f(x_\epsilon) \neq f(y_\nu) \). Then, we have \( (x_\epsilon, f(y_\nu)) \in (X \times Y) \setminus G(f) \). By Lemma 3.20, there exist a fuzzy \( \gamma \)-open set \( \mu \) in \( X \) containing \( x_\epsilon \) and a fuzzy regular closed set \( \rho \) in \( Y \) containing \( f(y_\nu) \) such that \( f(\mu) \nsubseteq \rho \). Since \( f \) is fuzzy almost contra \( \gamma \)-continuous, \( f^{-1}(\rho) \) is fuzzy \( \gamma \)-open in \( X \) such that \( y_\gamma \in f^{-1}(\rho) \). As \( f(\mu) \nsubseteq \rho \), we have \( \mu \nsubseteq f^{-1}(\rho) \). Taking \( \eta = f^{-1}(\rho) \), we have fuzzy \( \gamma \)-open sets \( \mu \) and \( \eta \) in \( X \) such that \( x_\epsilon \in \mu, y_\nu \in \eta \) respectively, whereas \( x_\epsilon \notin \eta y_\nu \notin \mu \). This proves that \( X \) is \( \gamma \)-\( T_1 \).

4. The Relationships

In this section, the relationships between fuzzy almost contra \( \gamma \)-continuity and other forms of continuity are investigated.

Definition 4.1. A function \( f : X \rightarrow Y \) is called fuzzy weakly almost contra \( \gamma \)-continuous if for each \( x \in X \), and each fuzzy regular closed set \( \eta \) of \( Y \) containing \( f(x_\epsilon) \), there exists a fuzzy \( \gamma \)-open set \( \mu \) in \( X \) containing \( x_\epsilon \) such that \( \text{int}(f(\mu)) \leq \eta \).

Definition 4.2. A function \( f : X \rightarrow Y \) is called fuzzy \((\gamma, s)\)-open if the image of each fuzzy \( \gamma \)-open set is fuzzy semi-open.

Theorem 4.3. If a function \( f : X \rightarrow Y \) is fuzzy weakly almost contra \( \gamma \)-continuous and fuzzy \((\gamma, s)\)-open, then \( f \) is fuzzy almost contra \( \gamma \)-continuous.

Proof. Let \( x_\epsilon \in X \) and \( \eta \) be a fuzzy regular closed set containing \( f(x_\epsilon) \). Since \( f \) is fuzzy weakly almost contra \( \gamma \)-continuous, there exists a fuzzy \( \gamma \)-open set \( \mu \) in \( X \) containing \( x_\epsilon \) such that \( \text{int}(f(\mu)) \leq \eta \). Since \( f \) is fuzzy \((\gamma, s)\)-open, \( f(\mu) \) is a semi-open set in \( Y \) and so \( f(\mu) \leq \text{cl}(\text{int}(f(\mu))) \leq \eta \). This shows that \( f \) is fuzzy almost contra \( \gamma \)-continuous.

Definition 4.4. Let \( X \) and \( Y \) be fuzzy topological spaces. A fuzzy function \( f : X \rightarrow Y \) is said to be

1) fuzzy almost contra precontinuous [5] if the inverse image of each fuzzy regular open set in \( Y \) is fuzzy preclosed in \( X \),

2) fuzzy almost contra semicontinuous [5] if the inverse image of each fuzzy regular open set in \( Y \) is fuzzy semi-closed in \( X \),
(3) fuzzy almost contra continuous \([5]\) if the inverse image of each fuzzy regular open set in \(Y\) is fuzzy closed in \(X\),

(4) fuzzy almost contra \(\alpha\)-continuous if the inverse image of each fuzzy regular open set in \(Y\) is fuzzy \(\alpha\)-closed in \(X\),

(5) fuzzy almost contra \(\beta\)-continuous \([5]\) if the inverse image of each fuzzy regular open set in \(Y\) is fuzzy \(\beta\)-closed in \(X\).

**Remark 4.5.** We have the following diagram describing the properties of a fuzzy function \(f: X \to Y:\)

![Diagram](image)

where the numbers represent the properties noted against them.

