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ON $(\in, \in \vee q)$-FUZZY ESSENTIAL IDEAL OF NEAR-RING

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Abstract: In this paper, our attempt is to define fuzzy essential ideal of near-ring using notions of belongingness $(\in)$ and quasi-coincidence (q) of fuzzy points of sets. We study $(\in, \in \vee q)$-fuzzy essential ideals of near-rings and investigate different characterizations of such ideals in terms of their level ideals.

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1. Introduction

The concept of a fuzzy set was introduced by Zadeh [10] in 1965, utilizing which Rosenfeld [8] in 1971 defined fuzzy subgroups. Since then, the different aspects of algebraic systems in fuzzy settings had been studied by several authors. The notion of fuzzy subnear-ring and fuzzy ideals of near-rings was introduced by Abou Zaid Salah [1]. The concept of quasi-coincidence of a fuzzy point with a fuzzy subset was used by Bhakat and Das [2], [3, [4], B. Davvaz [5], A.Narayanan and T. Manikantan [6] to discuss a lot on fuzzy subgroup, subnear-rings, ideals of rings and near-rings. In their work, they defined $(\in, \in \vee q)$-fuzzy subrings and $(\in, \in \vee q)$-fuzzy ideals of a ring and a near ring and they characterized such fuzzy ideals. K.K. Rajkhowa et al [7] recently studied the $(\in, \in \vee q)$-fuzzy essential ideal of rings using the notion of belongingness $(\in)$ and quasi-coincidence (q) of fuzzy points and investigated different characterizations of such ideals in terms of level subsets. In the present paper, we try to discuss further properties of $(\in, \in \vee q)$-fuzzy ideals of near-rings as $(\in, \in \vee q)$-fuzzy essential ideals of near-

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rings. We also review the characterizations of $(\in, \in \vee q)$-fuzzy ideals as done in Zhan Jiang-ming et al [9] by their level ideals μ_t for various values of t for which μ_t is an empty set or a subnear-ring (ideal) of a near-ring. In this respect, he defined the $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy subnear-ring (ideals).

2. Preliminaries

We recall the following definition of near-rings as given by different authors. A nonempty set N with two binary operations "+" and "." is called a near-ring if

(i) $(N, +)$ is a group;

(ii) $(N, .)$ is a semigroup;

(iii) $(x + y).z = x.z + y.z$ for all $x, y, z \in N$.

We will use the word 'near-ring' to mean 'right near-ring' for which it satisfies the right distributive law. N is said to be zero symmetric if $0.x = x.0 = 0$ for all $x \in N$. We denote $x.y$ by xy.

Example 1. Let G be a group, $M(G)$, the set of all mappings from G to G. We define"+" and"." on $M(G)$ by pointwise addition and composition respectively. Then $(M(G), +, .)$ is a right near-ring.

An ideal I of a near-ring N is a subset of N such that

(i) $(I$,+) is a normal subgroup of $(N, +)$;

(ii) $IN \subseteq I$;

(iii) $a(b + i) - ab \in I \quad \forall\, a, b \in N, \quad i \in I.$

Note that I is a right ideal if it satisfies (i) and (ii) and a left ideal if it satisfies (i) and (iii).

A function μ from a non-empty set X to the unit interval [0,1] is called a fuzzy subset of X. Here it is to be noted that "μ is non-zero" is not related to the fact that " $\mu(x) \neq 0$ for some $x \neq 0$", contradictory to the remark 2.1 of [7].

Throughout this paper, N will denote a near-ring unless otherwise specified. We denote χ_I the characteristic function of a subset of I of N.

Definition 2.1. A fuzzy subset μ of a set X and $t \in [0, 1]$, the subset $\mu_t = \{x \in X \mid \mu(x) \geq t\}$ is called a level subset of X determined by μ and t.

