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CONE METRIC SPACES AND COMMON FIXED POINT THEOREMS FOR CERTAIN CONTRACTIVE MAPPINGS

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Abstract: The purpose of this paper is to establish some common fixed point results for two Banach pairs of mappings which satisfy T-Reich and T-Rhoades contraction conditions in cone metric spaces without the assumption of normality condition of the cone.

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Key Words: fixed point, cone metric spaces, contraction mapping

1. Introduction and Preliminary Notes

Recently, Huang and Zhang [1] introduce the notion of cone metric spaces. He replaced real number system by ordered Banach space. He also gave the condition in the setting of cone metric spaces. These authors also described the convergence of sequences in the cone metric spaces and introduce the corresponding notion of completeness. Subsequently, many authors have generalized the results of Huang and Zhang and have studied fixed point theorems for normal and non-normal cones, see for instance [4], [8], [10], [11], [13], [14], etc.

In 2009, A. Beiranvand [2] et al introduced new classes of contractive functions T-contraction and T-contractive mappings and then they established and

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extended the Banach contraction principle. J.R. Morales and the E. Rojas [5], [6] obtained sufficient conditions for the existence of a unique fixed point of T-Kannan contractive, T-Zamfirescu, T-contractive mappings on complete cone metric spaces.

In [8], authors have proved some common fixed point theorems for a Banach pair of mappings satisfying T-Hardy Rogers type contraction condition in cone metric spaces. In sequel M. Ozturk and M. Basarir [3] proved some common fixed point theorems for f-contraction mapping in cone metric spaces without the assumption of normality condition of the cone. Subrahmanyam [9] introduced Banach operator of type k. Recently Chen and Li [7] extended the concept of Banach operator of type k to Banach operator pair and proved various best approximation results using common fixed point theorems for f-nonexpansive mappings.

The aim of this paper is to prove common fixed point theorems for two Banach pairs of mappings which satisfy T-Reich and T-Rhoades contraction conditions in cone metric spaces without the assumption of normality condition of the cone.

First, we recall some standard definitions and other results that will be needed in the sequel.

Definition 1.1. A self mapping T of a metric space (X, d) is said to be a contraction mapping, if there exists a real number $0 \leqslant k < 1$ such that for all $x, y \in X$,

$$d(Tx, Ty) \leqslant kd(x, y). \tag{1.1}$$

Definition 1.2. (see [2]) Let T and f be two self-mappings of a metric space (X, d). The self mapping f of X is said to be T-contraction, if there exists a real number $0 \leqslant k < 1$ such that

$$d(Tfx, Tfy) \leqslant kd(Tx, Ty), \tag{1.2}$$

for all $x, y \in X$.

If $T = I$, the identity mapping, Definition 1.2 reduces to Banach contraction mapping.

Example 1. Let $X = [0, \infty)$ be with the usual metric. Let define two mappings $T, f : X \to X$ as

$$fx = \beta x, \quad \beta > 1,$$

$$Tx = \frac{\propto}{x^2}, \quad \propto \in R.$$

It is clear that, f is not contraction but f is T-contraction, since,

$$d(Tfx, Tfy) = \left| \frac{\propto}{\beta^2 x^2} - \frac{\propto}{\beta^2 y^2} \right| = \frac{1}{\beta^2} \mid Tx - Ty \mid .$$

Definition 1.3. (see [2] Let T be a self mapping of a metric space (X, d). Then:

(i) A mapping T is said to be sequentially convergent, if the sequence $\{y_n\}$ in X is convergent whenever $\{Ty_n\}$ is convergent.

(ii) A mapping T is said to be subsequentially convergent, if $\{y_n\}$ has a convergent subsequence whenever $\{Ty_n\}$ is convergent.

Definition 1.4. (see [9]) Let T be a self mapping of a normed space X. Then T is called a Banach operator of type k if

$$\| T^2x - Tx \| \leqslant k \| Tx - x \|,$$

for some $k \geqslant 0$ and for all $x \in X$.