(1) fuzzy almost contra continuous function

(2) fuzzy almost contra \(\alpha\)-continuous function

(3) fuzzy almost contra precontinuous function

(4) fuzzy almost contra semi-continuous function

(5) fuzzy almost contra \(\gamma\)-continuous function

(6) fuzzy almost contra \(\beta\)-continuous function

None of the above implications is reversible.

**Example 4.6.** Fuzzy almost contra \(\alpha\)-continuity \(\not\Rightarrow\) fuzzy almost contra continuity.

Let \(X\) be a nonempty set and \(C_a : X \to [0,1]\) be defined as \(C_{a}(x) = a \forall x \in X \text{ and } a \in [0,1]\). Then \(\tau_1 = \{C_0, C_{6/10}, C_1\}\) and \(\tau_2 = \{C_0, C_{3/10}, C_1\}\) are fuzzy topologies and \((X, \tau_1), (X, \tau_2)\) are fuzzy topological spaces. The identity function \(f : (X, \tau_1) \to (X, \tau_2)\) is fuzzy almost contra \(\alpha\)-continuous but not fuzzy almost contra continuous.
In \((X, \tau_2), C_{7/10}\) is the only fuzzy regular closed set other than \(C_0\) and \(C_1\). Also \(f^{-1}(C_{7/10}) = C_{7/10}\), \(f\) being the identity function. In \((X, \tau_1), C_{7/10}\) is fuzzy \(\alpha\)-open since \(C_{7/10} \leq C_1 = int(cl(int(C_{7/10})))\). Thus \(f\) is fuzzy almost contra \(\alpha\)-continuous.

Obviously \(C_{7/10}\) is not fuzzy open in \((X, \tau)\). Hence \(f\) is not fuzzy almost contra continuous.

**Example 4.7.** Fuzzy almost contra semi-continuity \(\leftrightarrow\) fuzzy almost contra continuity.

In Example 4.6, \(C_{7/10} \in FRC(X)\) in \((X, \tau_2)\). This is the only fuzzy regular closed set other than \(C_0\) and \(C_1\). And \(f^{-1}(C_{7/10}) = C_{7/10}\) is fuzzy \(\alpha\)-open and hence fuzzy semi-open in \((X, \tau_1)\) but not fuzzy open. This proves that \(f\) is fuzzy almost contra semi-continuous but not fuzzy almost contra continuous.

**Example 4.8.** Fuzzy almost contra semi continuity \(\leftrightarrow\) fuzzy almost contra \(\alpha\)-continuity.

Let \(X\) be a nonempty set and \(C_a : X \to [0, 1]\) be defined as \(C_a(x) = 0\forall x \in X\) and \(a \in [0, 1]\). Then \(\tau_1 = \{C_0, C_{2/10}, C_1\}\) and \(\tau_2 = \{C_0, C_{3/10}, C_1\}\) are fuzzy topologies. Then \(f : (X, \tau_1) \to (X, \tau_2)\), the identity function is fuzzy almost contra semi-continuous but not fuzzy almost contra \(\alpha\)-continuous.

In \((X, \tau_2)\), the only fuzzy regular closed set other than \(C_0\) and \(C_1\) is \(C_{7/10}\). And \(f^{-1}(C_{7/10}) = C_{7/10}\) in \((X, \tau_1)\), \(f\) being the identity function. \(C_{7/10} \leq C_{8/10} = cl(int(C_{7/10}))\). Thus \(C_{7/10}\) is fuzzy semi open in \((X, \tau_1)\) proving that \(f\) is fuzzy almost contra semicontinuous. But \(C_{7/10} \notin C_{2/10} = int(cl(int(C_{7/10})))\) and so \(C_{7/10}\) is not fuzzy \(\alpha\)-open in \((X, \tau_1)\). This proves that \(f\) is not fuzzy almost contra \(\alpha\)-continuous.