The set $\{x \in N \mid \mu(x) > 0\}$ is called the support of μ and is denoted by Supp μ. A fuzzy subset of the form

$$\mu(y) = \begin{cases} t \neq 0 & \text{if } y = x \\ 0 & \text{if } y \neq x. \end{cases}$$

is said to be a fuzzy point denoted by x_t. Here x is called the support point and t is called its value. A fuzzy point x_t is said to belong to (respectively quasi coincident with) a fuzzy set μ written as $x_t \in \mu$ (resp. $x_t q \mu$) if $\mu(x) \geq t$ (resp. $\mu(x) + t > 1$). If $x_t \in \mu$ or $x_t q \mu$, then we write $x_t \in \vee q \mu$. The symbols $x_t \bar{\in} \mu$, $x_t \bar{q} \mu$, $x_t \overline{\in \vee q} \mu$ mean that $x_t \in \mu$, $x_t q \mu$, $x_t \in \vee q \mu$ do not hold respectively.

Definition 2.2. (see [2], [3]) A fuzzy subset μ of a group G is said to be an $(\in, \in \vee q)$-fuzzy subgroup of G if $\forall x, y \in G$ and $t, r \in (0, 1]$,

(i) $x_t, y_r \in \mu \Rightarrow (x + y)_{\min\{t,r\}} \in \vee q \mu$

(ii) $x_t \in \mu \Rightarrow (-x)_t \in \vee q \mu$

Remark 1. The conditions (i) and (ii) of definition 2.2 are respectively equivalent to

(i) $\mu(x + y) \geq \min\{\mu(x), \mu(y), 0.5\}$

(ii) $\mu(-x) \geq \min\{\mu(x), 0.5\}$ for all $x, y \in G$.

Remark 2. For any $(\in, \in \vee q)$-fuzzy subgroup μ of G such that $\mu(x) \geq 0.5$ for some $x \in G$, then $\mu(0) \geq 0.5$ and if $\mu(0) < 0.5$ then $\mu(x) < 0.5$ for all $x \in G$.In the later case, μ is just the fuzzy subgroup in the sense of Rosenfeld.

Remark 3. It is to be noted that if μ is a fuzzy subgroup then μ is an $(\in, \in \vee q)$-fuzzy subgroup of G. However, the converse may not be true as we see from example 3.11 and 3.14 of Bhakat, Das [3].

Here onwards we assume that μ is an $(\in, \in \vee q)$-fuzzy subgroup in the non-trivial sense for which case we have $\mu(0) \geq 0.5$, and there exists some $x \neq 0$ such that $\mu(x) < 0.5$.

Definition 2.3. (see [4]) An $(\in, \in \vee q)$-fuzzy subgroup μ of a group G is said to be $(\in, \in \vee q)$-fuzzy normal subgroup if for any $x, y \in G$ and $t \in (0, 1]$,

$$x_t \in \mu \Rightarrow (y + x - y)_t \in \vee q \mu.$$

Remark 4. (see [4]) The condition of $(\in, \in \vee q)$-fuzzy normal subgroup is given in the equivalent forms as

(i) $\mu(y+x-y) \geq \min\{\mu(x), 0.5\}$

(ii) $\mu(x+y) \geq \min\{\mu(y+x), 0.5\}$

(iii) $\mu([x,y]) \geq \min\{\mu(x), 0.5\}$ for all $x, y \in G$ where $[x,y]$ denotes the commutators of x, y in G.

Definition 2.4. (see [5], [6]) A fuzzy set μ of a near-ring N is called a $(\in, \in \vee q)$- fuzzy subnear-ring of N if for all $x, y \in N$, and $t, r \in (0,1]$

(i) $x_t, y_r \in \mu \Rightarrow (x+y)_{\min\{t,r\}} \in \vee q\mu$

(ii) $x_t \in \mu \Rightarrow (-x)_t \in \vee q\mu$

(iii) $x_t, y_r \in \mu \Rightarrow (xy)_{\min\{t,r\}} \in \vee q\mu$

Definition 2.5. (see [5], [6]) A fuzzy set μ of a near-ring N is called a $(\in, \in \vee q)$- fuzzy ideal of N if