This concept was introduced by Subrahmanyam [9] then Chen and Li [7] extended this as following.

Definition 1.5. (see [7]) Let T and f be two self mappings of a non-empty subset M of a normed linear space X. Then (T, f) is a Banach operator pair, if any one of the following conditions is satisfied:

(i) $T[F(f)] \subseteq F(f)$, i.e. $F(f)$ is T-invariant;

(ii) $fTx = Tx$ for each $x \in F(f)$;

(iii) $fTx = Tfx$ for each $x \in F(f)$;

(iv) $\| Tfx - fx \| \leqslant k \| fx - x \|$ for some $k \geqslant 0$.

Definition 1.6. (see [1]) Let E be a real Banach space and P a subset of E. P is called a cone if and only if:

(i) P is closed, non-empty and $P \neq \{0\}$;

(ii) $ax + by \in P$ for all $x, y \in P$ and non-negative real numbers a, b;

(iii) $x \in P$ and $-x \in P \Rightarrow x = 0 \Leftrightarrow P \cap (-P) = \{0\}$.

Given a cone $P \subset E$, a partial ordering is defined as $\leqslant$ on E with respect to P by $x \leqslant y$ if and only if $y - x \in P$. It is denoted as $x \ll y$ will stand for

$y-x \in int\,P$ where $int\,P$ denotes the interior of P. The cone P is called normal if there is a number $K>0$ such that for all $x, y \in E$,

$$0 \leqslant x \leqslant y \text{ implies } \| x \| \leqslant K \| y \| . \tag{1.3}$$

The least positive number K satisfying (1.3) is called normal constant of P.

Definition 1.7. (see [1]) Let X be a non-empty set. Suppose E is a real Banach space, P is a cone with $int\,P \neq \phi$ and $\leqslant$ is a partial ordering with respect to P. If the mapping $d : X \times X \to E$ satisfies:

(i) $0 \leqslant d(x,y)$ for all $x, y \in X$ and $d(x,y) = 0$ if and only if $x = y$;

(ii) $d(x,y) = d(y,x)$ for all $x, y \in X$;

(iii) $d(x,y) \leqslant d(x,z) + d(z,y)$ for all $x, y, z \in X$.

Then d is called a cone metric on $X$and (X,d) is called a cone metric space.

Definition 1.8. (see [5]) Let (X,d) be a cone metric space and $\{x_n\}$be a sequence in X. Then:

(i) $\{x_n\}$ converges to $x \in X$, if for every $c \in E$ with $0 \ll c$, there is $n_o \in N$, the set of all natural numbers such that for all $n \geqslant n_o, d(x_n, x) \ll c$. It is denoted by $\lim\limits_{n\to\infty} x_n = x$ or $x_n \to x, (n \to \infty)$;

(ii) If for any $c \in E$, there is a number $n_o \in N$ such that for all $m, n \geqslant n_o, d(x_n, x_m) \ll c$,then $\{x_n\}$is called a Cauchy sequence in X; (X,d) is a complete cone metric space, if every Cauchy seuence in X is convergent;

A self mapping $T : X \to X$ is said to be continuous at a point $x \in X$, if $\lim\limits_{n\to\infty} x_n = x$ implies that $\lim\limits_{n\to\infty} Tx_n = Tx$ for every $\{x_n\}$ in X.

Lemma 1.1. (see [11]) Let (X,d) be a cone metric space, $u, v, w \in X$ Then:

(i) If $u \ll v$ and $v \ll w$, then $u \ll w$;

(ii) If $u \leqslant v$ and $v \ll w$, then $u \ll w$;

(iii) If $0 \leqslant u \ll c$ for each $c \in int\,P$, then $u = 0$;

(iv) If $c \in int\,P, 0 \leqslant a_n$ and $a_n \to 0$,then there exists n_o such that for all $n > n_o$, it follows that $a_n \ll c$.

2. Main Results

First, we give definitions of T-Reich contractive and T-Rhoades contractive mappings on cone metric spaces which are based on the ideas of Morales and Rojas [5].

