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A NEW HYBRID BLOCK METHOD FOR THE SOLUTION OF GENERAL THIRD ORDER INITIAL VALUE PROBLEMS OF ORDINARY DIFFERENTIAL EQUATIONS

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Abstract: In this paper, we develop an order six block method using method of collocation and interpolation of power series approximate solution to give a system of non linear equations which is solved to give a continuous hybrid linear multistep method . The continous hybrid linear multistep method is solved for the independent solutions to give a continous hybrid block method which is then evaluated at some selected grid points to give a discrete block method . The basic properties of the discrete block method was investigated and found to be zero stable, consistent and convergent. The derived scheme was tested on some numerical examples and was found to give better approximation than the existing method.

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1. Introduction

This paper considers an approximate method for the solution of general third order initial value problems of the form

$$y^{'''} = f(x, y, y', y^{''}), \quad y^k(x_n) = y_n^k, \quad k = 0, 1, 2, \tag{1}$$

where x_n is the initial point ,y_n is the solution at x_n, f is contimuous within the interval of integration.

Equation (1) has a wide application in Enginerring, Thermodynamics and other real life problems, hence the study of methods of solving (1) is important to researchers.

Direct method for solving (1) has been reported to be more effcient than the method of reduction to system of first order ordinary differential equations (see[1], [2], [3], [4]).

Implicit linear multistep method which has better stability condition than explict method are solved using predictor corrector method. [5,6,7,8], among others proposed multi-derivative linear multistep method which is implemented in predictor-corrector mode. Despite the success recorded by this method, the major setback is that the predictors are in reducing order of accuracy, hence, the method does not give better approximation apart from the computational burden associated with the method.

Scholars later developed block method which cater for some of the setbacks of the predictor corrector method.[4,9,10,11], individually developed block method using different approximate solutions. It was found out that block method is more efficient in term of time of execution, cost effectiveness and accuracy than the predictor-corrector method.

In this paper, we propose an hybrid method with constant step-size implemented in block method. The paper is organised as follows: Chapter two considers the method and the materials for the development of method. Chapter three considers the analysis of the basic properties of the method which include, zero stability, consistency and convergent. Chapter four considers numerical examples where the effciency of the derived method is tested on some numerical examples. Chapter five considers the discussion of results and the finally chapter six is the conclusion.

2. Methods and Materials

We consider a power series approximate solution in the form

$$y(x) = \sum_{j=0}^{r+s-1} a_j x^j. \tag{2}$$

Substituting the third derivative of (2) into (1) gives

$$f\left(x, y, y', y''\right) = \sum_{j=2}^{r+s-1} j\,(j-1)\,(j-2) a_j x^{j-3}. \tag{3}$$

In this paper, we consider a grid of steplenght of two with constant step size (h) where $h = x_{n+i} - x_i, i = 0(1)2$ and off step points at $x_{n+\frac{1}{2}}$ and $x_{n+\frac{3}{2}}$

Collocating (3) at all points and interpolating (2) at $x_{n+\frac{1}{2}}, x_{n+1}$ and $x_{n+\frac{3}{2}}$ give a systems of non linear equation in the form

$$AX = U, \tag{4}$$

where

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \end{bmatrix}^T,$$

$$U = \begin{bmatrix} y_{n+\frac{1}{2}} & y_{n+1} & y_{n+\frac{3}{2}} & f_n & f_{n+\frac{1}{2}} & f_{n+1} & f_{n+\frac{3}{2}} & f_{n+2} \end{bmatrix}^T,$$