**Example 4.9.** Fuzzy almost contra precontinuity \(\leftrightarrow\) fuzzy almost contra \(\alpha\)-continuity.

Let \(X\) be a nonempty set and \(C_a : X \to [0, 1]\) be defined as \(C_a(x) = 0\forall x \in X\) and \(a \in [0, 1]\). The identity function \(f : (X, \tau_1) \to (X, \tau_2)\) where \(\tau_1 = \{C_0, C_{5/10}, C_1\}\) and \(\tau_2 = \{C_0, C_{3/10}, C_1\}\) is fuzzy almost contra precontinuous but not fuzzy almost contra \(\alpha\)-continuous. In \((X, \tau_2), C_{7/10}\) is the only fuzzy regular closed set other than \(C_0\) and \(C_1\). And \(f^{-1}(C_{7/10}) = C_{7/10}C_1 \leq C_1int(cl(C_{7/10}))\). Thus \(C_{7/10}\) is fuzzy preopen in \((X, \tau_1)\) and this proves that \(f\) is fuzzy almost contra precontinuous. But \(C_{7/10} \notin C_{5/10} = int(cl(int(C_{7/10})))\) which proves that \(C_{7/10}\) is not fuzzy \(\alpha\)-open in \((X, \tau_1)\) and hence \(f\) is not fuzzy almost contra \(\alpha\)-continuous.

**Example 4.10.** fuzzy almost contra \(\gamma\)-continuity \(\leftrightarrow\) fuzzy almost contra precontinuity.

In Example 4.8, the only fuzzy regular closed set in \((X, \tau_2)\) is \(C_{7/10}\), other
than $C_0$ and $C_1$. Also, $f$ being the identity $f^{-1}(C_{7/10}) = C_{7/10}$ in $(X, \tau_1). C_{7/10}$ is fuzzy semiopen in $(X, \tau_1)$ and hence fuzzy $\gamma$-open in $(X, \tau_1)$. Thus $f$ is fuzzy almost contra $\gamma$-continuous. But $C_{7/10} \not\subseteq \text{int}(\overline{\text{int}(C_{7/10})})$ which means that $C_{7/10}$ is not fuzzy preopen in $(X, \tau_1)$. This proves that $f$ is not fuzzy almost contra precontinuous.

**Example 4.11.** Fuzzy almost contra $\gamma$-continuity $\Rightarrow$ fuzzy almost contra semi-continuity.

In Example 4.9, $C_{7/10}$ is the only fuzzy regular closed set in $(X, \tau_2)$ other than $C_0$ and $C_1$. Also, $f$ being the identity $f^{-1}(C_{7/10}) = C_{7/10}$ in $(X, \tau_1). C_{7/10}$ is fuzzy preopen in $(X, \tau_1)$ and hence fuzzy $\gamma$-open in $(X, \tau_1)$. This proves that $f$ is fuzzy almost contra $\gamma$-continuous. But $C_{7/10} \not\subseteq C_{5/10} = \text{int}(\overline{\text{int}(C_{7/10})})$ which means that $C_{7/10}$ is not fuzzy semi-open. Hence $f$ is not contra fuzzy semi-continuous.

**Example 4.12.** In Example 4.8, $C_{7/10}$ is the only fuzzy regular closed set in $(X, \tau_2)$. Also $f$ being the identity function $f^{-1}(C_{7/10}) = C_{7/10}$ in $(X, \tau_1). C_{7/10} \leq C_{8/10} = \overline{\text{int}(\overline{\text{int}(C_{7/10})})}$ and hence $C_{7/10}$ is fuzzy $\beta$-open in $(X, \tau_1)$. This proves that $f$ is fuzzy almost contra $\beta$-continuous. But $C_{7/10} \not\subseteq C_{2/10} = \text{int}(\overline{\text{int}(C_{7/10})})$ which means that $C_{7/10}$ is not fuzzy preopen. Hence $f$ is not fuzzy almost precontinuous.

**Definition 4.13.** [5] A fuzzy space is said to be fuzzy $P_\Sigma$ if for any fuzzy open set $\mu$ of $X$ and each $x_\epsilon \in \mu$, there exists fuzzy regular closed set $\rho$ containing $x_\epsilon$ such that $x_\epsilon \in \rho \leq \mu$.