Definition 2.1. Let (X, d) be a cone metric space and $T, S : X \to X$ two functions:

(i) A mapping S is said to be T-Reich contraction, if there is $a + b + c < 1$ such that

$$d(TSx, TSy) \leqslant ad(Tx, TSx) + bd(Ty, TSy) + cd(Tx, Ty), \tag{2.1}$$

for all $x, y \in X$ and $a, b, c \geqslant 0$.

(ii) A mapping S is said to be T-Rhoades contraction, if there is $a+b+c < 1$ such that

$$d(TSx, TSy) \leqslant ad(Tx, TSy) + bd(Ty, TSx) + cd(Tx, Ty), \tag{2.2}$$

for all $x, y \in X$ and $a, b, c \geqslant 0$.

Theorem 2.1. *Let T, f and g be three continuous self mappings of a complete cone metric space (X, d). Assume that T is a injective mapping. If the mappings T, f and g satisfy*

$$d(Tfx, Tgy) \leqslant a_1 d(Tx, Tfx) + a_2 d(Ty, Tgy) + a_3 d(Tx, Ty), \tag{2.3}$$

for all $x, y \in X$ where $a_i, i = 1, 2, 3$ are all non-negative constants such that $a_1+a_2+a_3 < 1$ then f and g have a unique common fixed point in X. Moreover, if (T, f) and (T, g) are Banach pairs, then T, f and g have a unique common fixed point in X.

Proof. Let $x_o \in X$ as an arbitrary element and define the sequences $x_{2n+1} = fx_{2n}$ and $x_{2n+2} = gx_{2n+1}$ for each $n = 0, 1, 2, \ldots, \infty$. Then, by using (2.3) and triangle inequality

$$\begin{aligned} d(Tx_{2n+1}, Tx_{2n}) =& d(Tfx_{2n}, Tgx_{2n-1}) \\ \leqslant& a_1 d(Tx_{2n}, Tfx_{2n}) + a_2 d(Tx_{2n-1}, Tgx_{2n-1}) \\ &+ a_3 d(Tx_{2n}, Tx_{2n-1}) \\ =& a_1 d(Tx_{2n}, Tx_{2n+1}) + a_2 d(Tx_{2n-1}, Tx_{2n}) \\ &+ a_3 d(Tx_{2n}, Tx_{2n-1})(1 - a_1) d(Tx_{2n+1} Tx_{2n}) \end{aligned}$$

$$\begin{aligned}&\leqslant(a_2+a_3)d(Tx_{2n},Tx_{2n-1})\\&\quad d(Tx_{2n+1},Tx_{2n})\\&=\frac{a_2+a_3}{1-a_1}d(Tx_{2n},Tx_{2n-1}).\end{aligned}$$

Similarly:

$$\begin{aligned}d(Tx_{2n+3},Tx_{2n+2})=&d(Tfx_{2n+2},Tgx_{2n+1})\\\leqslant&a_1d(Tx_{2n+2},Tfx_{2n+2})+a_2d(Tx_{2n+1},Tgx_{2n+1})\\&+a_3d(Tx_{2n+2},Tx_{2n+1})\\=&a_1d(Tx_{2n+2},Tx_{2n+3})+a_2d(Tx_{2n+1},Tx_{2n+2})\\&+a_3d(Tx_{2n+2},Tx_{2n+1}),\end{aligned}$$

$$d(Tx_{2n+3},Tx_{2n+2})=\frac{a_2+a_3}{1-a_1}d(Tx_{2n+2},Tx_{2n+1}).$$

Thus

$$d(Tx_{n+1},Tx_n)\leqslant\lambda d(Tx_n,Tx_{n-1})\leqslant\cdots\leqslant\lambda^n d(Tx_1,Tx_o),$$

for all $n\geqslant 0$, where $\lambda=\frac{a_2+a_3}{1-a_1}<1$.