and

$$X = \begin{bmatrix} 1 & x_{n+\frac{1}{2}} & x^2_{n+\frac{1}{2}} & x^3_{n+\frac{1}{2}} & x^4_{n+\frac{1}{2}} & x^5_{n+\frac{1}{2}} & x^6_{n+\frac{1}{2}} & x^7_{n+\frac{1}{2}} \\ 1 & x_{n+1} & x^2_{n+1} & x^3_{n+1} & x^4_{n+1} & x^5_{n+1} & x^6_{n+1} & x^7_{n+1} \\ 1 & x_{n+\frac{3}{2}} & x^2_{n+\frac{3}{2}} & x^3_{n+\frac{3}{2}} & x^4_{n+\frac{3}{2}} & x^5_{n+\frac{3}{2}} & x^6_{n+\frac{3}{2}} & x^7_{n+\frac{3}{2}} \\ 0 & 0 & 0 & 6 & 24x_n & 60x^2_n & 120x^3_n & 210x^4_n \\ 0 & 0 & 0 & 6 & 24x_{n+\frac{1}{2}} & 60x^2_{n+\frac{1}{2}} & 120x^3_{n+\frac{1}{2}} & 210x^4_{n+\frac{1}{2}} \\ 0 & 0 & 0 & 6 & 24x_{n+1} & 60x^2_{n+1} & 120x^3_{n+1} & 210x^4_{n+1} \\ 0 & 0 & 0 & 6 & 24x_{n+\frac{3}{2}} & 60x^2_{n+\frac{3}{2}} & 120x^3_{n+\frac{3}{2}} & 210x^4_{n+\frac{3}{2}} \\ 0 & 0 & 0 & 6 & 24x_{n+2} & 60x^2_{n+2} & 120x^3_{n+2} & 210x^4_{n+2} \end{bmatrix}.$$

Solving (5) for $a_j's$ which are constant to be determined and putting back into (2) gives a continous hybrid multistep method of the form

$$y(x) = \alpha_{\frac{1}{2}} y_{n+\frac{1}{2}} + \alpha_1 y_{n+1} + \alpha_{\frac{3}{2}} y_{n+\frac{3}{2}}$$

$$+\, h^3 \left[\sum_{j=0}^{2} \beta_j f_{n+j} + \beta_{\frac{1}{2}} f_{n+\frac{1}{2}} + \beta_{\frac{3}{2}} f_{n+\frac{3}{2}} \right], \quad (5)$$

where

$$\alpha_{\frac{1}{2}} = 2t^2 - 5t + 13,\ \ \alpha_1 = -4t^2 + 8t - 3,\ \ \alpha_{\frac{3}{2}} = 2t^2 - 3t + 1,$$

$$\beta_0 = \frac{1}{40320}\left(128t^7 - 1120t^6 + 3920t^5 - 7000t^4 + 6720t^3 - 3304t^2 + 677t - 21\right),$$

$$\beta_{\frac{1}{2}} = -\frac{1}{10080}\left(128t^7 - 1008t^6 + 2912t^5 + 2912t^4 - 3339t^3 - 2620t + 609\right),$$

$$\beta_1 = \frac{1}{6720}\left(128t^7 - 896t^6 + 2128t^5 - 1680t^4 - 448t^2 + 1209t - 411\right),$$

$$\beta_{\frac{3}{2}} = -\frac{1}{10080}\left(128t^7 - 784t^6 + 1568t^5 - 1120t^4 + 245t^2 - 16t - 21\right),$$

$$\beta_2 = \frac{1}{240320}\left(128t^7 - 672t^6 + 1232t^5 - 840t^4 + 168t^2 + 5t - 21\right),$$

where $t = \frac{x-x_n}{h}$.

Solving (6) for the independent solution gives the continuous hybrid block method of the form

$$y(x) = \sum_{i=0}^{2} \frac{(jh)^{(i)}}{i!} y_n^{(i)} + h^3 \left(\sum_{k=0}^{2} \sigma_k f_{n+k} + \sigma_{\frac{1}{2}} f_{n+\frac{1}{2}} + \sigma_{\frac{3}{2}} f_{n+\frac{3}{2}} \right),$$

$$j = \frac{1}{2}, 1, \frac{3}{2}, 2, \quad (6)$$

where the coefficient of f_{n+k} give

$$\sigma_0 = \frac{1}{5040}\left(16t^7 - 140t^6 + 490t^5 - 875t^4 + 840t^3\right),$$

$$\sigma_{\frac{1}{2}} = -\frac{1}{630}\left(8t^7 - 63t^6 + 182t^5 - 210t^4\right),$$

$$\sigma_1 = -\frac{1}{420}\left(8t^7 - 56t^6 + 133t^5 - 105t^4\right),$$

$$\sigma_{\frac{3}{2}} = -\frac{1}{630}\left(8t^7 - 49t^6 + 98t^5 - 70t^4\right),$$

$$\sigma_2 = \frac{1}{5040}\left(16t^7 - 84t^6 + 154t^5 - 105t^4\right).$$

Evaluating (7) at $t = \frac{1}{2}, 1, \frac{3}{2}$ and 2 gives a discrete block formula of the form

