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PERISTALTIC TRANSPORT OF A FOURTH GRADE FLUID BETWEEN POROUS WALLS WITH SUCTION AND INJECTION

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Abstract: The Peristaltic flow of a fourth grade fluid between two porous walls with suction and injection is investigated. The perturbation technique in terms of small Deborah number is employed to determine the expressions for the velocity, the stream function, the pressure rise and friction force under long wavelength and low Reynolds number assumptions. The effects of different parameters on the pumping characteristics and frictional forces are discussed graphically.

Key Words: peristaltic transport, fourth grade fluid, suction, injection, pressure rise

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1. Introduction

Peristalsis is a well-known mechanism for pumping biological and industrial fluids. This mechanism generally occurs in the gastrointestinal, urinary and reproductive tracts in the living body. Sreenadh and Arunachalam [1] studied the Couette flow between two permeable beds with suction and injection. Mishra and Ramachandra Rao [2] made a detailed analysis on the peristaltic transport with permeable walls. Haroun [3] have studied the non-linear peristaltic flow of a fourth grade fluid in an inclined asymmetric channel. Recently, Kavitha et al. [4] and Hemadri Reddy et al. [5] discussed the peristaltic pumping of non-Newtonian Jeffrey and Carreau fluids between porous walls with suction and injection In view this, it will be interesting to study the peristaltic transport of a non-Newtonian fourth grade fluid in a channel between two porous beds with suction and injection is investigated, under long wavelength and low Reynolds number assumptions. The velocity, the stream function, the pressure rise and friction force are obtained. The results are deduced and discussed.

2. Mathematical Formulation

Consider the peristaltic flow of an incompressible fourth grade fluid of half width a. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower permeable walls of the channel. The fluid is injected into the channel perpendicular to the lower permeable wall with a constant velocity v_0 and is sucked out of the upper permeable wall with the same velocity v_0 as shown in figure 1. For simplicity, we restrict our discussion to the half width of the channel.

The wall deformation is given by

$$Y = H(X,t) = a + b \sin \frac{2\pi}{\lambda}(X - ct) \tag{1}$$

where b is the amplitude, λ is the wave length and c is the wave speed.

We introduce a wave frame of reference (x, y) moving with the velocity c in which the motion becomes independent of time when the channel length is an integral multiple of the wave length and the pressure difference at the ends of the channel is a constant. The transformation from the fixed frame of reference (X, Y) to the wave frame of reference (x, y) is given by

$$x = X - ct,\ y = Y,\ u = U - c,\ v = V,\ p'(x) = P'(X,t) \tag{2}$$

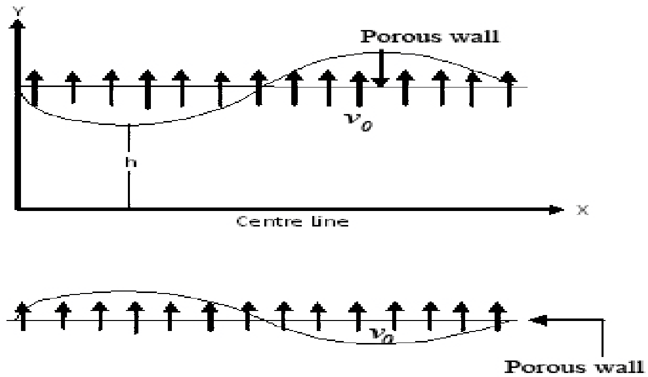

Figure 1: Physical Model

where (u, v) and (U, V) are the velocity components, p and P are the pressures in the wave and fixed frames of reference respectively. The equations governing the flow field, in the wave frame of reference are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \tag{4}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y} \tag{5}$$

