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EMPIRICAL LIKELIHOOD INFERENCE FOR A HETEROSCEDASTIC PARTIALLY LINEAR EV MODEL

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Abstract: The purpose of this article is to use the empirical likelihood method to study the confidence regions construction for the parameters of interest in a heteroscedastic partially linear EV model. When error variances is known and unknown cases, the two different empirical log-likelihood ratio for the parametric components were proposed and nonparametric version of Wilks' theorem were derived, respectively. Simulation study shows that the proposed empirical likelihood confidence regions have satisfactory coverage.

AMS Subject Classification: 62G15, 62G20
Key Words: empirical likelihood, confidence region, heteroscedastic, partially linear EV model

1. Introduction

Consider the following partially linear model

$$Y_i = X_i^\tau \beta + g(T_i) + \epsilon_i, \quad (i = 1, \dots, n) \tag{1}$$

where Y_i is the response, X_i and T_i are random regressors. Furthermore, due to the curse of dimensionality, it is assumed that T_i is univariate, $\beta = (\beta_1, \dots, \beta_p)^\tau$ is a vector p-dimensional unknown parameters, $g(\cdot)$ is the unknown regression

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function, ϵ_i is random error. Since such semiparametric model introduced by Engle et al.(1986) [1] to study the effect of weather on electricity demand, this partially linear regression model has been studied by many authors. Various estimators for β and $g(\cdot)$ were given by using different methods such as the kernel method, the smoothing splines method and local polynomial method. However, covariate measurement error exist in many applications. The classical measurement model assumes that covariate X_i is measured with additive error, that means X_i cannot be observed, but an unbiased measure of X_i, denoted by W_i, can be obtained such that

$$W_i = X_i + U_i \tag{2}$$

where U_i is the measurement error with mean zero and covariance matrix Σ_U and is independent of (Y_i, W_i, T_i). We also assume that Σ_U is known; otherwise, we can estimate it by repeatedly observing W_i.

Model (1) and (2) is partially linear EV model and has been studied by many researchers. Liang et al.(1999) [2] studied the properties of the parametric and nonparametric estimators in the partially linear EV model, and overcome the inconsistency caused by the measurement error by applying the so- called "correction for attenuation". Liang and Li (2009) [3] studied variable selection for partially linear models with measurement errors. Zhou et al.(2010) [4] gave the estimation and inference for fixed-effects partially linear EV regression model.

In some applications, we are often interested in the confidence intervals of the interesting parametric. We recommend using the empirical likelihood method to construct the confidence intervals for β. The empirical likelihood was introduced by Owen (1988) [5] and its general properties were studied Owen (1990) [6]. Empirical likelihood has advantages over other methods such as those based on normal approximations or the bootstrap. Owen (1991) [7] applied the empirical likelihood method to linear models and Kolaczyk (1994) [8] made further extensions to generalized linear models. Shi and Lau (2000) [9] and Wang and Jing (1999,2003) [10] and [11] applied the empirical likelihood method to partially linear models, respectively. Xue and Zhu (2008) [12] studied the partially linear model with longitudinal data. For partially linear EV model, Cui and Chen (2003) [13] and Li and Xue (2008) [14] constructed the confidence region for parameter by using different ideas. An important feature that above papers were discussed when errors are assumed to be independent and identically distributed random variables. However, in many cases, the homoscedastic assumption for the errors is strong. Heteroscedasticity is often found in residuals form both cross-sectional and time series modeling in applications. As we all known, the ordinary least squares estimator become inconsistent when the errors are heteroscedastic. Lu (2009) [15] and Fan et al. (2011) [16] studied the heteroscedastic partially linear model based on empirical likelihood approach, respectively.

Suppose that $\{Y_i, W_i, T_i, Z_i, 1 \le i \le n\}$ is a sample from (Y, W, T, Z) in the following model with $\varepsilon = \sqrt{f(Z)}e$, where $f(\cdot)$ is a univariate function,

$$\begin{cases} Y_i &= X_i^\tau \beta + g(T_i) + \varepsilon_i \\ W_i &= X_i + U_i \end{cases} \tag{3}$$

where $\varepsilon_i = \sqrt{f(Z_i)}e_i := \sigma_i e_i$, Z_i is univariate. The model (3) is heteroscedastic partially linear EV model. Let $\{W_i, T_i, Z_i, 1 \le i \le n\}$ be i.i.d. random variable, and e_i be independent of $\{W_i, T_i, Z_i\}$ with $E(e_i|W_i, T_i, Z_i) = 0$ and $Var(e_i|W_i, T_i, Z_i) = 1$.