$$\mathbf{A}^{(0)}\mathbf{Y}_m^{(i)} = \sum_{i=0}^{2} \frac{(jh)^{(i)}}{i!} e_i y_n^{(i)} + h^{(3-i)} \left[\mathbf{d}_i f(y_n) + \mathbf{b_i F}\left(\mathbf{Y}_m\right)\right], \tag{7}$$

where

$$\begin{aligned} \mathbf{Y}_m^{(i)} &= \begin{bmatrix} y_{n+\frac{1}{2}}^{(i)} & y_{n+1}^{(i)} & y_{n+\frac{3}{2}}^{(i)} & y_{n+2}^{(i)} \end{bmatrix}^T, \\ F(\mathbf{Y}_m) &= \begin{bmatrix} f_{n+\frac{1}{4}} & f_{n+\frac{1}{2}} & f_{n+\frac{3}{4}} & f_{n+\frac{3}{4}} \end{bmatrix}^T, \\ \mathbf{Y}_n^{(i)} &= \begin{bmatrix} y_{n-\frac{1}{2}}^{(i)} & y_{n+1}^{(i)} & y_{n+\frac{3}{2}}^{(i)} & y_{n+2}^{(i)} \end{bmatrix}^T, \end{aligned}$$

and $A^{(0)} = 4 \times 4$ Identity matrix.

When $i = 0$:

$$\mathbf{e}_0 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{e}_1 = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{3}{2} \\ 0 & 0 & 0 & 2 \end{bmatrix}, \mathbf{e}_2 = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{8} \\ 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{9}{8} \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\mathbf{d}_0 = \begin{bmatrix} 0 & 0 & 0 & \frac{133}{8960} \\ 0 & 0 & 0 & \frac{331}{630} \\ 0 & 0 & 0 & \frac{1431}{8960} \\ 0 & 0 & 0 & \frac{31}{105} \end{bmatrix}, \quad \mathbf{b_0} = \begin{bmatrix} \frac{107}{8064} & -\frac{103}{13440} & \frac{43}{13440} & \frac{-47}{80640} \\ \frac{83}{630} & \frac{-1}{21} & \frac{13}{630} & \frac{-19}{5040} \\ \frac{1863}{4480} & \frac{-243}{4480} & \frac{45}{896} & \frac{-81}{8960} \\ \frac{272}{315} & \frac{4}{105} & \frac{16}{105} & \frac{-1}{63} \end{bmatrix}.$$

When $i = 1$:

$$\mathbf{e}_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{3}{2} \\ 0 & 0 & 0 & 2 \end{bmatrix} \mathbf{d_1} = \begin{bmatrix} 0 & 0 & 0 & \frac{367}{5760} \\ 0 & 0 & 0 & \frac{53}{360} \\ 0 & 0 & 0 & \frac{147}{640} \\ 0 & 0 & 0 & \frac{14}{45} \end{bmatrix},$$

$$\mathbf{b_1} = \begin{bmatrix} \frac{3}{32} & \frac{-47}{960} & \frac{29}{1440} & \frac{-7}{1920} \\ \frac{2}{5} & \frac{-1}{12} & \frac{2}{45} & \frac{-1}{120} \\ \frac{117}{160} & \frac{27}{320} & \frac{3}{32} & -\frac{9}{640} \\ \frac{16}{15} & \frac{4}{15} & \frac{16}{45} & 0 \end{bmatrix}.$$

When $i = 2$:

$$\mathbf{e}_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{d_2} = \begin{bmatrix} 0 & 0 & 0 & \frac{251}{1440} \\ 0 & 0 & 0 & \frac{29}{180} \\ 0 & 0 & 0 & \frac{27}{160} \\ 0 & 0 & 0 & \frac{7}{45} \end{bmatrix},$$

$$\mathbf{b_2} = \begin{bmatrix} \frac{323}{720} & \frac{-11}{60} & \frac{53}{720} & \frac{-19}{1440} \\ \frac{31}{45} & \frac{2}{15} & \frac{1}{45} & \frac{-1}{180} \\ \frac{51}{80} & \frac{9}{20} & \frac{21}{80} & \frac{-3}{160} \\ \frac{32}{45} & \frac{4}{15} & \frac{32}{45} & \frac{7}{45} \end{bmatrix}.$$

3. Implementation of the Method

In order to implement the method, we propose a prediction equation of the form

$$Y_m^{(0)} = \sum_{i-0}^{2} \frac{(jh)^i}{i!} y_n^{(i)} + h^3 \sum_{\lambda=0}^{2} \frac{\partial\lambda}{\partial x^\lambda} f(x, y, y', y'')_{(x_0, y_0, y_0', y_0'')}, \tag{8}$$

where $Y_m^{(0)} = Y_{m_{(x_0, y_0, y_0', y_0'')}}$.