Due to symmetry, the problem is studied only for upper half of the channel. The boundary conditions for the velocity are

$$\frac{\partial u}{\partial y} = 0 \ \ at \ \ y = 0 \tag{6}$$

$$u = -c \ \ at \ \ y = H \tag{7}$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\bar{x} = \frac{x}{\lambda},\ \bar{y} = \frac{y}{a},\ \bar{u} = \frac{u}{c},\ \ \bar{v} = \frac{v}{c},\ h = \frac{H}{a},\ \bar{p} = \frac{p a^2}{\lambda\mu c},\ \bar{S} = \frac{a}{\mu c}S,$$
$$\bar{t} = \frac{ct}{\lambda},\ \ delta = \frac{a}{\lambda},\ \ \phi = \frac{b}{a},\ \ \mathrm{Re} = \frac{\rho c a}{\mu},\ \lambda_i = \frac{\alpha_i c}{\mu a}\,(i = 1, 2)\,,$$
$$\xi_j = \frac{\beta_j c^2}{\mu a^2}(j = 1, 2, 3),\ \ \eta_k = \frac{\gamma_k c^3}{\mu a^3}\,(k = 1 - 8) \tag{8}$$

where Re and δ represents the Reynolds number and wave number respectively.

In view of dimensionless quantities (8), the Equations (3) - (5), after dropping bars, reduce to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{9}$$

$$\mathrm{Re}\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p'}{\partial x}+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y} \tag{10}$$

$$\mathrm{Re}\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p'}{\partial y}+\delta^2\frac{\partial S_{yx}}{\partial x}+\delta\frac{\partial S_{yy}}{\partial y} \tag{11}$$

Under lubrication approach, neglecting the terms of order , from Equations (10) we get.

$$\frac{\partial p}{\partial x}=\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial y}\left(1+2\Gamma\left(\frac{\partial u}{\partial y}\right)^2\right)\right]-k\frac{\partial u}{\partial y} \tag{12}$$

where $k=\mathrm{Re}v_0$, $v_0$is suction/injection velocity and $\Gamma=\xi_2+\xi_3$ is the Deborah number. The corresponding dimensionless boundary conditions in the wave frame of reference are given by

$$\frac{\partial u}{\partial y}=0 \;\; at \;\; y=0 \tag{13}$$

$$u=-1 \;\; at \;\; y=\; h=1+\phi\sin 2\pi\, x \tag{14}$$

The volume flow rate q in a wave frame of reference is given by

$$q=\int_0^{h(x)} u\, dy. \tag{15}$$

The instantaneous flux $Q(X,t)$ in a fixed frame is

$$Q(X,t)=\int_0^h U\, dY=\int_0^h (u+1)\, dy\;=q+h. \tag{16}$$

The time average flux $\bar{Q}$ over one period $T\left(=\lambda/_c\right)$ of the peristaltic wave is

$$\bar{Q}=\frac{1}{T}\int_0^T Q\, dt\;=\int_0^1 (q+h)\, dx\;=q+1. \tag{17}$$

3. Perturbation Solution

The equation (12) is non-linear and its closed form solution is not possible. So, we expand u, p and q in terms of G as

$$\begin{aligned} u &= u_0 + \Gamma\, u_1 + O(\Gamma^2) \\ \frac{\partial p}{\partial x} &= \frac{\partial p_0}{\partial x} + \Gamma \frac{\partial p_1}{\partial x} + O(\Gamma^2) \\ q &= q_0 + \Gamma\, q_1 + O(\Gamma^2) \end{aligned} \tag{18}$$

Substituting the equations of (18) in (12) and solving the resulting systems, we get

$$u_0 = -1 + \frac{hP_0}{k} - \frac{P_0}{k^2}e^{kh} + \frac{P_0}{k^2}e^{ky} - \frac{yP_0}{k} \tag{19}$$

$$\begin{aligned} u_1 = {}& P_1\left[\frac{h}{k} - \frac{e^{kh}}{k^2}\right] + \frac{P_0^3}{k^3}\left[\frac{8e^{kh}}{k} + \frac{e^{3kh}}{k} + he^{kh} - \frac{6e^{2kh}}{k}\right] \\ & + \left[\frac{P_1}{k^2} - \frac{8P_0^4}{k^4}\right]e^{ky} - \frac{yP_1}{k} - \frac{P_0^3}{k^3}\left[\frac{e^{3ky}}{k} + ye^{ky} - \frac{6e^{2ky}}{k}\right] \end{aligned} \tag{20}$$