**Definition 4.14.** [9] A fuzzy function $f : X \rightarrow Y$ is said to be fuzzy $\gamma$-continuous if $f^{-1}(\mu)$ is fuzzy $\gamma$-open in $X$ for every fuzzy open set $\mu$ in $Y$.

**Theorem 4.15.** Let $f : X \rightarrow Y$ be a fuzzy function. Then, if $f$ is fuzzy almost contra $\gamma$-continuous and $Y$ is fuzzy $P_\Sigma$, then $f$ is fuzzy $\gamma$-continuous.

**Proof.** Let $\mu$ be any fuzzy open set in $Y$. Since $Y$ is fuzzy $P_\Sigma$, there exists a family $\psi$ whose members are fuzzy regular closed sets of $Y$ such that $\mu = \vee \{ \rho : \rho \in \psi \}$. Since $f$ is fuzzy almost contra $\gamma$-continuous, $f^{-1}(\rho)$ is fuzzy $\gamma$-open in $X$ for each $\rho \in \psi$ and $f^{-1}(\mu)$ is fuzzy $\gamma$-open in $X$. Therefore, $f$ is fuzzy almost contra $\gamma$-continuous.

**Definition 4.16.** [5] A space is said to be fuzzy weakly $P_\Sigma$ if for any fuzzy regular open set $\mu$ and each $x_\epsilon \in \mu$, there exists a fuzzy regular closed set $\rho$ containing $x_\epsilon$ such that $x_\epsilon \in \rho \leq \mu$.

**Definition 4.17.** A fuzzy function $f : X \rightarrow Y$ is said to be fuzzy almost
\(\gamma\)-continuous at \(x_\epsilon \in X\) if for each fuzzy open set \(\eta\) containing \(f(x_\epsilon)\), there exists a fuzzy \(\gamma\)-open set \(\mu\) containing \(x_\epsilon\) such that \(f(\mu) \subseteq \text{int}(\text{cl}(\eta))\).

**Theorem 4.18.** Let \(f : X \to Y\) be a fuzzy almost contra \(\gamma\)-continuous function. If \(Y\) is fuzzy weakly \(P_\Sigma\), then \(f\) is fuzzy almost \(\gamma\)-continuous.

**Proof.** Let \(\mu\) be any fuzzy regular open set of \(Y\). Since \(Y\) is fuzzy weakly \(P_\Sigma\), there exists a family \(\psi\) whose members are fuzzy regular closed sets of \(Y\) such that \(\mu \vee \{\rho : \rho \in \psi\}\). Since \(f\) is fuzzy almost contra \(\gamma\)-continuous, \(f^{-1}(\rho)\) is fuzzy \(\gamma\)-open in \(X\) for each \(\rho \in \psi\) and \(f^{-1}(\mu)\) is fuzzy \(\gamma\)-open in \(X\). Hence, \(f\) is fuzzy almost \(\gamma\)-continuous.

**Definition 4.19.** [9] A fuzzy function \(f : X \to Y\) is called fuzzy \(\gamma\)-irresolute if the inverse image of each fuzzy \(\gamma\)-open set is fuzzy \(\gamma\)-open.

**Theorem 4.20.** Let \(X, Y, Z\) be fuzzy topological spaces and let \(f : X \to Y\) and \(g : Y \to Z\) be fuzzy functions. If \(f\) is fuzzy \(\gamma\)-irresolute and \(g\) is fuzzy almost contra \(\gamma\)-continuous, then \(g \circ f : X \to Z\) is a fuzzy almost contra \(\gamma\)-continuous function.

**Proof.** Let \(\mu \leq Z\) be any fuzzy regular closed set. Since \(g\) is fuzzy almost contra \(\gamma\)-continuous, \(g^{-1}(\mu)\) is fuzzy \(\gamma\)-open in \(Y\). But \(f\) is fuzzy \(\gamma\)-irresolute \(\Rightarrow f^{-1}(g^{-1}(\mu))\) is fuzzy \(\gamma\)-open in \(X\). Thus \((g \circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))\) is fuzzy \(\gamma\)-open in \(X\) and this proves that \(g \circ f\) is a fuzzy almost contra \(\gamma\)-continuous function.