Take those issues into consideration, the main goal of this article to study the heteroscedastic partially linear EV model. The empirical likelihood method is adopted to investigate this model for constructing the confidence regions of β when f is known and unknown, and then derive nonparametric version of Wilks' theorem, respectively.

The rest of this article is organized as follows. The empirical log-likelihood ratio for β are proposed in Section 2. Assumption conditions and main results are given in Section 3. Simulation studies are conducted in Section 4. The proofs of the main results are related to Section 5.

2. Methodology

In this section we construct the empirical likelihood confidence region of the parameter vector β when f is known and unknown, respectively. To apply the EL method for the heteroscedastic partially linear EV model, at first, assume that β is known, then model (3) is reduced to a nonparametric regression model

$$Y_i - W_i^\tau \beta = g(T_i) + \omega_i, \quad \omega_i = \varepsilon_i - U_i^\tau \beta \tag{4}$$

The fact that without considering heteroscedasticity, $g(T_i)$ can be estimated by one of the following methods: kernel smoothing, polynomial approximation, smoothing spline and wavelet. Fan (1993) [17] and Fan and Gijbels (1996) [18] showed that the local polynomial smoother has many attractive properties. Therefore, we adopt the local polynomial method in this article to estimate the nonparametric component $g(\cdot)$, although any other nonparametric method will still work.

If T_i is in a small neighborhood of t, we can approximate $g(T_i)$ locally by a linear function as follows:

$$g(T_i) \approx g(t) + g'(t)(T_i - t) \equiv a + b(T_i - t) \tag{5}$$

for a given β, we can get the weighted local least-squares estimator of $g(t)$, termed $\hat{g}(t, \beta)$, by minimising

$$\sum_{i=1}^{n} \{Y_i - W_i^{\tau}\beta - a - b(T_i - t)\}^2 \cdot K_{h_1}(T_i - t) \tag{6}$$

where $K_{h_1}(\cdot) = h_1^{-1}K(\cdot/h_1)$, $K(\cdot)$ is kernel function, h_1 is a sequence of positive numbers tending to zero, called bandwidth. Simple calculation yields

$$\hat{a} = \frac{\sum_{i=1}^{n} \omega_i(t)(Y_i - W_i^{\tau}\beta)}{\sum_{j=1}^{n} \omega_j(t)} \tag{7}$$

where

$$\omega_j(t) = K_{h_1}(T_j - t)[S_{n,2}(t) - (T_j - t)S_{n,1}(t)]$$

$$S_{n,l}(t) = \frac{1}{n}\sum_{j=1}^{n} K_{h_1}(T_j - t)(T_j - t)^l, \quad l = 0, 1, 2.$$

Then we can estimate $g(t)$ by

$$\hat{g}(t, \beta) = \sum_{i=1}^{n} \omega_{h_1 i}(t)(Y_i - W_i^{\tau}\beta) \tag{8}$$

where

$$\omega_{h_1 i}(t) = \omega_i(t) / \sum_{j=1}^{n} \omega_j(t) \tag{9}$$

Let

$$\widetilde{Y}_i = Y_i - \sum_{j=1}^{n} \omega_{h_1 j}(T_i)Y_j, \quad \widetilde{W}_i = W_i - \sum_{j=1}^{n} \omega_{h_1 j}(T_i)W_j.$$

When the function $f(Z_i) = \sigma_i^2$ is unknown, the local linear estimator of $f(\cdot)$ is defined by $E(Y_i - X_i^{\tau}\beta - \hat{g}(T_i, \beta))^2 = \sigma_i^2$ and $E(Y_i - W_i^{\tau}\beta - \hat{g}(T_i, \beta))^2 = \sigma_i^2 + \beta^{\tau}\Sigma_U\beta$. Then we can get the estimator of σ_i^2, thus

$$\hat{\sigma}_{ni}^2 = \hat{f}_n(Z_i) = \sum_{j=1}^{n} \omega_{h_2 j}(t)(Y_j - W_j^{\tau}\beta - \hat{g}(T_i, \beta))^2 - \beta^{\tau}\Sigma_U\beta \tag{10}$$

where the weight functions $W_{h_2j}(\cdot)$ have the same from as $\omega_{h_1j}(\cdot)$ except that h_1 is replaced by the bandwidth h_2.