Substituting (8) into (7) gives

$$\mathbf{A}^{(0)}\mathbf{Y}_m^{(i)} = \sum_{i=0}^{2} \frac{(jh)^{(i)}}{i!} y_n^{(i)} + h^{(3-i)} \left[\mathbf{d}_i f(y_n) + \mathbf{b_i}\mathbf{F}(\mathbf{Y}_m)\right]. \tag{9}$$

Equation (9) is our block method which is implemented as a simultaneous integrator in this paper.

It should be noted that (10) is different from the self starting method proposed by [12]. In this case, our method can be referred to as a non self starting block method because the prediction equation required partial differential hence it is not gotten directly from (7) as claimed by [12].

4. Analysis of our Block Method

4.1. Order of the Block

Let the linear operator $\Delta\{y(x) : h\}$ be defined on (7) when $i = 0$ such that

$$\Delta\{y(x) : h\} = \mathbf{A}^{(0)}\mathbf{Y}_m - \sum_{i=0}^{2} \frac{(jh)^{(i)}}{i!} y_n^{(i)} - h^{(3-i)} \left[\mathbf{d}_i f(y_n) + \mathbf{b_i}\mathbf{F}(\mathbf{Y}_m)\right]. \tag{10}$$

Expand (11) $\mathbf{Y}_m$ and $\mathbf{F}(\mathbf{Y}_m)$ in Taylor series and comparing the coefficients of h gives

$$\begin{aligned}\Delta\{y(x):h\} &= C_0 y(x) + C_1 y'(x) \\ &\quad + \ldots + C_p h^p y^p(x) + C_{p+1} h^{p+1} y^{p+1}(x) + C_{p+2} h^{p+2} y^{p+2}(x) + \ldots\end{aligned}$$

(see [13])

Definition 1. The linear operator Δ and associated block formula (7) are said to be of order p if $C_0 = C_1 = \ldots = C_p = C_{p+1} = 0$, $C_{p+2} \neq 0$. C_{p+2} is called the error constant and implies that the truncation error is given by $t_{n+k} = C_{p+2} h^{p+2} y^{p+2}(x) + O\left(h^{p+3}\right)$.

For our method, expanding (7) in Taylor series gives:

$$\begin{bmatrix} \sum\limits_{j=0}^{\infty} \frac{\left(\frac{1}{2}h\right)^j}{j!} - y_n - \frac{1}{2}hy_n' - \frac{1}{8}h^2 y_n'' - \frac{113}{8960}h^3 y_n''' - \sum\limits_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \begin{bmatrix} \frac{107}{8064}\left(\frac{1}{2}\right)^j - \frac{103}{13440}(1)^j \\ + \frac{43}{13440}\left(\frac{3}{2}\right)^j - \frac{47}{80640}(2)^j \end{bmatrix} \\ \sum\limits_{j=0}^{\infty} \frac{(h)^j}{j!} - y_n - hy_n' - \frac{1}{2}h^2 y_n'' - \frac{331}{630}h^3 y''' - \sum\limits_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \begin{bmatrix} \frac{83}{630}\left(\frac{1}{2}\right)^j - \frac{1}{21}(1)^j \\ + \frac{13}{630}\left(\frac{3}{2}\right)^j - \frac{19}{5040}(2)^j \end{bmatrix} \\ \sum\limits_{j=0}^{\infty} \frac{\left(\frac{3}{2}h\right)^j}{j!} - y_n - \frac{3}{2}hy_n' - \frac{9}{8}h^2 y_n'' - \frac{1431}{8969}h^3 y_n''' - \sum\limits_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \begin{bmatrix} \frac{1863}{4480}\left(\frac{1}{2}\right)^j - \frac{243}{4480}(1)^j \\ + \frac{45}{896}\left(\frac{3}{2}\right)^j - \frac{81}{8960}(2)^j \end{bmatrix} \\ \sum\limits_{j=0}^{\infty} \frac{(2h)^j}{j!} y_n^{(j)} - y_n - 2hy_n' - 2h^2 y_n'' - \frac{31}{105}h^3 y_n'' - \sum\limits_{j=0}^{\infty} \frac{h^{j+3}}{j!} y_n^{j+3} \begin{bmatrix} \frac{272}{315}\left(\frac{1}{2}\right)^j + \frac{4}{105}(1)^j \\ + \frac{16}{105}\left(\frac{3}{2}\right)^j - \frac{1}{63}(2)^j \end{bmatrix} \end{bmatrix} = 0.$$

