International Journal of Pure and Applied Mathematics
Volume 90 No. 3 2014, 309-320
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v90i3.5

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FUZZY (r, s)-MINIMAL SEMIOPEN SETS AND FUZZY (r, s)-M SEMICONTINUOUS MAPPINGS ON FUZZY (r, s)-MINIMAL SPACES

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Abstract: We introduce the concept of fuzzy (r, s)-minimal semiopen set on a fuzzy (r, s)-minimal space. We also introduce the concept of fuzzy (r, s)-M semicontinuous mapping which is a generalization of fuzzy (r, s)-M continuous mapping, and investigate characterization of fuzzy (r, s)-M semicontinuity.

AMS Subject Classification: 54C08
Key Words: fuzzy minimal structures, (r, s)-minimal open, (r, s)-minimal semiopen, (r, s)-minimal preopen, fuzzy (r, s)-M continuous, fuzzy (r, s)-M semicontinuous

1. Introduction

The concept of fuzzy set was introduced by Zadeh [16]. Chang [3] introduced the concept of fuzzy topology in terms of fuzzy sets defined by Zadeh. Kubiak [10]

Received: July 5, 2013

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and Šostak [15] also introduced the fundamental concept of a fuzzy topological structure. Chattopadhyay et al [4] have redefined the same concept under the name gradation of openness. A general approach to the study of topological type structures on fuzzy power sets was developed in [7]-[11].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [2]. By using intuitionistic fuzzy sets, Çoker and his colleagues [5], [6] introduced intuitionistic fuzzy topological spaces. In [4], Çoker and Demirci introduced intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces. Recently, Samanta and Mondal [14], introduced the notion of intuitionistic gradation of openness of fuzzy sets, where to each fuzzy subsets there is a definite grade of openness and there is a grade of non-openness. Thus, the concept of intuitionistic gradation of openness is a generalization of the concept of gradation of openness and the topology of intuitionistic fuzzy sets. In [12], we introduced the concept of fuzzy (r, s)-minimal space which is an extension of the intuitionistic fuzzy topological space in Šostak's sense. We also introduced and studied the concept of fuzzy (r, s)-M continuity. In this paper, we introduce the concepts of fuzzy (r, s)-minimal semiopen set, fuzzy (r, s)-minimal semi-interior operator, fuzzy (r, s)-minimal semi-closure operator and fuzzy (r, s)-M semicontinuous mapping. We also study some basic properties for fuzzy (r, s)-minimal semi-interior operators and fuzzy (r, s)-minimal semi-closure operators. Moreover, we investigate characterizations for the fuzzy (r, s)-M semicontinuity in terms of fuzzy (r, s)-minimal semi-interior operators and fuzzy (r, s)-minimal semi-closure operators.

2. Preliminaries

Let I be the unit interval $[0, 1]$ of the real line. A member μ of I^X is called a fuzzy set of X. For any $\mu \in I^X$, μ^c denotes the complement $1 - \mu$. By $\underline{0}$ and $\underline{1}$ we denote constant maps on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \underline{1} - \mu)$.

Throughout this paper, let X be a nonempty set, $I = [0,1]$ and $I_0 = (0,1]$ and $I_1 = [0,1)$. For $\alpha \in I$, $\underline{\alpha}(x) = \alpha$ for all $x \in X$.

Definition 2.1. (see [14]) An intuitionistic gradation of openness (IGO, for short) on X is an ordered pair (τ, τ^*) of functions from I^X to I such that

(IGO1) $\tau(\lambda) + \tau^*(\lambda) \leq 1$, $\forall \lambda \in I^X$,

(IGO2) $\tau(\underline{0}) = \tau(\underline{1}) = 1$, $\tau^*(\underline{0}) = \tau^*(\underline{1}) = 0$,

(IGO3) $\tau(\lambda_1 \wedge \lambda_2) \geq \tau(\lambda_1) \wedge \tau(\lambda_2)$ and $\tau^*(\lambda_1 \wedge \lambda_2) \leq \tau^*(\lambda_1) \vee \tau^*(\lambda_2)$, for each $\lambda_1, \lambda_2 \in I^X$,

(IGO4) $\tau(\bigvee_{i\in\Delta} \lambda_i) \geq \bigwedge_{i\in\Delta} \tau(\lambda_i)$ and $\tau^*(\bigvee_{i\in\Delta} \lambda_i) \leq \bigvee_{i\in\Delta} \tau^*(\lambda_i)$, for each $\lambda_i \in I^X, i \in \Delta$.

The triplet (X, τ, τ^*) is called an intuitionistic fuzzy topological space (IFTS, for short). τ and τ^* may be interpreted as gradation of openness and gradation of nonopenness, respectively.