(i) μ is a $(\in, \in \vee q)$-fuzzy subnear-ring of N

(ii) $x_t \in \mu \Rightarrow (y+x-y)_t \in \vee q\mu$ for all $x, y \in N$

(iii) $x_t \in \mu, y \in N \Rightarrow (xy)_t \in \vee q\mu$ for all $x, y \in N$

(iv) $a_t \in \mu$, $x, y \in N \Rightarrow (x(y+a)-xy)_t \in \vee q\mu$ for all $x, y, a \in N$.

A fuzzy subset with condition (i), (ii), (iii) is called an $(\in, \in \vee q)$- fuzzy right ideal of N. If μ satisfies (i), (ii), (iv), then it is called an $(\in, \in \vee q)$- fuzzy left ideal of N.

Lemma 2.6. **[6]** *Let μ be a fuzzy subset of N. Then*

(i) μ is a $(\in, \in \vee q)$-fuzzy subnear-ring of N iff
$\mu(x-y), \mu(xy) \geq \min\{\mu(x), \mu(y), 0.5\}$ *for all $x, y \in N$*

(ii) $x_t \in \mu, y \in N \Rightarrow (y+x-y)_t \in \vee q\mu$ iff $\mu(y+x-y) \geq \min\{\mu(x), 0.5\}$ for all $x, y \in N$

(iii) $x_t \in \mu, y \in N \Rightarrow (xy)_t \in \vee q\mu$ iff $\mu(xy) \geq \min\{\mu(x), 0.5\}$ $\forall x, y \in N$

(iv) $a_t \in \mu, x, y \in N \Rightarrow (y(x+a)-yx)_t \in \vee q\mu$ iff $\mu(y(x+a)-yx) \geq \min\{\mu(a), 0.5\}$ $\forall x, y, a \in N$

Theorem 2.7. **[5],[6]** *A fuzzy subset μ of N is an $(\in, \in \vee q)$-fuzzy ideal of N iff $\forall x, y, a \in N$*

(i) $\mu(x-y) \geq \min\{\mu(x), \mu(y), 0.5\}$

(ii) $\mu(y+x-y) \geq \min\{\mu(x), 0.5\}$

(iii) $\mu(xy) \geq \min\{\mu(x), 0.5\}$

(iv) $\mu\{y(x+a)-yx\} \geq \min\{\mu(a), 0.5\}$

Remark 5. A fuzzy subnear-ring or ideal of N due to definition of [1] is an $(\in, \in \vee q)$-fuzzy subnear-ring or ideal of N but the converse, in general, is not true, as we can see from the following example.

Example 2: $N = \{0, a, b, c\}$ be the near-ring with $(N, +)$ as the Klien's four group and $(N, .)$ as defined below.

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	b
c	a	a	a	c

Define a fuzzy subset $\mu : N \to [0, 1]$ by $\mu(0) = 0.8$, $\mu(a) = 0.4$, $\mu(b) = 0.9$, $\mu(c) = 0.4$. Then μ is a $(\in, \in \vee q)$-fuzzy subnear-ring of N but it is not a fuzzy subnear-ring of N as $\mu(0) = \mu(b+b) \not\geq \min\{\mu(b), \mu(b)\}$=0.9.

Theorem 2.8. **[5],[6]** *A non-empty subset I of N is a subnear-ring (ideal) of N iff χ_I is an $(\in, \in \vee q)$-fuzzy subnear-ring (ideal) of N.*

Theorem 2.9. **[6]** *A non-empty subset I of N is an $(\in, \in \vee q)$-fuzzy subnear-ring (ideal) of N iff the level subset μ_t is a subnear-ring (ideal) of N for all $0 \leq t \leq 0.5$.*

Remark 6. For $t \notin$[0,0.5], μ may be an $(\in, \in \vee q)$-fuzzy ideal of N but μ_t may not be an ideal of N. Let t=0.9 in the above example, μ_t is not a subgroup of N.