Comparing the coefficient efficient of h, the order of the block is six with error constant of $\begin{bmatrix} \frac{139}{10321920} & \frac{1}{11520} & \frac{243}{1146880} & \frac{1}{2520} \end{bmatrix}^T$.

4.2. Zero Stability

A block method is said to be zero stable if as $h \longrightarrow 0$, the roots $r_j, = 1\,(1)\,k$ of the first characteristic polynomial $\rho(r) = 0$ that is $\rho(r) = \det\left[\sum A^{(0)} R^{k-1}\right] = 0$ satisfying $|\, R \,| \leq 1$,must have multiplicity equal to unity

For our method

$$\rho(R) = \left[R \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right] = 0,$$

$$R = 0, 0, 0, 1.$$

Hence the method is zero stable.

x	$\Delta_1; h = 0.1$	$\Delta_2; h = 0.01$	$\Delta_3; h = 0.001$	$\Delta_4; h = 0.0001$
0.1	0.004999997916611	0.004999997916667	0.004999997916667	0.004999997916667
0.2	0.019999866666859	0.019999866667262	0.019999866667259	0.019999866667259
0.3	0.044998481293978	0.044998481284190	0.044998481284171	0.044998481284171
0.4	0.079991467388617	0.079991467273523	0.079991467273444	0.079991467273444
0.5	0.124967454367055	0.124967453567459	0.124967453567221	0.124967453567221
0.6	0.179902837409194	0.179902834981683	0.179902834981102	0.179902834981101
0.7	0.244755067600357	0.244755061293070	0.244755061291842	0.244755061291842
0.8	0.319454500640289	0.319454487436109	0.319454487433778	0.319454487433777
0.9	0.403894871267148	0.403894845891351	0.403894845887279	0.403894845887278
1.0	0.49792248311043	0.497922439830212	0.497922439823557	0.497922439823554

Table 1: Result of Experiment I

4.3. Consistency

A block method is said to be consistent if it has order $p \geq 1$.Hence our method is consistent.

4.4. Convergence

A block method is said to be convergent if and only if it is consistent and zero stable. from our method it is shown clearly that our method is convergent.

5. Numerical Examples

Experiment 1. We consider the solution to Blassius equation in fluid dynamics given by

$$2y''' + yy'' = 0, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 1.$$

The exact solution does not exist.

The results are shown in Tables 1 and 2.

We now state a condition which enable us to give the numerical solution to Experiment 1.

Condition 1. *Let $y(x_n)$ be the exact solution to the differential equation (1). Let $\lim_{h \to h_i} \Delta_n \to y(x_n)$ and $\lim_{h \to h_{i+1}} \Delta_{i+1} \to y(x_n)$ such that $\mid \Delta_{i+1} - \Delta_i \mid \to 0$, then Δ_{i+1} is the numerical solution of (1) where $h_i > h_{i+1} \to 0$, i denotes the stepsize and n is the evaluation point.*

x	$\mid \Delta_2 - \Delta_1 \mid$	$\mid \Delta_3 - \Delta_2 \mid$	$\mid \Delta_4 - \Delta_3 \mid$
0.1	5.600(−14)	0.0000(+00)	0.0000(+00)
0.2	1.3280(−13)	3.0000(−15)	0.0000(+00)
0.3	9.7880(−12)	1.9000(−14)	0.0000(+00)
0.4	1.1509(−10)	7.9000(−14)	0.0000(+00)
0.5	7.9960(−10)	2.3800(−13)	0.0000(+00)
0.6	2.4275(−10)	2.3800(−13)	1.0000(−15)
0.7	6.3073(−09)	1.2280(−12)	0.0000(+00)
0.8	1.3204(−08)	2.3310(−12)	1.0000(−15)
0.9	2.5376(−08)	4.0720(−12)	1.0000(−15)
1.0	4.3280(−08)	6.6550(−12)	3.0000(−15)