Definition 2.2. (see [12]) Let X be a nonempty set, $r \in I_0$, $s \in I_1$ and $r + s \leq 1$. The pair $(\mathcal{M}, \mathcal{M}^*)$ of maps $\mathcal{M}, \mathcal{M}^* : I^X \to I$ on X is said to have (r,s)-fuzzy minimal structure if the family

$$\mathcal{M}_{r,s} = \{\lambda \in I^X \mid \mathcal{M}(\lambda) \geq r \text{ and } \mathcal{M}^*(\lambda) \leq s\}$$

contains $\underline{0}$ and $\underline{1}$. Then $(X, \mathcal{M}, \mathcal{M}^*)$ is called (r,s)-fuzzy minimal space (simply, (r,s)-FMS).

Then every member of $\mathcal{M}_{(r,s)}$ is called a *fuzzy (r,s)-minimal open* set. A fuzzy set λ is called a *fuzzy (r,s)-minimal closed* set if the complement of λ is a fuzzy (r,s)-minimal open set.

Definition 2.3. (see [12]) Let $(X, \mathcal{M}, \mathcal{M}^*)$ be an (r,s)-FMS. The (r,s)-fuzzy minimal closure and (r,s)-fuzzy minimal interior of λ, denoted by $\mathcal{C}_m(\lambda, r, s)$ and $\mathcal{I}_m(\lambda, r, s)$, respectivity, are defined as

$$\mathcal{C}_m(\lambda, r, s) = \bigwedge\{\mu \in I^X \mid \underline{1} - \mu \in \mathcal{M}_{r,s} \text{ and } \lambda \leq \mu\},$$

$$\mathcal{I}_m(\lambda, r, s) = \bigvee\{\mu \in I^X \mid \mu \in \mathcal{M}_{r,s} \text{ and } \lambda \geq \mu\}.$$

Theorem 2.4. (see [12]) *Let $(X, \mathcal{M}, \mathcal{M}^*)$ be an (r,s)-FMS and $\lambda, \mu \in I^X$. Then:*

(1) $\mathcal{I}_m(\lambda, r, s) \leq \lambda$ and if $\lambda \in \mathcal{M}_{r,s}$ then $\mathcal{I}_m(\lambda, r, s) = \lambda$.

(2) $\mathcal{C}_m(\lambda, r, s) \geq \lambda$ and if $\underline{1} - \lambda \in \mathcal{M}_{r,s}$ then $\mathcal{C}_m(\lambda, r, s) = \lambda$.

(3) If $\lambda \leq \mu$ *then* $\mathcal{I}_m(\lambda, r, s) \leq \mathcal{I}_m(\mu, r, s)$ *and* $\mathcal{C}_m(\lambda, r, s) \leq \mathcal{C}_m(\mu, r, s)$.

(4) $\mathcal{I}_m(\lambda \wedge \mu, r, s) = \mathcal{I}_m(\lambda, r, s) \wedge \mathcal{I}_m(\mu, r, s)$ *and* $\mathcal{C}_m(\lambda \vee \mu, r, s) = \mathcal{C}_m(\lambda, r, s) \vee \mathcal{C}_m(\mu, r, s)$.

(5) $\mathcal{I}_m(\mathcal{I}_m(\lambda, r, s), r, s) = \mathcal{I}_m(\lambda, r, s)$ *and* $\mathcal{C}_m(\mathcal{C}_m(\lambda, r, s), r, s) = \mathcal{C}_m(\lambda, r, s)$.

(6) $\underline{1} - \mathcal{C}_m(\lambda, r, s) = \mathcal{I}_m(\underline{1} - \lambda, r, s)$ *and* $\underline{1} - \mathcal{I}_m(\lambda, r, s) = \mathcal{C}_m(\underline{1} - \lambda, r, s)$

Definition 2.5. (see [12]) Let $(X, \mathcal{M}, \mathcal{M}^*)$ and $(Y, \mathcal{N}, \mathcal{N}^*)$ be two (r, s)-FMS's. Then $f : X \to Y$ is said to be (r, s)-fuzzy M-continuous if for every $\mu \in \mathcal{N}_{r,s}$, $f^{-1}(\mu) \in \mathcal{M}_{r,s}$.

Theorem 2.6. (see [12]) *Let* $f : (X, \mathcal{M}, \mathcal{M}^*) \to (Y, \mathcal{N}, \mathcal{N}^*)$ *be a function,*

(1) f *is* (r, s)*-fuzzy* M*-continuous.*

(2) $\underline{1} - f^{-1}(\mu) \in \mathcal{M}_{r,s}$, *for each* $\underline{1} - \mu \in \mathcal{N}_{r,s}$.