Consequently, Zhan Jian ming et al [9] considered another type of fuzzy ideal of N such that the level sets μ_t is an ideal (sub-near ring) of N for all $t \in$ (0.5, 1]. These fuzzy subnear-ring or ideals satisfy the following conditions

(i) $max\{\mu(x+y), 0.5\} \geq min\{\mu(x), \mu(y)\}$

(ii) $max\{\mu(-x), 0.5\} \geq \mu(x)$

(iii) $max\{\mu(xy), 0.5\} \geq min\{\mu(x), \mu(y)\}$

(iv) $max\{\mu(y+x-y), 0.5\} \geq \mu(x)$

(v) $max\{\mu(xy), 0.5\} \geq \mu(x)$

(vi) $max\{\mu(y(x+a)-yx), 0.5\} \geq \mu(a)$ for all $x, y, a \in N$.

Also in [9], they defined subnear-rings or ideals with thresholds (α, β) of N for $\alpha, \beta \in$(0, 1],$\alpha < \beta$ as a tool for making the level sets μ_t to be subnear-rings or ideals for all $t \in (\alpha, \beta]$ and vice-versa, In view of [2], we define the $(\in \vee q)$-level subset as follows.

Definition 2.10. The subset $\overline{\mu}_t = \{x \in X \mid \mu(x) \geq t \ \ or \ \ \mu(x)+t > 1\} = \{x \in X \mid x_t \in \vee q\mu\}$ is called $(\in \vee q)$-level subset of X determined by μ and t.

Remark 7. It follows from the definition 2.10 that $(\in \vee q)$-level subset, $\mu_t = \{x \in X \mid \mu(x) \geq t\} \subseteq \{x \in X \mid x_t \in \vee q\mu\}$. However the reverse set inclusion may not be true.

Remark 8. If μ is a fuzzy subset of X, $t > r$, the level set μ_t satisfy $\mu_t \subseteq \mu_r$. However for the case of $(\in \vee q)$-level set $\overline{\mu_t}$ does not satisfy this condition.

Example 3. Let $X = \{a, b, c\}$. $\mu(a) = 0.7, \mu(b) = 0.3, \mu(c) = 0.4$ then $b \in \overline{\mu}_{0.9}$ but $b \notin \overline{\mu}_{0.7}$ that means $\overline{\mu}_{0.9} \nsubseteq \overline{\mu}_{0.7}$

Theorem 2.11. **[2]** *A fuzzy subset μ of a group G is a fuzzy subgroup of G iff $\overline{\mu_t}$ is a subgroup for all $t \in (0, 1]$.*

3. Main Results

As we observe from the above section, the studies undertaken by Davvaz [5], Narayanan [6] were confined to the properties of $(\in, \in \vee q)$-fuzzy subnear-rings or ideals and their characterization by the level sets to give similar properties of the crisp set theory for $t \in (0, 0.5]$ or $t \in (0.5, 1]$ and the investigations of Bhakat and Das {[3],[4]} were confined to fuzzy subgroups but the corresponding property of level set in crisp set theory covers the whole domain (0,1]. We give an attempt to extend their work in case of the later level set $\overline{\mu_t}$ of N.

Theorem 3.1. *A fuzzy subset μ of N is an $(\in, \in \vee q)$-fuzzy subnear-ring (ideal) iff the level subset $\overline{\mu_t}$ is a subnear-ring (ideal) of N for all $t \in (0, 1]$.*

Proof. We provide the proof of the result in case of $(\in, \in \vee q)$-fuzzy ideals. Let μ be an $(\in, \in \vee q)$-fuzzy ideal of N. Let $x, y \in \overline{\mu_t}$. Then $\mu(x) \geq t$ or

$\mu(x)+t > 1$ and $\mu(y) \geq t$ or $\mu(y)+t > 1$. Then $\mu(x-y) \geq \min\{\mu(x), \mu(y), 0.5\}$ as μ is a $(\in, \in \vee q)$-fuzzy ideal.