Table 2: Error of Experiment I

x	$Error$; $h = 0.1$	$Error$; $h = 0.05$	$Error$; $h = 0.025$	$Error$; $h = 0.01$	$Error$ in [12]
0.1	1.6613(−12)	3.0184(−14)	6.9625(−14)	1.0270(−15)	1.5405(−09)
0.2	7.5411(−12)	1.7771(−11)	8.3819(−13)	1.0353(−14)	9.8455(−09)
0.3	1.3843(−09)	7.9592(−11)	3.1088(−12)	3.5527(−14)	2.3652(−08)
0.4	4.5006(−09)	2.0970(−10)	7.6137(−12)	8.4377(−14)	4.3273(−08)
0.5	1.0520(−08)	4.2962(−10)	1.4997(−11)	1.6298(−13)	3.9018(−08)
0.6	1.9715(−08)	7.5781(−10)	2.5804(−11)	2.7700(−13)	6.9700(−08)
0.7	3.2968(−08)	1.2092(−09)	4.0468(−11)	4.3032(−13)	5.2032(−08)
0.8	5.0419(−08)	1.7949(−09)	5.9301(−11)	6.2572(−13)	1.3527(−07)
0.9	7.2608(−08)	2.5220(−09)	8.2484(−11)	8.6553(−13)	4.7483(−07)
1.0	9.9511(−08)	3.3932(−09)	1.1007(−10)	1.1500(−12)	1.0693(−07)

Table 3: Error of Experiment II, Error in [12] is at $h = 0.1$

Experiment II. We consider a linear third order initial

$$y''' + (y') = 0, \quad y(0) = 0,\ y'(0) = 1,\ y''(0) = -2,\ x \in [0,1]\,.$$

Exact solution: $y(x) = 2(1 - \cos x) + \sin x$.
The result is given in Table 3.
Source: Adesanya et al (2012).

Experiment III. We consider a special third order problems

$$y''' = 3\sin x, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = -2,\ 0 \le x \le 1.$$

Exact solution : $y(x) = 3\cos x + \frac{x^2}{2} - 2$.
The result is given in Table 4.
Source: Adesanya et al (2012).
It must be noted that $Error = \mid \Delta_{i+1} - \Delta_i \mid$.

5.1. Discussion of Result

We consider three numerical examples to test the efficiency of our derived method. Experiment one considered Blassius equation in thermodynamics.

x	$Error$; $h = 0.1$	$Error$; $h = 0.05$	$Error$; $h = 0.025$	$Error$; $h = 0.01$	$Error$ in [12]
0.1	2.5934(−12)	4.6185(−14)	4.4409(−16)	1.1102(−16)	0.0000
0.2	1.1857(−11)	1.8563(−13)	3.1086(−15)	0.0000(+00)	9.9920(−16)
0.3	2.6224(−11)	4.1578(−13)	6.4393(−15)	4.4409(−16)	1.5543(−15)
0.4	4.7034(−11)	7.3574(−13)	1.1546(−14)	4.4409(−16)	3.1086(−15)
0.5	7.2700(−11)	1.1424(−12)	1.7764(−14)	7.7716(−16)	4.6629(−15)
0.6	1.0437(−10)	1.6327(−12)	2.5535(−14)	5.5511(−16)	6.8833(−15)
0.7	1.4049(−10)	2.2020(−12)	3.4306(−14)	7.7716(−16)	9.1035(−15)
0.8	1.8197(−10)	2.8458(−12)	4.4131(−14)	1.1657(−15)	1.1490(−14)
0.9	2.2736(−10)	3.5596(−12)	5.5123(−14)	1.4433(−15)	1.4210(−14)
1.0	2.7729(−10)	4.3369(−12)	6.7404(−14)	1.3184(−15)	1.7458(−14)

Table 4: Error of Experiment III, Error in [12] is at $h = 0.01$

This problem does not have an analytical solution, hence condition I is necessary in order to give the numerical solution. It is evident from table II that the solution is consistent and converges. Experiments II and III considered a linear and special third order initail value problems respectively. These problems are solved by Adesanya *et al*. (2012) where a block method of order six was proposed. Tables III and IV show clearly that our method gave better approximation than the existing methods

6. Conclusion

We have proposed an hybrid block method for the solution of third order initial value problems in this paper. The derived method is zero stable, consistent and convergent. the method derived gave low error constant, hence it gave better approximation than the existing methods.

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