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γ-STABLE TREE

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Abstract: For a given non-adjacent pair $\{x, y\}$ in a graph G, we denote by G_{xy} the graph obtained by deleting x and y and adding a new vertex xy adjacent to precisely those vertices of $G - x - y$ which were adjacent to at least one of x or y in G. We say that G_{xy} is obtained by contracting on $\{x, y\}$. In this paper we introduce γ-stable graphs and we have established that γ-stable trees have a unique structure.

AMS Subject Classification: 05C69
Key Words: domination, γ-stable

1. Introduction

We consider only simple connected undirected graphs$G = (V, E)$. We say that H is a subgraph of G, if $V(H) \subseteq V(G)$ and $uv \in E(H)$ implies $uv \in E(G)$. If a subgraph H satisfies the added property that for every pair u, v of vertices, $uv \in E(H)$ if and only if $uv \in E(G)$, then H is called an induced subgraph of G and is denoted by $< H >$. We indicate that u is adjacent to v by writing $u \perp v$. A set of vertices D in a graph $G = (V, E)$ is a dominating set if every vertex of $V - D$ is adjacent to some vertex of D. If D has the smallest possible cardinality of any dominating set of G, then D is called a minimum dominating set-abbreviated

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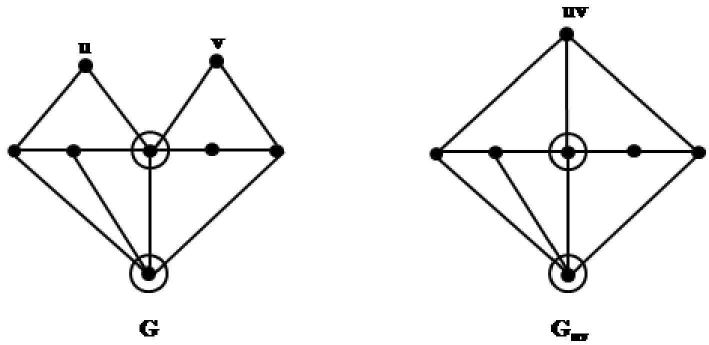

Figure 1: In Figure 1, $\gamma(G) = \gamma(G_{uv}) = 2$, this is true for all $x, y \in V(G)$, where x not adjacent to y.

MDS. The cardinality of any MDS for G is called the domination number of G and it is denoted by $\gamma(G)$. γ- set denotes a dominating set for G with minimum cardinality. A set of vertices D in a graph G is called a clique dominating set if every two vertices in D are adjacent. The open neighborhood of vertex $v \in V(G)$ is denoted by $N(v) = \{u \in V(G)|(uv) \in E(G)\}$ while its closed neighborhood is the set $N[v] = N(v) \cup \{v\}$. The private neighborhood of $v \in D$ is denoted by $pn[v, D]$, is defined by $pn[v, D] = N(v) - N(D - \{v\})$. A vertex v is said to be a, down vertex if $\gamma(G-u) < \gamma(G)$, level vertex if $\gamma(G-u) = \gamma(G)$, up vertex if $\gamma(G - u) > \gamma(G)$. For details of on domination we refer to [2]. For a given non-adjacent pair $\{x, y\}$ in a graph G, we denote by G_{xy} the graph obtained by deleting x and y and adding a new vertex xy adjacent to precisely those vertices of $G - x - y$ which were adjacent to at least one of x or y in G. We say that G_{xy} is obtained by contracting on $\{x, y\}$[1].

In this paper we introduce γ-stable graphs and initiate a study on them.

2. γ-Stable Graphs

A γ-set $D \subseteq V$ is said to graph domination set if D is a clique dominating set for G, that is $\gamma(G_{xy}) = \gamma(G)$. In all the figures encircled vertices denote a γ-set for G

Theorem 1. *A graph G is γ- stable if and only if every γ- set D of G is clique dominating.*

Proof. Let G be a γ-stable graph. If there is a γ-set D of G such that $< D >$ is not a clique, then we mean that there exist $u, v \in D, u$ not adjacent to v, such that $D^{'} = D - \{u\} - \{v\} \cup \{uv\}$ is a γ- set for G_{uv}, $|D^{'}| < |D|$, a contradiction as G is γ-stable, which implies any γ-set of G is a clique dominating set.

Assume that, G is a graph such that for every γ-set D is a clique dominating set. We claim that G is γ-stable.

Let D be a γ-set for G. Let $x, y \in V(G)$ such that x not adjacent y. We claim that $\gamma(G_{xy}) = |D|$. If possible let $\gamma(G_{xy}) < |D|$. Let $D^{'}$ be a γ-set for G_{xy}.

Case i. $x, y \in V - D$.

Subcase i. $xy \in D^{'}$.

$D^{''} = D^{'} - \{xy\} \cup \{x\} \cup \{y\}$ is a γ-set for G such that x is not adjacent to y. This is not possible, since every γ- set of G is clique dominating.

Subcase ii. $xy \notin D^{'}$.

It is not possible that, every vertex in D' is adjacent to xy else $D^{'}$ itself will be a γ-set for G such that $|D^{'}| < |D|$ a contradiction. Hence $D^{''} = D^{'} \cup \{y\}$ is a γ-set for G, if D' dominates x or $D^{'} \cup \{x\} = D^{''}$ is a γ-set for G, if D' dominates y such that $D^{''}$ is not a clique. Hence $\gamma(G_{xy}) = |D|$.

