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SETTING CONTROLLER PARAMETERS THROUGH A MINIMUM STRATEGY WITH A WEIGHTED LEAST SQUARES METHOD

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Abstract: This paper presents a feasible real time self-tuning of a controller. The main contribution of this paper consists of presenting a minimum variance control strategy together with a weighted least squares method to adapt the parameters of this approximated controller. Robustness in the proposed loop control is achieved. For that the technique is quite general and can be applied to any kind of system.

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1. Introduction and Motivation

Adaptive controllers are widely used in modern control systems. In particular, in many applications an adaptation of parameters of the controller is needed to overcome changes in the model to be controlled or to handle with noise signals. Controllers based on signal processing algorithms are often used for solving practical problems as in [1]. Minimum Variance approach is known and often used in control theory to overcome such kind of problems. This paper proposes a Minimum Variance Control strategy together with a weighted least squares method to adapt the parameters of a controller of the first order. A discrete

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solution is proposed to be used in practical applications. This contribution emphasizes some mathematical aspects of an algorithm which the author used in practical applications such as for instance in [2] and in [3]. In particular, in [2] this algorithm is used in designing a controller in the context of a throttle valve control. In [3] a similar algorithm is used to minimize the decoupling error in order to improve the parameters identification performances in an application in which a synchronous motor is considered. Section 2 of the paper is devoted to the problem formulation and a procedure for a possible solution of the proposed problem is given.

2. An Adaptive Controller Using Minimum Variance and Weighted Least Squares Method

The novelty of the presented method consists of combining a minimum variance control strategy with a weighted least squares method to adapt the parameters of a PD controller.

Definition 1. Let

$$\eta(k) = \frac{\eta(k-1)}{(1 + t_s\frac{k_{app}}{k_d})} + \frac{t_s\frac{k_{app}}{k_d}(k_p - k_{app})e_y(k)}{(1 + t_s\frac{k_{app}}{k_d})} \tag{1}$$

$$u(k) = \eta(k) + k_{app}e_y(k), \tag{2}$$

be a discrete differential equation, where t_s indicates the sampling time with $t \in \mathbb{R}$, $u(k)$, $\eta(k)$ and $e(k)$ are discrete variables with $k \in \mathbb{N}$. k_{app}, k_d and k_p are parameters to be set belonging to $\mathbb{R}$. □

In [2] it was shown that for $k_{app} \to \infty$ equations (1) and (2) approximate a PD controller characterised by coefficients k_p and k_d. Signal $u(k)$ can represent the output of the controller, $e_y(k)$ represents the error of a possible system to be controlled and in the meantime $e_y(k)$ states the input of the controller. Signal $\eta(k)$ is the state of the controller. Transforming the controller represented by (1) and (2) with Z-transform, the following equations are obtained:

$$U(z) = \frac{(t_s\frac{k_{app}}{k_d}(k_p - k_{app}))E_y(z)}{(1 + t_s\frac{k_{app}}{k_d}) - z^{-1}} + k_{app}E_y(z), \tag{3}$$

and

$$U(z) = \frac{(t_s\frac{k_{app}}{k_d}k_p + k_{app} - k_{app}z^{-1})E_y(z)}{(1 + t_s\frac{k_{app}}{k_d}) - z^{-1}}. \tag{4}$$

As described earlier, the objective of minimum variance control is to minimize the variation of an output of a system with respect to a desired output signal in the presence of noise. To realise that a discrete input $u(k)$ is chosen to minimize

$$J = E\{e_y^2(k+d)\}, \tag{5}$$

where d is the delay time and E is the expected value. This optimization technique is signal based and it is used to adapt the parameters of the proposed controller. Assuming $d = 2$, then

$$\begin{aligned} e_y(k) = a_1 e_y(k-1) + a_2 e_y(k-2) + b_1 u(k-1) + b_2 u(k-2) \\ + n(k) + c_1 n(k-1) + c_2 n(k-2), \end{aligned} \tag{6}$$

where coefficients a_1, a_2, b_1, b_2 and c_1, c_2 belong to $\mathbb{R}$ and they need to be estimated, $n(k)$ denotes the white noise. At the next sample equation (6) becomes:

$$\begin{aligned} e_y(k+1) = a_1 e_y(k) + a_2 e_y(k-1) + b_1 u(k) + b_2 u(k-1) \\ + n(k+1) + c_1 n(k) + c_2 n(k-1). \end{aligned} \tag{7}$$

The prediction at time "k" is:

$$\begin{aligned} \hat{e}_y(k+1/k) = a_1 e_y(k) + a_2 e_y(k-1) + b_1 u(k) + b_2 u(k-1) \\ + c_1 n(k) + c_2 n(k-1). \end{aligned} \tag{8}$$

Considering that:

$$J = E\{e_y^2(k+1/k)\} = E\{[\hat{e}_y(k+1/k) + n(k+1)]^2\}$$

and assuming that the noise is not correlated to signal $e_y(k)$, it follows:

$$\begin{aligned} E\{[\hat{e}_y(k+1/k) + n(k+1)]^2\} = E\{[\hat{e}_y(k+1/k)]^2\} \\ + E\{[n(k+1)]^2\} = E\{[\hat{e}_y(k+1/k)]^2\} + \sigma_n^2, \end{aligned} \tag{9}$$

where σ_n is defined as the variance of the white noise. The goal is to find $u(k)$ such that:

$$\hat{e}_y(k+1/k) = 0. \tag{10}$$

It is possible to write (6) as:

$$n(k) = e_y(k) - a_1 e_y(k-1) - a_2 e_y(k-2) - b_1 u(k-1)$$

$$- b_2u(k-2) - c_1n(k-1) - c_2n(k-2). \quad (11)$$

Considering the effect of the noise as follows:

$$c_1n(k-1) + c_2n(k-2) \approx c_1n(k-1), \quad (12)$$

and transforming equation (11) using Z-transform, then:

$$N(z) = E_y(z) - a_1z^{-1}E_y(z) - a_2z^{-2}E_y(z) - b_1z^{-1}U(z) \\ - b_2z^{-2}U(z) - c_1z^{-1}N(z) \quad (13)$$

and

$$N(z) = \frac{(1 - a_1z^{-1} - a_2z^{-2})E_y(z)}{1 + c_1z^{-1}} \frac{-(b_1z^{-1} + b_2z^{-2})U(z)}{1 + c_1z^{-1}}, \quad (14)$$

where $z \in \mathbb{C}$ and represents the well known complex variable. The approximation in equation (12) is equivalent to consider the following assumption:

$$\|c_2\| << \|c_1\|. \quad (15)$$

In other words, the assumption stated in (15) means that the noise model of equation (11) is assumed to be a model of the first order. Considering Z-transform of equation (8) with $c_1n(k-1) + c_2n(k-2) \approx c_1n(k-1)$, then

$$z\hat{E}_y(z) = a_1E_y(z) + a_2z^{-1}E_y(z) + b_1U(z) + b_2z^{-1}U(z) + c_1N(z). \quad (16)$$

Considering that:

$$N(z) = \frac{(1 - a_1z^{-1} - a_2z^{-2})E_y(z)}{1 + c_1z^{-1}} \frac{-(b_1z^{-1} + b_2z^{-2})U(z)}{1 + c_1z^{-1}}, \quad (17)$$

Inserting equation (17) into equation (16) it follows:

$$z\hat{E}_y(z) = a_1E_y(z) + a_2z^{-1}E_y(z) + b_1U(z) + b_2z^{-1}U(z) + \\ c_1\Big(\frac{(1 - a_1z^{-1} - a_2z^{-2})E_y(z)}{1 + c_1z^{-1}} \frac{-(b_1z^{-1} + b_2z^{-2})U(z)}{1 + c_1z^{-1}}\Big). \quad (18)$$

According to equation (10), then $\hat{E}_y(z) = 0$ and through some calculations the following expression is obtained:

$$zE_y(z) + c_1E_y(z) = a_1E_y(z) + c_1a_1z^{-1}E_y(z) + a_2z^{-1}E_y(z) + c_1a_2z^{-2}E_y(z) + \\ b_1U(z) + c_1b_1z^{-1}U(z) + b_2z^{-1}U(z) + c_1b_2z^{-2}U(z) +$$

$$c_1(1 - a_1 z^{-1} - a_2 z^{-2})E_y(z) - c_1(b_1 z^{-1} + b_2 z^{-2})U(z), \quad (19)$$

from (19) it follows:

$$U(z) = -\frac{(a_1 + c_1 + a_2 z^{-1})E_y(z)}{b_1(1 + c_1 z^{-1}) + b_2(1 + c_1 z^{-1})}. \quad (20)$$

Comparing (20) with (4), it is left with a straightforward diophantine equation to be solved. The diophantine equation gives the relationship between the parameters $Y = [a_1, a_2, b_1, b_2, c_1]$ and the parameters of the controller k_{app}, k_d, and k_p as follows:

$$-a_2 = k_{app} \quad (21)$$

$$a_1 + c_1 = k_{app} + t_s \frac{k_{app}}{k_d} k_p \quad (22)$$

$$b_1 + b_2 = 1 + t_s \frac{k_{app}}{k_d} \quad (23)$$

$$b_1 c_1 + b_2 c_1 = -1. \quad (24)$$

Considering that $b_1 c_1 + b_2 c_1 = -1$ and that $a_2 = -k_{app}$ the transfer function defined by (20) becomes:

$$U(z) = -\frac{(a_1 + c_1 - k_{app} z^{-1})E_y(z)}{b_1 + b_2 - z^{-1}}. \quad (25)$$

Heuristic values for parameters k_{app}, k_d, and k_p are set in order to control the system according to the desired dynamic performance. This yields initial values for parameters $Y = [a_1, a_2, b_1, b_2, c_1]$. New values for vector $Y = [a_1, a_2, b_1, b_2, c_1]$ are calculated at each sampling time using the recursive least squares method. Once parameters $Y = [a_1, a_2, b_1, b_2, c_1]$ are estimated through the least squares method at each sampling time, k_{app}, k_d, and k_p represent the solution of an algebraic system with three equations and three variables. The presented analysis can be summarised in some steps which can establish a possible procedure as in the following.

Procedure.

- **Step 0**: Set heuristic values for k_{app}, k_d, and k_p. k_{app} is a value big enough to guarantee the asymptotic approximation of the considered PD regulator. The heuristic initial values are based on the Ziegel-Nichols method, see [4].

- **Step 1**: Calculate the new Y parameters of the ARMAX model using the recursive least squares method.

- **Step 2**: Calculate a new k_{app}, k_d, and k_p from the parameterization of the controller.

- **Step 3**: Calculate the new control signal.

- **Step 4**: Update the regressor, $e_y(k) \rightarrow e_y(k-1)$, $u(k-1) \rightarrow u(k-2)$, ...

Steps 1-4 are repeated for each sampling period.

3. Conclusions

This paper presents a feasible real time self-tuning of a PD controller. The main contribution of this paper consists of presenting a minimum variance control strategy together with a weighted least squares method to adapt the parameters of the considered PD controller. The technique presented does not depend on any model structure in which the controller can be used. For that the technique is quite general and can be applied to any kind of system to be controlled.

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