**Definition 4.21.** A fuzzy function \(f : X \to Y\) is called always fuzzy \(\gamma\)-open [20] if the image of each fuzzy \(\gamma\)-open set is fuzzy \(\gamma\)-open.

**Theorem 4.22.** If \(f : X \to Y\) is a surjective always fuzzy \(\gamma\)-open function and \(g : Y \to Z\) is a fuzzy function such that \(g \circ f : X \to Z\) is fuzzy almost contra \(\gamma\)-continuous, then \(g\) is fuzzy almost contra \(\gamma\)-continuous.

**Proof.** Let \(\mu \leq Z\) be any fuzzy regular closed set. Since \(g \circ f\) is fuzzy almost contra \(\gamma\)-continuous, \((g \circ f)^{-1}(\mu)\) is fuzzy \(\gamma\)-open in \(X\). Therefore \(f^{-1}(g^{-1}(\mu)) = (g \circ f)^{-1}(\mu)\) is fuzzy \(\gamma\)-open in \(X\). \(f\) is always fuzzy \(\gamma\)-open surjection implies \(f(f^{-1}(g^{-1}(\mu))) = g^{-1}(\mu)\) is fuzzy \(\gamma\)-open in \(Y\). Thus \(g\) is fuzzy almost contra \(\gamma\)-continuous.

**Corollary 4.23.** Let \(f : X \to Y\) be a surjective fuzzy \(\gamma\)-irresolute and always fuzzy \(\gamma\)-open function and let \(g : Y \to Z\) be a fuzzy function. Then, \(g \circ f : X \to Z\) is fuzzy almost contra \(\gamma\)-continuous if and only if \(g\) is fuzzy almost contra \(\gamma\)-continuous.

**Proof.** It can be obtained from Theorem 4.20 and Theorem 4.22.
Definition 4.24. A space $X$ is said to be fuzzy $\gamma$-compact [20] (fuzzy $S$-closed [3]) if every fuzzy $\gamma$-open (respectively fuzzy regular closed) cover of $X$ has a finite subcover.

Theorem 4.25. The fuzzy almost contra $\gamma$-continuous image of a fuzzy $\gamma$-compact space is fuzzy $S$-closed.

Proof. Suppose that $f: X \to Y$ is a fuzzy almost contra $\gamma$-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular closed cover of $Y$. Since $f$ is fuzzy almost contra $\gamma$-continuous, $\{f^{-1}(\eta_i) : i \in I\}$ is a fuzzy $\gamma$-open cover of $X$ and $X$ being fuzzy $\gamma$-compact, there exists a finite subset $I_o$ of $I$ such that $X = \bigvee \{f^{-1}(\eta_i) : i \in I_o\}$. Since $f$ is surjective, we have $Y = \bigvee \{\eta_i : i \in I_0\}$ and thus $Y$ is fuzzy $S$-closed.

Definition 4.26. A space $X$ is said to be

1. fuzzy $\gamma$-closed-compact [20] if every fuzzy $\gamma$-closed cover of $X$ has a finite subcover,

2. fuzzy nearly compact [7] if every fuzzy regular open cover of $X$ has a finite subcover.

Theorem 4.27. The fuzzy almost contra $\gamma$-continuous image of a fuzzy $\gamma$-closed-compact space is fuzzy nearly compact.

Proof. Suppose that $f: X \to Y$ is a fuzzy almost contra $\gamma$-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular open cover of $Y$. Since $f$ is fuzzy almost contra $\gamma$-continuous, $\{f^{-1}(\eta_i) : i \in I\}$ is a fuzzy $\gamma$-closed cover of $X$. Since $X$ is fuzzy $\gamma$-closed-compact, there exists a finite subset $I_0$ of $I$ such that $X = \bigvee \{f^{-1}(\eta_i) : i \in I_0\}$. Thus, we have $Y = \bigvee \{\eta_i : i \in I_0\}$ and $Y$ is fuzzy nearly compact.