In order to define the empirical log-likelihood ratio functions with model (3), let $\mathcal{L}_n^2(\beta) = \frac{1}{n}\sum_{i=1}^n \sigma_i^{-2}[(Y_i - W_i^\tau\beta - \hat{g}(t_i,\beta))^2 - \beta^\tau\Sigma_U\beta]$. An estimator of the true parameter β should let $\mathcal{L}_n^2(\beta)$ attain minimum, hence we have

$$\sum_{i=1}^n \sigma_i^{-2}[\widetilde{W}_i(Y_i - \widetilde{W}_i^\tau\beta - \hat{g}(t_i,\beta)) + \Sigma_U\beta] = 0 \tag{11}$$

Next, we introduce two auxiliary random variables $\Lambda_{ki}(\beta)(k=1,2)$ under f is known and unknown,respectively,

$$\Lambda_{1i}(\beta) = \sigma_i^{-2}\widetilde{W}_i(Y_i - W_i^\tau\beta - \hat{g}(t_i,\beta)) + \sigma_i^{-2}\Sigma_U\beta = \sigma_i^{-2}\widetilde{W}_i(\widetilde{Y}_i - \widetilde{W}_i^\tau\beta) + \sigma_i^{-2}\Sigma_U\beta, \tag{12}$$

$$\Lambda_{2i}(\beta) = \hat{\sigma}_{ni}^{-2}\widetilde{W}_i(Y_i - W_i^\tau\beta - \hat{g}(t_i,\beta)) + \hat{\sigma}_{ni}^{-2}\Sigma_U\beta = \hat{\sigma}_{ni}^{-2}\widetilde{W}_i(\widetilde{Y}_i - \widetilde{W}_i^\tau\beta) + \hat{\sigma}_{ni}^{-2}\Sigma_U\beta. \tag{13}$$

Using such information, we can define an empirical log-likelihood ratio function for β under the conditions that f is known and unknown, respectively.

$$\mathscr{L}_{kn}(\beta) = -2\max\left\{\sum_{i=1}^n \log(np_{ki})|p_{ki}\geq 0, \sum_{i=1}^n p_{ki} = 1, \sum_{i=1}^n p_{ki}\Lambda_{ki}(\beta) = 0\right\}, \quad k=1,2. \tag{14}$$

where $p_{ki} = p_{ki}(\beta), i = 1,\ldots,n, k = 1,2$. Based on the method of Lagrange multipliers to find the optimal $p_{ki} = \frac{1}{n[1+\lambda_k^\tau\Lambda_{ki}(\beta)]}$, then the empirical log-likelihood ratio can be represent as

$$\mathscr{L}_{kn}(\beta) = 2\sum_{i=1}^n \log\{1 + \lambda_k^\tau\Lambda_{ki}(\beta)\}, \quad k=1,2. \tag{15}$$

where $\lambda_k = \lambda_k(\beta)$ is a $p\times 1$ vector, which satisfies

$$\sum_{i=1}^n \frac{\Lambda_{ki}(\beta)}{1+\lambda_k^\tau\Lambda_{ki}(\beta)} = 0, \quad k=1,2. \tag{16}$$

3. Main Results

In this section, it can be shown later that if β is the true parameter vector when f is known and unknown, $\mathscr{L}_{kn}(\beta)$ is asymptotically chi-square distributed and derived the nonparametric Wilks' theorem for $\Lambda_{kn}(\beta)(k=1,2)$, respectively. Let C denote positive constants whose values may vary at each occurrence. Before stating the main results, we list the following conditions.

A1 For some $s>2$ such that $E\|W\|^{2s}<\infty$, $E\|Z\|^{2s}<\infty$, $E\|e\|^{2s}<\infty$ and for some $\delta<2-s^{-1}$, there is $n^{2\delta-1}h_1\to\infty$, where $\|\cdot\|$ is Euclidean norm.