Now for $n>m$ we have

$$\begin{aligned}d(Tx_n,Tx_m)\leqslant&d(Tx_n,Tx_{n-1})+d(Tx_{n-1},Tx_{n-2})+\cdots+d(Tx_{m+1},Tx_m)\\\leqslant&(\lambda^{n-1}+\lambda^{n-2}+\cdots+\lambda^m)d(Tx_1,Tx_o)\\\leqslant&\frac{\lambda^m}{1-\lambda}d(Tx_1,Tx_o).\end{aligned}$$

Let $0\ll c$ be given. Choose $\delta>0$ such that $c+N_\delta(0)\subseteq P$, where

$$N_\delta(0)=\{y\in E:\parallel y\parallel<\delta\}.$$

Also, choose a natural number N_1 such that $\frac{\lambda^m}{1-\lambda}d(Tx_1,Tx_o)\in N_\delta(0)$, for all $m\geqslant N_1$. Then

$$\frac{\lambda^m}{1-\lambda}d(Tx_1,Tx_o)\ll c,\ \text{for all } m\geqslant N_1.$$

Thus

$$d(Tx_n,Tx_m)\leqslant\frac{\lambda^m}{1-\lambda}d(Tx_1,Tx_o)$$

and

$$\frac{\lambda^m}{1-\lambda}d(Tx_1,Tx_o)\ll c,$$

for all $m > n$. Then we get $d(Tx_n, Tx_m) \ll c$ for all $n > m$. Therefore, $\{Tx_n\}$ is a Cauchy sequence in (X, d). As X is complete, there exists $z \in X$ such that $\lim_{n\to\infty} Tx_n = z$.

Since T is sub-sequentially convergent, $\{x_n\}$ has a convergent subsequence $\{x_m\}$ such that $\lim_{m\to\infty} x_m = u$. As T is continuous

$$\lim_{m\to\infty} Tx_m = Tu.$$

By the uniqueness of the limit, $z = Tu$. Since f and g are continuous, $\lim_{m\to\infty} gx_m = gu$ and $\lim_{m\to\infty} fx_m = fu$. Agains since T is continuous, $\lim_{m\to\infty} Tgx_m = Tgu$ and $\lim_{m\to\infty} Tfx_m = Tfu$.

Therefore, if m is odd, then

$$\lim_{n\to\infty} Tgx_{2n+1} = Tgu.$$

Choose a natural number N_2 such that

$$d(Tx_{2n+1}, Tu) \ll [\frac{c}{2}(\frac{a_2 + a_3}{1 - a_1})]$$

for all $n \geqslant N_2$.

Now consider

$$\begin{aligned}
d(Tgu, Tu) \leqslant& d(Tgu, Tx_{2n+1}) + d(Tx_{2n+1}, Tu)\\
=& d(Tgu, Tfx_{2n}) + d(Tfx_{2n}, Tu)\\
\leqslant& a_1 d(Tu, Tgu) + a_2 d(Tx_{2n}, Tfx_{2n})\\
&+ a_3 d(Tu, Tx_{2n}) + d(Tx_{2n+1}, Tu)\\
=& a_1 d(Tu, Tgu) + a_2 d(Tx_{2n}, Tx_{2n+1})\\
&+ a_3 d(Tu, Tx_{2n}) + d(Tx_{2n+1}, Tu).
\end{aligned}$$

So

$$d(Tu, Tgu) \leqslant (\frac{a_2 + a_3}{1 - a_1}) d(Tx_{2n}, Tu) + (\frac{1 + a_2}{1 - a_1}) d(Tu, Tx_{2n+1}) \ll c,$$

for all $n \geqslant N_2$. Therefore, $d(Tu, Tgu) \ll \frac{c}{i}$ for all $i \geqslant 1$. Hence, $\frac{c}{i} - d(Tu, Tgu) \in P$ for all $i \geqslant 1$. Since P is closed, $-d(Tu, Tgu) \in P$ and so $d(Tu, Tgu) = 0$. Hence $Tu = Tgu$. As T is injective, $u = gu$. Thus u in the fixed point of g. And if m is even, then we have

