International Journal of Pure and Applied Mathematics
Volume 86 No. 1 2013, 1-6
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v86i1.1

ijpam.eu

IRREGULAR VECTORS AND SUBSPACE-HYPERCYCLIC OPERATORS

Mansooreh Moosapoor
Payame Noor University
P.O. Box 19395-4697, Tehran, IRAN

Abstract: In this paper we show that if T is subspace-hypercyclic or subspace-transitive with respect to a subspace M, then T^n has a dense set of irregular vectors in M for every $n \in N$. Also, we prove that if T is hyponormal or subnormal or normal or compact, for every $n \in N$, T^n can not be subspace-hypercyclic and subspace-chaotic.

AMS Subject Classification: 47A16, 47B37, 37B99
Key Words: irregular vectors, subspace-hypercyclic operators, subspace-transitive operators, subspace-chaotic operators

1. Introduction

In this paper we will be concerned with a certain type of behavior of orbits of bounded linear operators on a complex separable Hilbert space H.We show the algebra of all bounded linear operators on H by $B(H)$.

If $T \in B(H)$ and $x \in H$, then the set

$$orb(T, x) = \{x, Tx, T^2x, ...\}$$

is called the orbit of the vector x under the operator T.

Let M be a nonzero and closed subspace of H. We say that T is subspace-hypercyclic for M if there exists $x \in H$ such that $orb(T, x) \cap M$ is dense in M. Such a vector x is called a subspace-hypercyclic vector for T. We say T is M -transitive, if for any non-empty open sets $U, V \subseteq M$, both relatively open, there exists $n \in N_0$ such that $U \cap T^{-n}(V)$ contains a relatively open, non-empty

Received: October 23, 2012

subset of M. These operators introduced by Madore and Avendano in [4] in 2011. They proved in [4] that if T is subspace-hypercyclic with respect to a subspace M, then M must be infinite dimensional. You can also see [3] and [7] fore more information.

As in [5] an irregular vector of an operator T is a vector x such that $limsup_n||T^n x|| = \infty$ and $liminf_n||T^n x|| = 0$ or equivalently $sup_n||T^n x|| = \infty$ and $inf_n||T^n x|| = 0$. You can find some properties of these vectors in [5] and [2]. Obviously subspace-hypercyclic vectors are irregular vectors.

In all of this paper, M is a closed and infinite dimensional subspace of H.

2. Preliminaries

Theorem 2.1. (see [4]) *Let $T \in B(H)$. The following conditions are equivalent:*

(i) T is subspace-transitive with respect to M.

(ii) For all non-empty sets $U \subseteq M$, $V \subseteq M$ both relatively open, there exists $n \in N_0$ such that $T^{-n}(U) \cap V$ is a relatively open non-empty subset of M.

(iii) For all non-empty sets $U \subseteq M$, $V \subseteq M$ both relatively open, there exists $n \in N_0$ such that $T^{-n}(U) \cap V$ is non-empty and $T^n(M) \subseteq M$.

Lemma 2.2. (see [4]) *Let $T \in B(X)$ be M- transitive. Then T has a dense set of M-hypercyclic vectors in M.*

Remark 2.3. By the previous theorem if T is M-transitiv, then it has a dense subset of irregular vectors in M, since as you see M- hypercyclic vectors are irregular vectors.

Theorem 2.4. (see [4]) *Let $T \in B(H)$. If T is subspace-transitive with respect to M then T is subspace-hypercyclic with respect to M.*

Lemma 2.5. (see [5]) *x is an irregular vector of T if and only if $T^k x$ is an irregular vector of T for some k if and only if $T^k x$ is an irregular vector of T for every k.*

Theorem 2.6. (see [5]) *x is an irregular vector of T if and only if x is an irregular vector of T^m for every m.*

By Theorem 2.6, we have the following corollary.

Corollary 2.7. *Let $T \in B(H)$. If T is M-transitive then T^n has a dense set of irregular vectors in M for every $n \in N$.*

Proof. By Remark 2.3, T has a dense subset of irregular vectors in M. Now theorem2.6 says us these irregular vectors are irregular vectors for every T^n. So every T^n has a dense set of irregular vectors in M. □

Definition 2.8. (see [6]) Let $T \in B(X)$ where (X, d) is a metric space. We say T is M-chaotic if:

1) T is M- transitive;

2) T has a dense set of periodic points in M;

3) T has a M-sensitive dependense on initial conditions.