A2 $E(W_i|T_i)$ satisfy the Lipschitz condition of order 1.

A3 T has density function $m(t)$ on $[0,1]$, and

$$0<\inf_{0\le t\le 1} m(t)\le \sup_{0\le t\le 1} m(t)<\infty.$$

A4 $g(\cdot)$ and $f(\cdot)$ have the continuous second derivative on $[0,1]$ and

$$0<C_1\le \min_{1\le i\le n} f(Z_i)\le \max_{1\le i\le n} f(Z_i)\le C_2<\infty.$$

A5 The kernel $K(\cdot)$ is a symmetric probability density function, and is a bounded variation function on its support set $[-1,1]$.

A6 The bandwidth h_1 satisfies $nh_1^2/\log^2 n\to\infty$ and $nh_1^8\to 0$.

A7 The bandwidth h_1 and h_2 satisfy that $n^{1-\frac{3}{2s}}h_1/\log^2 n\to\infty$, $n^{3/s}h_1^8\log^2 n = o(1)$, $n^{\frac{1}{2}-\frac{1}{s}}h_2/\log n\to\infty$, $n^{1-\frac{1}{2s}}h_1^{\frac{1}{2}}h_2/\log^{\frac{3}{2}} n\to\infty$ and $n^{\frac{1}{2}-\frac{1}{s}}h_1^{-2}h_2/\log^2 n\to\infty$.

Theorem 3.1. *Suppose that Conditions A1-A6 holds. If β is the true parameter, then $\mathscr{L}_{1n}(\beta)\xrightarrow{\mathcal{L}}\chi_p^2$, where $\xrightarrow{\mathcal{L}}$ represent the convergence in distribution, and χ_p^2 means the chi-square distribution with p degree of freedom.*

Theorem 3.2. *Suppose that Conditions A1-A7 holds. If β is the true parameter, then $\mathscr{L}_{2n}(\beta)\xrightarrow{\mathcal{L}}\chi_p^2$.*

Let $\chi_p^2(\alpha)$ be the $1-\alpha$ quantile of χ_p^2 for $0<\alpha<1$. By using Theorems 1-2, we obtain an approximate $1-\alpha$ confidence region for β as $R_{k\alpha}=\{\beta: \mathscr{L}_{2n}(\beta)\le C_\alpha\}, k=1,2$ under $f(\cdot)$ is known and unknown, respectively.

4. Simulation Results

In this section, we carry out a Monte Carlo experiments to investigate the finite sample performance of the empirical likelihood approach for the above model in the cases $p = 1$ and $p = 2$, respectively.

First, we conduct a simulation study with $p = 1$. Simplicity, we consider the following heteroscedastic partially linear EV regression model

$$\begin{cases} Y_i &= X_i\beta_0 + \sin(2\pi T_i) + \sqrt{f(Z_i)}e_i \\ W_i &= X_i + U_i \end{cases}$$

where $\beta = 1$. Take $f(Z) = 1 - 0.5\sin(4\pi Z)$ where $f(\cdot)$ is known. The design point $X_i \sim N(0,1)$, $T_i \sim U(0,1)$, $Z_i \sim N(0,1)$, $e_i \sim N(0,1)$ and $U_i \sim N(0,\Sigma_U)$, $\Sigma_U = 0.2^2$. When $f(\cdot)$ is unknown, we can estimate $f(\cdot)$ by (10).

In the simulations, we draw 1000 random sample of sizes 50, 100 or 150 from the above model. The average lengths (AI) of the confidence intervals and their coverage probabilities (CP), with the nominal level $1 - \alpha = 0.90$ and 0.95,respectively. The kernel function is taken as the Epanechnikov kernel $K(t) = \frac{3}{4}(1 - t^2)I_{|t|\le 1}$. The "leave-one-sample-out" method is used to select the bandwidth h. Therefore, we can select the bandwidths as $h_1 = h_2 = \hat{h}_{opt}$. Some representive coverage probabilities and coverage confidence intervals are reported in Table 1 and Table 2 when f is known and unknown, respectively. Simulation results show that empirical likelihood method performs well.