$$\lim_{n\to\infty} Tfx_{2n} = Tfu.$$

Now, by using (2.3) and triangle inequality, we have

$$d(Tu,Tfu) \leqslant (\frac{a_2+a_3}{1-a_1})d(Tx_{2n+1},Tu) + (\frac{1+a_2}{1-a_1})d(Tu,Tx_{2n+2}) \ll c,$$

for all $n \geqslant N_2$. Therefore, $d(Tu,Tfu) \ll \frac{c}{i}$ for all $i \geqslant 1$. Hence, $\frac{c}{i} - d(Tu,Tfu) \in P$ for all $i \geqslant 1$. Since P is closed, $-d(Tu,Tfu) \in P$ and so $d(Tu,Tfu) = 0$. Hence $Tu = Tfu$. As T is injective, $u = fu$. Thus u in the fixed point of f too.

For the uniqueness suppose that u^* is another common fixed point of f amd g,

$$\begin{aligned} d(Tu,Tu^*) =& d(Tfu,Tgu^*) \\ \leqslant& a_1 d(Tu,Tfu) + a_2 d(Tu^*,Tgu^*) \\ &+ a_3 d(Tu,Tu^*), \end{aligned}$$

$$d(Tu,Tu^*) \leqslant (a_1+a_2+a_3)d(Tu,Tu^*).$$

Since $a_1+a_2+a_3 < 1, d(Tu,Tu^*) = 0$ which implies that $Tu = Tu^*$. We know that T is injective, $u = u^*$ is the unique common fixed point of f and g. Since we have assumed that $\{T,f\}$ and $\{T,g\}$ are Banach pairs; $\{T,f\}$ and $\{T,g\}$ commutes at the fixed point of f and g, respectively. This implies that $Tfu = fTu$ for $u \in F(f)$. So $Tu = fTu$ which gives that Tu is another fixed point of f. It is true for g, too. By the uniquesness of fixed point of f, $Tu = u$. Hence $u = Tu = fu = gu, u$ is unique common fixed point of T, f and g in X.

Theorem 2.2. *Let T, f and g be three continuous self mappings of a complete cone metric space (X,d). Assume that T is a injective mapping. If the mappings T, f and g satisfy*

$$d(Tfx,Tgy) \leqslant a_1 d(Tx,Tgy) + a_2 d(Ty,Tfx) + a_3 d(Tx,Ty), \tag{2.4}$$

for all $x,y \in X$ where $a_i, i = 1,2,3$ are all non-negative constants such that $a_1+a_2+a_3 < 1$ and $a_1 = a_2$ then f and g have a unique common fixed point in X. Moreover, if (T,f) and (T,g) are Banach pairs, then T, f and g have a unique common fixed point in X.

Proof. Let $x_o \in X$ as an arbitrary element and define the sequences $x_{2n+1} = fx_{2n}$ and $x_{2n+2} = gx_{2n+1}$ for each $n = 0,1,2,\ldots,\infty$. Then by using (2.4) and triangle inequality

$$\begin{aligned} d(Tx_{2n+1},Tx_{2n}) =& d(Tfx_{2n},Tgx_{2n-1}) \\ \leqslant& a_1 d(Tx_{2n},Tgx_{2n-1}) + a_2 d(Tx_{2n-1},Tfx_{2n}) \end{aligned}$$

$$\begin{aligned}
&+ a_3 d(Tx_{2n}, Tx_{2n-1}) \\
=&a_1 d(Tx_{2n}, Tx_{2n}) + a_2 d(Tx_{2n-1}, Tx_{2n+1}) \\
&+ a_3 d(Tx_{2n}, Tx_{2n-1}) \\
\leqslant&a_2 [d(Tx_{2n-1}, Tx_{2n}) + d(Tx_{2n}, Tx_{2n+1})] \\
&+ a_3 d(Tx_{2n}, Tx_{2n-1}),
\end{aligned}$$