Corollary 2.9. *If $T \in B(H)$ is a M-chaotic operator then T^n has a dense set of irregular vectors in M for every $n \in N$.*

Proof. By the definition: T is M-transitive. So the proof is clear by using Corollary 2.7. □

3. Main Results

Theorem 3.1. *If $T \in B(H)$ is subspace-hypercyclic with respect to M, then T has a dense set of irregular vectors in M.*

Proof. Let x be a M -hypercyclic vector for T. So x is an irregular vector for T. By lemma2.5 for every k, $T^k x$ is an irregular vector for T. Therefore every member of $orb(T, x) \cap M$ is an irregular vector for T and as we know $orb(T, x) \cap M$ is dense in M. □

Theorem 3.2. (see [5]) *If $T \in B(H)$ has irregular vectors, then the spectrum of T must intersect the unit circle, S^1.*

Madore and avendano proved in [4] that if is subspace-hypercyclic with respect to a subspace, then $\sigma(T) \cap S^1 \neq \phi$. Now we extend this statement as follows:

Theorem 3.3. *If $T \in B(H)$ is subspace-hypercyclic or subspace-transitive with respect to a subspace, then $\sigma(T^n) \cap S^1 \neq \phi$ for every $n \in N$.*

Proof. As we show before, if T is subspace-hypercyclic or subspace-transitive with respect to a subspace, for every $n \in N$, T^n has irregular vectors. So by theorem3.2, the spectrum of T^n must intersect the unit circle, S^1. □

Corollary 3.4. (see [5]) *Hyponormal operators, subnormal operators and normal operators, don't have irregular vectors.*

Theorem 3.5. (see [5]) *Compact operators, do not have irregular vectors.*

Theorem 3.6. *If T is hyponormal or subnormal or normal or compact, then T^n can not be subspace- hypercyclic for every $n \in N$.*

Proof. First suppose that T is hyponormal. Suppose that there is $m \in N$ and a closed subspace M of H such that T^m is M-hypercyclic. So T is M-hypercyclic too and hence T has a dense set of irregular vectors in M by theorem 3.1. But this is a cotradiction since by corollary3.4, hyponormal operators don't have irregular vectors.

The remain of proof is similar, by using Corollary 3.4 and Theorem 3.5. □

Theorem 3.7. *Let $T \in B(H)$ and M be a closed subspace of H. If T^n is M-chaotic for some $n \in N$, then:*

1) T is not compact.

2) T is not normal.

3) T is not subnormal.

4) T is not hyponormal.

5) $\sigma(T) \cap S^1 \neq \phi$.

Proof. Suppose that there is $n \in N$ such that T^n is M-chaotic. So T^n and hence T is M -transitive. Therefore T has a dense set of irregular vectors in M. So by theorem3.6, T can not be hyponormal or subnormal or normal or compact and by theorem3.3, $\sigma(T) \cap S^1 \neq \phi$. □

Lemma 3.8. (see [5]) *If $S \oplus T$ has irregular vectors, then at least one of S or T has irregular vectors.*

Definition 3.9. (see [6]) Let $T \in B(H)$ and M be a closed non-zero subspace of H. We say T is M-weakly mixing, if $T \oplus T$ is M-transitive.

By Lemma 3.8 and Definition 3.9, the proof of following corollary is clear.

Corollary 3.10. *Subspace-weakly mixing operators have irregular vectors.*

Remark 3.11. By corollary3.10, if T is subspace-weakly mixing, then:

1) T is not compact.

2) T is not normal.

3) T is not subnormal.

4) T is not hyponormal.

5) $\sigma(T) \cap S^1 \neq \phi$.

Definition 3.12. (see [1]) A linear manifold $Y \subseteq H$ is an irregular manifold for $T \in B(H)$, if every vector $y \in Y - \{0\}$ is an irregular vector for T.

Theorem 3.13. (see [1]) *Let $T \in B(H)$. If $sup_n ||T^n|| = \infty$ and there is a dense subset $H_0 \subseteq H$ such that for each $x \in H_0$, $lim_{n\to\infty} T^n x = 0$, then T admits a dense irregular manifold.*

Theorem 3.14. *Let $T \in B(H)$ be a M-hypercyclic operator whose generalized kernel, $\bigcup_{n=1}^{\infty} ker(T^n)$, is dense in H. Then T has a dense irregular manifold in H.*

Proof. By hypothesis T is M-hypercyclic, so $sup_n ||T^n|| = \infty$. If $x \in \bigcup_{n=1}^{\infty} ker(T^n)$, we have $lim_{n\to\infty} T^n x = 0$. So by theorem3.13, T admits a dense irregular manifold. □

References

[1] T. Bermudes, A. Bonilla, F. Martinez-Gimenez, A. Peris, Li-Yorke and distributionally chaotic operators, *J. Math. Anal. Appl.*, **373** (2011), 83-93.

[2] K.-G. Grosse-Erdmann, A. Peris Manguillot, *Linear Chaos*, Springer (2011).

[3] C.M. Le, On subspace-hypercyclic operators, *Proc. Amer. Math. Soc.*, **B9** (2011), 2847-2852.

[4] B.F. Madore, R.A. Martinez-Avendano, Subspace hypercyclicity, *J. Math. Anal. Appl.*, **373**, No. 2 (2011), 502-511.

[5] Gabriel T. Prajitura, Irregular vectors of Hilbert space operators, *J. Math. Anal. Appl.*, **354** (2009), 689-697.

[6] S. Talebi, M. Moosapoor, Subspace-chaotic operators and subspace-weakly mixing operators, *International Journal of Pure and Applied Mathematics*, **78**, No. 6 (2012), 879-885.

[7] S. Talebi, M. Moosapoor, Some results on subspace-hypercyclic operators, *Pioneer Journal of Mathematics and Mathematical Sciences*, **5**, No. 1 (2012), 79-85.