$$\begin{aligned} Case(a)\ \mu(x-y) &\geq \min\{t, 0.5\} \textit{ in case } \mu(x) \geq t,\ \mu(y) \geq t. \\ Case(b)\ \mu(x-y) &\geq \min\{t, 1-t, 0.5\} \textit{ in case } \mu(x) \geq t,\ \mu(y)+t > 1 \\ & \quad \textit{or } \mu(x)+t > 1,\ \mu(y) \geq t. \\ Case(c)\ \mu(x-y) &\geq \min\{1-t, 0.5\} \textit{ in case } \mu(x)+t \geq 1,\ \mu(y)+t \geq 1. \end{aligned}$$

For all the above cases, we have the following results

$$\begin{aligned} \mu(x-y) &\geq t \textit{ or } \mu(x-y) > 1-t \textit{ according as } t \leq 0.5 \\ & \quad \textit{or } t > 0.5. \end{aligned}$$

In all the cases, $(x-y)_t \in \vee q\mu$ or $x-y \in \overline{\mu_t}$. Next let $x \in \overline{\mu_t}, y \in N$ then $\mu(y+x-y) \geq \min\{\mu(x), 0.5\}$ as μ is $(\in, \in \vee q)$-fuzzy normal subgroup of N. This implies $\mu(y+x-y) \geq \min\{t, 0.5\}$ if $\mu(x) \geq t$ or $\mu(y+x-y) > \min\{1-t, 0.5\}$ if $\mu(x) > 1-t$. In either case, $\mu(y+x-y) \geq t$ or $\mu(y+x-y) \geq 1-t$ according as $t \leq 0.5$ or $t > 0.5$ respectively. Thus $(y+x-y) \in \overline{\mu_t}$ i.e. $\overline{\mu_t}$ is a normal subgroup of N. Let $a \in \overline{\mu_t}$, $x, y \in N$, $\mu(x(y+a)-xy) \geq \min\{\mu(a), 0.5\} \Rightarrow \mu(x(y+a)-xy) \geq \min\{t, 0.5\}$ if $\mu(a) \geq t$ or $\mu(x(y+a)-xy) > \min\{1-t, 0.5\}$ if $\mu(a) > 1-t$. In either case, $\mu(x(y+a)-xy) \geq t$ or $1-t$ according as $t \leq 0.5$ or $t > 0.5$ respectively. So, $x(y+a)-xy \in \overline{\mu_t}$. Hence $\overline{\mu_t}$ is an ideal of N.

Conversely assume that μ is a fuzzy subset of N such that $\overline{\mu_t}$ is an ideal of N for all $t \in (0,1]$. If possible let $x, y \in N$ such that $\mu(x-y) < t < min\{\mu(x), \mu(y), 0.5\}$. Then $t \in (0, 0.5)$ and $x \in \overline{\mu_t}$, $y \in \overline{\mu_t}$ but $x-y \notin \overline{\mu_t}$ as $\mu(x-y) < t$ and $\mu(x-y)+t \leq 1$, a contradiction to the assumption that μ_t is an ideal of N in fact a subgroup of N. So, $\mu(x-y) \geq min\{\mu(x), \mu(y), 0.5\}$ for all $t \in (0,1]$ or μ is $(\in, \in \vee q)$-fuzzy subgroup of N. Again suppose there exist $x, y \in N$ and $t \in (0, 0.5)$ such that $\mu(y+x-y) < t < min\{\mu(x), 0.5\}$. Then $x \in \overline{\mu_t}$ but $\mu(y+x-y) < t$ and $\mu(y+x-y)+t \leq 1$ i.e. $y+x-y \notin \overline{\mu_t}$, a contradiction to the fact that $\overline{\mu_t}$ is a normal subgroup of N. So, μ is $(\in, \in \vee q)$-fuzzy normal subgroup of N. Similarly, we can prove that μ is an $(\in, \in \vee q)$-fuzzy ideal of N using condition (iii) and (iv) of theorem 2.7. □