$$d(Tx_{2n+1}, Tx_{2n}) \leqslant a_2 d(Tx_{2n+1}, Tx_{2n}) + (a_2 + a_3) d(Tx_{2n}, Tx_{2n-1}),$$

$$d(Tx_{2n+1}, Tx_{2n}) = \frac{a_2 + a_3}{1 - a_2} d(Tx_{2n,} Tx_{2n-1}).$$

Similarly,

$$\begin{aligned}
d(Tx_{2n+3}, Tx_{2n+2}) =&d(Tfx_{2n+2}, Tgx_{2n+1}) \\
\leqslant&a_1 d(Tx_{2n+2}, Tgx_{2n+1}) + a_2 d(Tx_{2n+1}, Tfx_{2n+2}) \\
&+ a_3 d(Tx_{2n+2}, Tx_{2n+1}) \\
=&a_1 d(Tx_{2n+2}, Tx_{2n+2}) + a_2 d(Tx_{2n+1}, Tx_{2n+3}) \\
&+ a_3 d(Tx_{2n+2}, Tx_{2n+1}) \\
\leqslant&a_2 [d(Tx_{2n+1}, Tx_{2n+2}) + d(Tx_{2n+2}, Tx_{2n+3})] \\
&+ a_3 d(Tx_{2n+2}, Tx_{2n+1}),
\end{aligned}$$

$$d(Tx_{2n+3}, Tx_{2n+2}) = \frac{a_2 + a_3}{1 - a_2} d(Tx_{2n+2,} Tx_{2n+1}).$$

Thus

$$d(Tx_{n+1}, Tx_n) \leqslant \lambda d(Tx_n, Tx_{n-1}) \leqslant \cdots \leqslant \lambda^n d(Tx_1, Tx_o),$$

for all $n \geqslant 0$ where $\lambda = \frac{a_2 + a_3}{1 - a_2} < 1$.

Now for $n > m$ we have

$$\begin{aligned}
d(Tx_n, Tx_m) \leqslant&d(Tx_n, Tx_{n-1}) + d(Tx_{n-1}, Tx_{n-2}) + \cdots + d(Tx_{m+1}, Tx_m) \\
\leqslant&(\lambda^{n-1} + \lambda^{n-2} + \cdots + \lambda^m) d(Tx_1, Tx_0) \\
\leqslant&\frac{\lambda^m}{1 - \lambda} d(Tx_1, Tx_o).
\end{aligned}$$

Let $0 \ll c$ be given. Choose $\delta > 0$ such that $c + N_\delta(0) \subseteq P$, where $N_\delta(0) = \{y \in E : \| y \| < \delta\}$.

Also, choose a natural number N_1 such that $\frac{\lambda^m}{1-\lambda} d(Tx_1, Tx_0) \in N_\delta(0)$, for all $m \geqslant N_1$. Then $\frac{\lambda^m}{1-\lambda} d(Tx_1, Tx_0) \ll c$, for all $m \geqslant N_1$. Thus

$$d(Tx_n, Tx_m) \leqslant \frac{\lambda^m}{1 - \lambda} d(Tx_1, Tx_0)$$

and

$$\frac{\lambda^m}{1-\lambda} d(Tx_1, Tx_0) \ll c,$$

for all $m > n$. Then we get $d(Tx_n, Tx_m) \ll c$ for all $n > m$. Therefore, $\{Tx_n\}$ is a Cauchy sequence in (X, d). As X is complete, there exists $z \in X$ such that $\lim_{n\to\infty} Tx_n = z$.

Since T is subsequentially convergent, $\{x_n\}$has a convergent subsequence $\{x_m\}$ such that $\lim_{m\to\infty} x_m = u$. As T is continuous $\lim_{m\to\infty} Tx_m = Tu$.

By the uniqueness of the limit, $z = Tu$. Since f and g are continuous, $\lim_{m\to\infty} gx_m = gu$ and $\lim_{m\to\infty} fx_m = fu$.

The rest of the proof is similar to the proof of Theorem 2.1.

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