Definition 4.28. Let $f: (X, \tau) \to (Y, \sigma)$ be a function. Then $f$ is said to be

1. fuzzy semi-open [22] if the image of every fuzzy open set of $X$ is fuzzy semiopen in $Y$.

2. fuzzy $\alpha$-open [19] if the image of every fuzzy open set of $X$ is fuzzy $\alpha$-open in $Y$.

3. fuzzy preopen [22] if the image of every fuzzy open set of $X$ is fuzzy preopen in $Y$.

4. fuzzy $\gamma$-open [9] if the image of every fuzzy open set of $X$ is fuzzy $\gamma$-open in $Y$. 
(5) fuzzy \((LC, s)\) if the image of every fuzzy open set of \(X\) is fuzzy \(LC\) set in \(Y\).

**Theorem 4.29.** For a fuzzy function \(f : X \rightarrow Y\), where \(Y\) is a fuzzy extremally disconnected space, the following properties are equivalent.

1. \(f\) is fuzzy open.
2. \(f\) is fuzzy \(\alpha\)-open and a fuzzy \((LC, s)\).
3. \(f\) is fuzzy preopen and a fuzzy \((LC, s)\).
4. \(f\) is fuzzy semi-open and a fuzzy \((LC, s)\).
5. \(f\) is fuzzy \(\gamma\)-open and a fuzzy \((LC, s)\).

**Definition 4.30.** Let \(f : X \rightarrow Y\) be a fuzzy function. Then \(f\) is said to be

1. fuzzy almost continuous \([1]\) if the inverse image of every fuzzy regular open set of \(Y\) is fuzzy open in \(X\).
2. fuzzy almost \(\alpha\)-continuous \([15]\) if the inverse image of every fuzzy regular open set of \(Y\) is fuzzy \(\alpha\)-open in \(X\).
3. fuzzy almost semicontinuous \([13]\) if the inverse image of every fuzzy regular open set of \(Y\) is fuzzy semi-open in \(X\).
4. fuzzy almost precontinuous \([21]\) if the inverse image of every fuzzy regular open set of \(Y\) is fuzzy preopen in \(X\).
5. fuzzy almost \(\gamma\)-continuous if the inverse image of every fuzzy regular open set of \(Y\) is fuzzy \(\gamma\)-open in \(X\).
6. fuzzy almost \((LC, s)\)-continuous if the inverse image of every fuzzy regular open set of \(Y\) is a fuzzy \(LC\) set in \(X\).

**Theorem 4.31.** For a fuzzy function \(f : X \rightarrow Y\), where \(X\) is a fuzzy extremally disconnected space, the following properties are equivalent.

1. \(f\) is a fuzzy almost continuous.
2. \(f\) is fuzzy almost \(\alpha\)-continuous and a fuzzy \((LC, s)\)-continuous.
3. \(f\) is fuzzy almost precontinuous and a fuzzy \((LC, s)\)-continuous.
(4) \( f \) is fuzzy almost semicontinuous and a fuzzy \((LC, s)\)-continuous.

(5) \( f \) is fuzzy almost \(\gamma\)-continuous and a fuzzy \((LC, s)\)-continuous.

**Definition 4.32.** Let \( f : X \rightarrow Y \) be a fuzzy function. Then \( f \) is said to be fuzzy almost contra \((LC, s)\)-continuous if the inverse image of every fuzzy regular closed set of \( Y \) is fuzzy \( LC \) set in \( X \).

**Theorem 4.33.** For a fuzzy function \( f : X \rightarrow Y \), where \( X \) is a fuzzy extremally disconnected space, the following properties are equivalent.

(1) \( f \) is fuzzy almost contra continuous.

(2) \( f \) is fuzzy almost contra \(\alpha\)-continuous and fuzzy contra \((LC, s)\)-continuous.

(3) \( f \) is fuzzy almost contra precontinuous and fuzzy contra \((LC, s)\)-continuous.

(4) \( f \) is fuzzy almost contra semicontinuous and fuzzy contra \((LC, s)\)-continuous.

(5) \( f \) is fuzzy almost contra \(\gamma\)-continuous and fuzzy contra \((LC, s)\)-continuous.

**References**