n	$1-\alpha = 0.10$		$1-\alpha = 0.05$	
	CP	AI	CP	AI
50	0.848	0.682	0.863	0.671
100	0.897	0.539	0.901	0.518
150	0.917	0.449	0.925	0.406

Table 1: CP and AL: when $f(\cdot)$ is known and $\beta = 1$

Next, we conduct a simulation study with $p = 2$. Consider the following semiparametric heteroscedastic partially linear EV model

$$\begin{cases} Y_i &= X_{ia}^{\tau}\beta_0 + \sin(2\pi T_i) + \sqrt{f(Z_i)}e_i \\ W_i &= X_i + U_i \end{cases}$$

where $\beta = (1,1)^{\tau}$, $X_{i1} \sim N(0,1)$, $X_{i2} \sim N(0,1)$, $U_i \sim N(0,\Sigma_U)$, where $\Sigma_U = 0.2^2 I_2$. The other settings are the same as in the case $p = 1$ above when $f(\cdot)$ is known and unknown, respectively.

n	$1-\alpha=0.10$		$1-\alpha=0.05$	
	C	A	C	A
50	0.842	0.689	0.857	0.677
100	0.871	0.595	0.885	0.573
150	0.907	0.504	0.912	0.491

Table 2: CP and AL: when $f(\cdot)$ is unknown and $\beta = 1$

n	$\alpha = 0.10$		$\alpha = 0.05$	
	f known	f unknown	f known	f unknown
50	0.834	0.829	0.898	0.891
100	0.865	0.857	0.914	0.907
150	0.893	0.882	0.943	0.936

Table 3: CP: when $\beta = (1,1)^T$ and $f(\cdot)$ is known and unknown respectively

In this simulations, the size of samples are the same as the first simulations. The coverage probabilities of the confidence regions, with a nominal level $1-\alpha = 0.90$ and 0.95, for β are presented in Table 3 when f is known and unknown, respectively.

The conclusions of the above simulated experiments are summarized below:

(i) the empirical likelihood approach performs well when f is known and unknown;

(ii) the coverage probabilities of the confidence regions tend to increase and the average lengths decrease when the sample size n become larger, it also show that the confidence regions for known f is better than unknown f.

5. Proof of the Theorems

To present the proofs plainly. First, we introduce several lemmas.

Lemma 5.1. *Let* $\{\eta_1, \ldots, \eta_n\}$ *be i.i.d. random variables which satisfy* $E(\eta_i) = 0$ *and* $E(|\eta_i|^\delta) < \infty$ *for some* $\delta > 1$. *Let* $\{a_{ij}, 1 \le i, j \le n\}$ *is a series of real numbers such that* $\max_{1\le j\le n} \sum_{i=1}^n |a_{ij}| \le C_1 < \infty$. *If* $d_n =$

$\max_{1\le i,j\le n}|a_{ij}|$. *Then*

$$\max_{1\le j\le n}\left|\sum_{i=1}^{n}a_{ij}\eta_i\right| = O((n^{1/\delta}d_n \vee d_n^{1/2})\log n)a.s. \tag{17}$$

Lemma 5.2. *Suppose $\{\eta_1,\ldots,\eta_n\}$ be i.i.d. random variables.*

(i) If $E\eta_i = 0$ and $E|\eta_i|^3 < \infty$, then $\max_{1\le m\le n}|\sum_{i=1}^{m}\eta_i| = O(n^{1/2}\log n)$ a.s.

(ii)If $E|\eta_i|^s$ is bounded for $s > 1$, then $\max_{1\le i\le n}|\eta_i| = o(n^{1/s})$ a.s.

Lemma 5.3. *Under the same conditions of Theorem 1, we have*

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\Lambda_{1i}(\beta) \xrightarrow{d} N(0,\Omega), \tag{18}$$

$$\frac{1}{n}\sum_{i=1}^{n}\Lambda_{1i}(\beta)\Lambda_{1i}^{\tau}(\beta) \xrightarrow{p} \Omega, \tag{19}$$

$$\max_{1\le i\le n}\|\Lambda_{1i}(\beta)\| = O_p(n^{1/2}), \tag{20}$$

where $\Omega = E\{f^{-1}(U)(\Sigma+\Sigma_U)\}+E(f^{-2}(U))\{\Sigma\beta^{\tau}\Sigma_U\beta+E[(\Sigma_U-UU^{\tau})\beta\beta^{\tau}(\Sigma_U-UU^{\tau})]\} > 0$ and $\Sigma = E\{[X_i - E(X_i|T_i)]^{\otimes 2}\}$, here $A^{\otimes 2} = AA^{\tau}$