Lemma 3.2. *A non-empty subset A of N is an ideal of N iff χ_A is an $(\in, \in \vee q)$-fuzzy ideal of N.*

Proof. It is straightforward. □

Let us now consider $(\in, \in \vee q)$-fuzzy subgroups of N in the non-trivial sense that is when $\mu(0) \geq 0.5$ and there exists a non-zero element $x \in N$ such that

$\mu(x) < 0.5$. So, we assume that an $(\in, \in \vee q)$-fuzzy subgroup of N is non-trivial if $\mu(x) \neq 0$ for some $x \neq 0$. This is valid assumption as $\mu(0) \geq 0.5$.

Definition 3.3. An ideal A of a near-ring N is called an essential ideal of N denoted by $A \subseteq_e N$, if for every non zero ideal B of N, $A \cap B \neq (0)$.

Definition 3.4. A non-trivial $(\in, \in \vee q)$-fuzzy ideal μ of N is called an $(\in, \in \vee q)$-fuzzy essential ideal of N denoted by $\mu \subseteq_e N$, if for every non trivial $(\in, \in \vee q)$-fuzzy ideal θ of N, there exists $x(\neq 0) \in N$, with $x_t \in \vee q\mu$ and $x_t \in \vee q\theta$ for every $t \in (0, \mu(0)]$ for which $\overline{\mu_t}$ is non zero.

Lemma 3.5. *A non-trivial $(\in, \in \vee q)$-fuzzy ideal μ of N is an $(\in, \in \vee q)$-fuzzy essential ideal of N iff for every non trivial $(\in, \in \vee q)$-fuzzy essential ideal θ of N, there exists $x(\neq 0) \in N$ with $\mu(x) \geq \min\{t, 0.5\}$ and $\theta(x) \geq \min\{t, 0.5\}$ for every $t \in (0, \mu(0)]$ for which $\overline{\mu_t}$ is non zero.*

Proof. Let μ be a $(\in, \in \vee q)$-fuzzy essential ideal of N, θ be any non-trivial $(\in, \in \vee q)$-fuzzy ideal of N. If possible suppose $t < 0.5$, and $\mu(x) < t$ or $\theta(x) < t$, where $x \neq 0$. Then $\mu(x) < 0.5$. So, $\mu(x) + t < 1$ or $\theta(x) < 0.5$ and $\theta(x) + t < 1$ i.e. $x_t \overline{\in \vee q}\mu$ or $x_t \overline{\in \vee q}\theta$. Also if $t \geq 0.5$ and $\mu(x) < \min\{0.5, t\}$ or $\theta(x) < \min\{0.5, t\}$. Then, $x_{0.5} \overline{\in \vee q}\mu$ or $x_{0.5} \overline{\in \vee q}\theta$.This implies that there exists an element $x(\neq 0) \in N$ with $\mu(x) \geq \min\{t, 0.5\}$ and $\theta(x) \geq \min\{t, 0.5\}$, where $t \in (0, \mu(0)]$.

Conversely, assume that there exists a non-zero element $x(\neq 0) \in N$ with $\mu(x) \geq \min\{t, 0.5\}$ and $\theta(x) \geq \min\{t, 0.5\}$. If $t \leq 0.5$ then $\mu(x) \geq t$ or $\theta(x) \geq t$. If $t > 0.5$ then $\mu(x) \geq 0.5 > 1 - t$ and $\theta(x) \geq 0.5 > 1 - t$ i.e. $x_t \in \vee q\mu$ and $x_t \in \vee q\theta$. Thus μ is a non-trivial $(\in, \in \vee q)$-fuzzy essential ideal of N. □