Case ii. $x \in D, y \in V - D$.

Subcase i, $xy \notin D^{'}$.

Proof is similar to Subcase i of Case i.

Subcase ii. $xy \notin D^{'}$.

Proof is similar to Subcase ii of Case i.

Hence $\gamma(G_{xy}) = |D|$, for all $x, y \in V(G)$, x is not adjacent to y, that is G is a γ-stable graph. □

Theorem 2. *If G is γ-stable, then $pn[u, D] \geq 2$, for all $u \in V(G)$.*

Proof. Let G be γ-stable and D be a γ-set for G. Let $u \in D$. Suppose if $pn[u, D] = 1$ or ϕ.

1. If $pn[u, D] = 1$, say $pn[u, D] = v$, then $D^{'} = D - \{u\} \cup \{v\}$ is a γ-set for G, a contradiction to theorem[1], that is D' does not form a clique.

2. If $pn[u, D] = \phi$, then $D^{'} = D - \{u\}$ itself is a γ- set for G. Hence $pn[u, D] \geq 2$. □

3. γ-Stable Tree

Theorem 3. *A γ-stable graph is a tree if G has a unique γ- set such that $\gamma(G) = 2$.*

Proof. Assume that T is a γ-stable tree. Since T has no cycles, every γ-set of T is K_2, which implies $\gamma(T) = 2$. If T has two different γ-sets say D_1 and D_2.

Case i. D_1 and D_2 are distinct.

Let $x, y \in D_1$ and $p, q \in D_2$. Since D_1 is a γ-set, every element of D_2 is adjacent to atleast one element of D_1 and vice - versa. If p and q are adjacent distinct elements of D_1, then cycle $< xypq >$ is C_4. If p and q are both adjacent to the same element of D_1(say x), then cycle $C_3 : xpq$ is created. In both cases, we get a contradiction as T is a tree.

Case ii. D_1 and D_2 are not distinct.

Let $x, y \in D_1$ and $x, z \in D_2$. Since T is γ-stable $(xy), (xz) \in E(T)$, that is there is a path of length 2 from z to y, via x. Since D_1 and D_2 are γ-sets, z dominates $pn[y, D_1]$ and y dominates $pn[z, D_2]$. Since T is γ-stable , $pn[y, D_1] \geq 2$ [2]. Assume that $x_1 \in pn[y, D_1]$, that is $z \perp x_1$ and $y \perp x_1$, already $x \perp y, z$. This implies $< xzx_1y >$ is C_4, a contradiction as T is a tree.
Hence if T is γ- stable, then T has a unique γ-set D such that $|D| = 2$. □

Converse need not be true, that is if G is a γ-stable graph, that has a unique γ-set such that $\gamma(G) = 2$, then G need not be a tree.

Theorem 4. *If G is a γ- stable tree, then every $v \in V - D$ is a pendant vertex.*

Proof. By theorem [3], we know that $\gamma(T) = 2 = \{u, v\}$(say), such that $u \perp v$. Assume that any $w \in V - D$ is an internal vertex. w cannot be a 2 - dominated vertex, since D is a unique γ-set [3], and G is γ-stable[else $< uvw >$ forms circute C_3], that is if w is an internal vertex, then w is dominated either by u or v, say w is dominated by u. w is an internal vertex adjacent to u. Since w is an internal vertex there is a vertex say $u_1 \perp w$. But we know that D is a unique γ-set. This implies either u or v dominates u_1 also, that is $< uu_1w >$ forms circute C_3 or $< vu_1wu >$ forms a circuit C_4, a contradiction as T is a tree. Hence w is not an internal vertex, that is every vertex in $V - D$ is a pendant vertex.

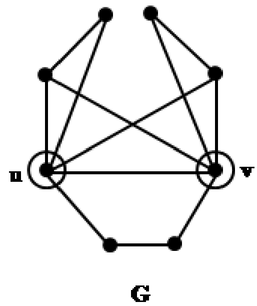

Figure 2: In Figure 2, $\gamma(G) = 2$, $\{uv\}$ is the only possible γ-set and G is not a tree.

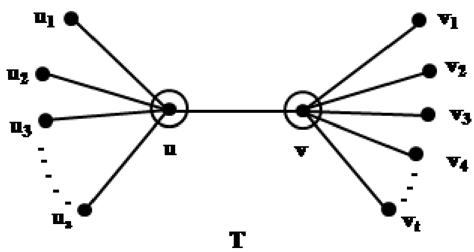

Figure 3: In Figure 3, T represent a γ stable tree.

4. Conclusion

From the Thorems 3 and 4, we concude that if T is a γ-stable tree, then:

- $\gamma(T) = 2$.
- $\gamma(T)$ is unique.
- Every vertex in $V - D$ is pendant.

  So, the tree structure of any γ stable tree is

□

References

[1] Hazel Everett et al, *Path Parity and Perfection*, http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.52.9669.

[2] T.W. Haynes, S.T. Hedetniemi, P.J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker, New York (1998).