$$\frac{dp_0}{dx} = \frac{(q_0 + h)}{k_1} \tag{21}$$

$$\frac{dp_1}{dx} = \frac{q_1}{k_1} - P_0^3\frac{k_2}{k_1} \tag{22}$$

$$q_0 = \int_0^h u_0\, dy = P_0 k_1 - h \tag{23}$$

$$q_1 = \int_0^h u_1\, dy = P_1 k_1 + P_0^3 k_2 \tag{24}$$

in which $P_0 = \frac{\partial p_0}{\partial x}$ and $P_1 = \frac{\partial p_1}{\partial x}$

$$k_1 = \frac{h^2}{k} - \frac{he^{kh}}{k^2} + \frac{e^{kh}}{k^3} - \frac{1}{k^3} - \frac{h^2}{2k}$$

$$k_2 = \frac{7he^{kh}}{k^4} - \frac{7e^{kh}}{k^5} + \frac{he^{3kh}}{k^4} + \frac{h^2e^{kh}}{k^3} - \frac{6he^{2kh}}{k^4} - \frac{e^{3kh}}{3k^5} + \frac{6e^{2kh}}{2k^5} + \frac{8}{k^5}$$

Substituting equations (21) and (22) into the equation (18) and using the relation $\frac{dp_0}{dx} = \frac{dp}{dx} - \Gamma\frac{dp_1}{dx}$ and neglecting terms greater than $O(\Gamma^2)$ we get

$$u = -1 + P\left[\frac{h}{k} - \frac{e^{kh}}{k^2} + \frac{e^{ky}}{k^2} - \frac{y}{k}\right] +$$

$$\Gamma P^3 \left[\frac{8e^{kh}}{k^4} + \frac{3e^{hk}}{k^4} + \frac{he^{kh}}{k^3} - \frac{6e^{2kh}}{k^4} - \frac{8e^{ky}}{k^4} - \frac{e^{3ky}}{k^4} - \frac{ye^{ky}}{k^3} + \frac{6e^{2ky}}{k^4}\right] \tag{25}$$

$$\frac{dp}{dx} = \frac{(q+h)}{k_1} - \Gamma\frac{(q+h)^3 k_2}{k_1^4} \tag{26}$$

The dimensionless pressure rise and frictional force per one wavelength in the wave frame are defined, respectively as

$$\Delta p = \int_0^1 \frac{dp}{dx}\, dx \tag{27}$$

$$F = \int_0^1 h\left(-\frac{dp}{dx}\right) dx. \tag{28}$$

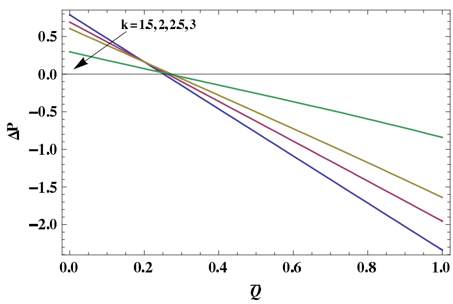

Figure 2: The variation of ΔP with $\bar{Q}$ for different values of k with $\phi = 0.4$, $\Gamma = 0.001$

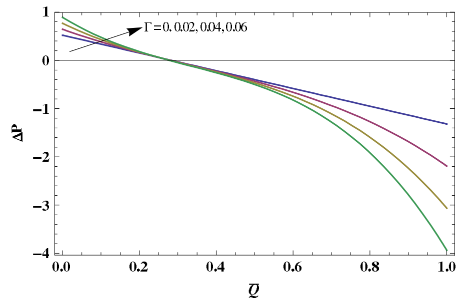

Figure 3: The variation of ΔP with $\bar{Q}$ for different values of Γ with $\phi = 0.4$, $k = 3$

4. Results and Discussion

The variation of pressure difference as a function of $\bar{Q}$ for different values of k is shown in Fig. 2. We observe that the larger the suction parameter k, the smaller the pressure rise against which the pump works. For a given flux $\bar{Q}$, the pressure rise decreases with increasing k. From Figs. (3) and (4), we find that the larger the Deborah number Γ and amplitude ratio ϕ, the greater the