Theorem 3.6. *Let μ be a non-trivial $(\in, \in \vee q)$-fuzzy ideal of N. Then μ is an $(\in, \in \vee q)$-fuzzy essential ideal of N for all $t \in (0, \mu(0)]$ for which $\overline{\mu_t}$ is non zero iff $\overline{\mu_t}$ is an essential ideal of N.*

Proof. Let μ be a non-trivial $(\in, \in \vee q)$-fuzzy ideal of N. Then $\overline{\mu_t}$ is an ideal of N by theorem 3.1 for all $t \in (0, 1]$. As μ is non-trivial, $\overline{\mu}_t$ is a non zero ideal of N for all $t \in (0, \mu(0)]$. Let A be a non zero ideal of N . Let us define a fuzzy subset θ of N as

$$\theta(x) = \begin{cases} \mu(x) & \text{if } x \in \text{A} \\ 0 & \text{if } x \notin A. \end{cases}$$

Then θ is a non-zero $(\in, \in \vee q)$-fuzzy ideal of N. So, by lemma 3.4 there exists $x(\neq 0) \in N$ such that $\mu(x) \geq \min\{t, 0.5\}$ and $\theta(x) \geq \min\{t, 0.5\}$ for all $t \in (0, \mu(0)]$. Thus $x \in \overline{\mu_t}$ and $\theta(x) \geq t$ or $\theta(x) \geq 0.5$. This implies that

$\theta(x) = \mu(x) > 0$ i.e. $x \in A$. So, $\overline{\mu_t} \cap A \neq (0)$ i.e. $\overline{\mu_t}$ is essential ideal of N for any $t \in (0, \mu(0)]$.

Conversely, assume that $\overline{\mu_t}$ is non zero essential ideal for every $t \in (0, \mu(0)]$. Let θ be any non-trivial $(\in, \in \vee q)$-fuzzy ideal of N with $\mu(0) = \theta(0)$. Thus there exists $x(\neq 0) \in N$ with $x \in \overline{\mu_t} \cap \overline{\theta_t}$ for all $t \in (0, \mu(0)]$, as $\overline{\mu_t}$, for every $t \in (0, \mu(0)]$ is an essential ideal of N and $\overline{\theta_t}$ for $t \in (0, \mu(0)]$ is non-zero. This implies that there exists $x \neq 0$ with $x_t \in \vee q\mu$ and $x_t \in \vee q\theta$ for all $t \in (0, \mu(0)]$. By definition 3.3, μ is a non-trivial $(\in, \in \vee q)$-fuzzy essential ideal of N. □

Theorem 3.7. *An ideal A of N is essential in N iff χ_A is an $(\in, \in \vee q)$-fuzzy essential ideal of N.*

Proof. Let A be an essential ideal of N. Then χ_A is a non-trivial $(\in, \in \vee q)$-fuzzy ideal of N. $(\overline{\chi}_A)_t$ is essential for all $t \in (0, 1]$ as $(\overline{\chi}_A)_t = A$ for $t \in (0, 1]$. Hence by theorem 3.6, χ_A is an $(\in, \in \vee q)$-fuzzy essential ideal of N. Conversely, let χ_A be an $(\in, \in \vee q)$-fuzzy ideal of N. Let B be any non-zero ideal of N. Then χ_B is also a non-trivial $(\in, \in \vee q)$-fuzzy ideal of N. Hence there exists $x \neq 0$ with $\chi_A(x) \geq \min\{t, 0.5\}, \chi_B(x) \geq \min\{t, 0.5\}$ for $t \in (0, 1]$. So, $x \in A \cap B$. Thus A is an essential ideal of N. □

Example 4. Let (N,+,.) be the near ring given in example 2. Let μ be the fuzzy subset of N defined by $\mu(0) = 0.8, \mu(a) = \mu(b) = \mu(c) = 0.4$. μ is $(\in, \in \vee q)$-fuzzy essential ideal of N as it is seen that $\overline{\mu}_t$ is N or 0 for all $t \in (0, \mu(0)]$ and {0,a} is the only non-zero ideal of N.

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