Proof. The proof is similar to Lemma 5.4, Lemma 5.5 and Lemma 5.6 of Fan et al. (2011) □

Proof of Theorem 2.1. Using the same arguments as were used in the proof of Owen (1990), we have

$$\lambda_1 = O_p(n^{-1/2}) \tag{21}$$

From (16), we have

$$0 = \frac{1}{n}\sum_{i=1}^{n}\frac{\Lambda_{1i}(\beta)}{1+\lambda_1^{\tau}\Lambda_{1i}(\beta)} = \frac{1}{n}\sum_{i=1}^{n}\Lambda_{1i}(\beta) - \frac{1}{n}\sum_{i=1}^{n}\Lambda_{1i}(\beta)\Lambda_{1i}^{\tau}(\beta)\lambda_1 + \frac{1}{n}\sum_{i=1}^{n}\frac{\Lambda_{1i}(\beta)[\lambda_1^{\tau}\Lambda_{1i}(\beta)]^2}{1+\lambda_1^{\tau}\Lambda_{1i}(\beta)}$$

By using Lemma 5.3, we obtain

$$\sum_{i=1}^{n}[\lambda_1^{\tau}\Lambda_{1i}(\beta)]^2 = \sum_{i=1}^{n}\lambda_1^{\tau}\Lambda_{1i}(\beta) + O_p(1) \tag{22}$$

$$\lambda_1 = \left[\sum_{i=1}^{n} \Lambda_{1i}(\beta)\Lambda_{1i}^{\tau}(\beta)\right]^{-1} \sum_{i=1}^{n} \Lambda_{1i}(\beta) + O_p(n^{-1/2}) \tag{23}$$

Applying the Taylor expansion for the empirical log-likelihood ratio, it follows that

$$\mathscr{L}_{1n}(\beta) = 2\sum_{i=1}^{n} \left\{\lambda_1^{\tau}\Lambda_{1i}(\beta) - [\lambda_1^{\tau}\Lambda_{1i}(\beta)]^2/2\right\} + O_p(1) \tag{24}$$

Then, by (22)-(24), we have

$$\mathscr{L}_{1n}(\beta) = \left[\frac{1}{\sqrt{n}}\sum_{i=1}^{n} \Lambda_{1i}(\beta)\right]^{\tau} \left[\frac{1}{n}\sum_{i=1}^{n} \Lambda_{1i}(\beta)\Lambda_{1i}^{\tau}(\beta)\right]^{-1} \left[\frac{1}{\sqrt{n}}\sum_{i=1}^{n} \Lambda_{1i}(\beta)\right] + O_p(1)$$

Together with Lemmas 5.3. This completes the proof of Theorem 1. □

Lemma 5.4. *With the same conditions of Theorem 2, for any $\delta > 0$, we have*

$$\max_{1\le k\le n} \left|\hat{f}_n(U_k) - f(U_k)\right| = O_p(n^{-1/4}h_2^{-3/4} + h_2^{2-\delta} + n^{-3/4}h_1^{-7/4}h_2^{-(1+\delta)})$$

Proof. The proof is similar to Lemma 5.3 of Fan et al. (2011) and we omit the details. □

Lemma 5.5. *Under the same conditions of Theorem 2, we have*

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n} \Lambda_{2i}(\beta) \xrightarrow{d} N(0, \Omega), \tag{25}$$

$$\frac{1}{n}\sum_{i=1}^{n} \Lambda_{2i}(\beta)\Lambda_{2i}^{\tau}(\beta) \xrightarrow{p} \Omega, \tag{26}$$

$$\max_{1\le i\le n} \|\Lambda_{2i}(\beta)\| = O_p(n^{1/2}), \tag{27}$$

Proof of Theorem 2.2. Together with Lemmas 5.4-5.5 and following the similar arguments in the proof of Theorem 1, we can easily verify Theorem 2 and hence is omitted here. □

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