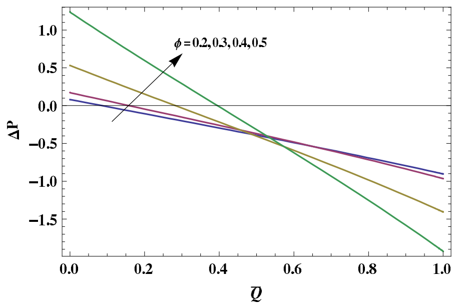

Figure 4: The variation of ΔP with $\bar{Q}$ for different values of ϕ with $k = 3$, $\Gamma = 0.002$

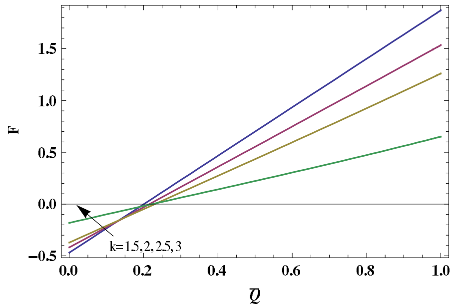

Figure 5: The variation of F with $\bar{Q}$ for different values of k with $\phi = 0.4$, $\Gamma = 0.001$

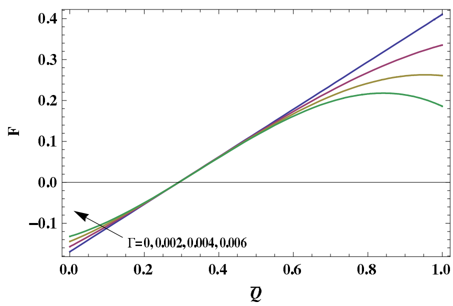

Figure 6: The variation of F with $\bar{Q}$ for different values of Γ with $\phi = 0.4$, $k = 5$

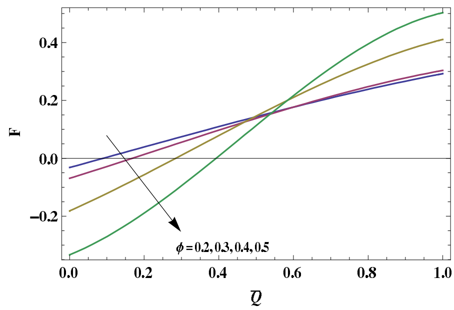

Figure 7: The variation of ΔP with $\bar{Q}$ for different values of ϕ with $k = 4.6$, $\Gamma = 0.002$

pressure rise against which the pump works. For a given flux $\bar{Q}$, the pressure rise increases with increasing Γ and ϕ. The frictional force F as a function of $\bar{Q}$ for different values of k and Γ are shown in figs. (5) and (6). We see that the friction force first increases and then decreases with an increase in k and Γ. From fig. (7) we observe that the friction force first decreases and then increases with an increase in ϕ. In general, it is observed that the frictional force F has opposite behavior compared to ΔP.

References

[1] S. Sreenadh, P.V. Arunachalam, Couette flow between two permeable beds

with suction and injection, *Proc. Nat. Acad. Sci.*, India IV, **56(A)** (1986), 229-235.

[2] M. Mishra, Rao Ramachandra, Peristaltic slip flow in a porous tube with a porous peripheral layer, In: *Proceedings of International Conference on Mathematical Biology*, Iit, Kanpur (2004).

[3] M.H. Haroun, Non-linear Peristaltic flow of a fourth grade fluid in an inclined asymmetric channel, *Comput. Mater. Sci.*, **39** (2007), 324.

[4] A. Kavitha, R.H. Reddy, A.N.S. Srinivas, S. Sreenadh, R. Saravana, Peristaltic Pumping of a Jeffrey fluid between porous walls with suction and injection, *International Journal of Mechanical and Materials Engineering (IJMME)*, **7**, No. 2 (2012), 152-157.

[5] R. Hemadri Reddy, A. Kavitha, S. Sreenadh, R. Saravana, Peristaltic Transport of a Carreau fluid in a porous channel with suction and injection, *IJMSE*, **2**, No. 2, 25-30.