(3) $f(\mathcal{C}_m(\lambda, r, s)) \leq \mathcal{C}_m(f(\lambda), r, s)$, *for* $\lambda \in I^X$.

(4) $\mathcal{C}_m(f^{-1}(\mu), r, s)) \leq f^{-1}(\mathcal{C}_m(\mu, r, s))$, *for* $\mu \in I^Y$.

(5) $f^{-1}(\mathcal{I}_m(\mu, r, s)) \leq \mathcal{I}_m(f^{-1}(\mu), r, s)$, *for* $\mu \in I^Y$.

Then, (1) $\Leftrightarrow$ (2) $\Rightarrow$ (3) $\Leftrightarrow$ (4) $\Leftrightarrow$ (5).

3. Fuzzy (r, s)-Minimal Semiopen Sets and Fuzzy (r, s)-M Semicontinuity

Definition 3.1. Let $(X, \mathcal{M}, \mathcal{M}^*)$ be an (r, s)-FMS and $\lambda \in I^X$. Then a fuzzy set λ is called a *fuzzy* (r, s)*-minimal semiopen set* in X if

$$\lambda \leq mC(mI(\lambda, r, s), r, s).$$

A fuzzy set λ is called a *fuzzy* (r, s)*-minimal semiclosed* set if the complement of λ is fuzzy (r, s)-minimal preopen.

Every fuzzy (r, s)-minimal open set is fuzzy (r, s)-minimal semiopen but the converse may not be true in general.

Example 3.2. Let $X = I$. For $0 < n < 1$, consider the following fuzzy sets

$$\mu_n(x) = \begin{cases} \frac{x}{n} & \text{if} \quad 0 < x \leq n \\ \frac{1-x}{1-n} & \text{if} \quad n < x \leq 1; \end{cases}$$

$$\lambda(x) = \begin{cases} \frac{x}{2} + \frac{3}{4} & \text{if } 0 \le x \le \frac{1}{2} \\ 1 & \text{if } \frac{1}{2} < x \le 1. \end{cases}$$

Define $\mathcal{M}, \mathcal{M}^* : I^X \to I$ on X by

$$\mathcal{M}(\mu) = \begin{cases} n & \text{if } \mu = \mu_n \\ \max(\{\alpha, 1-\alpha\}) & \text{if } \mu = \underline{\alpha} \\ 0 & \text{otherwise,} \end{cases}$$

$$\mathcal{M}^*(\mu) = \begin{cases} 1-n & \text{if } \mu = \mu_n \\ \min(\{1-\alpha, \alpha\}) & \text{if } \mu = \underline{\alpha} \\ 1 & \text{otherwise.} \end{cases}$$

Consider a $(\frac{3}{4}, \frac{1}{4})$-fuzzy minimal structure $(\mathcal{M}_{\frac{3}{4}}, \mathcal{M}^*_{\frac{1}{4}})$ as follows

$$\mathcal{M}_{\frac{3}{4},\frac{1}{4}} = \{\mu \in I^X \mid \mathcal{M}(\mu) \ge \frac{3}{4} \text{ and } \mathcal{M}^*(\mu) \le \frac{1}{4}\}.$$

Then

$$\mathcal{I}_m(\lambda, \frac{3}{4}, \frac{1}{4}) = \begin{cases} 2x & \text{if } 0 \le x \le \frac{1}{2} \\ 1 & \text{if } \frac{1}{2} < x < 1 \\ 0 & \text{if } x = 1, \end{cases}$$

and $\mathcal{C}_m(\mathcal{I}_m(\lambda, \frac{3}{4}, \frac{1}{4}), \frac{3}{4}, \frac{1}{4}) = \underline{1}$.

So λ is fuzzy (r,s)-minimal semiopen but it is not fuzzy (r,s)-minimal open.

Lemma 3.3. *Let $(X, \mathcal{M}, \mathcal{M}^*)$ be an (r,s)-FMS. Then:*

(1) a fuzzy set λ is fuzzy (r,s)-minimal semiclosed if and only if $mI(mC(\lambda, r,s), r,s) \le \lambda$.

(2) $\underline{0}$ and $\underline{1}$ are both fuzzy (r,s)-minimal semiopen.

Remark 3.4. Let $(X, \mathcal{M}, \mathcal{M}^*)$ be an (r,s)-FMS and $\lambda \in I^X$. Then a fuzzy set λ is called a *fuzzy (r,s)-minimal preopen set* [13] in X if

$$\lambda \le mI(mC(\lambda, r, s), r, s).$$

The concepts of fuzzy (r,s)-minimal semiopen sets and fuzzy (r,s)-minimal preopen sets are independent in an (r,s)-FMS.

$$\begin{array}{ccc} & & \text{fuzzy } (r,s)\text{-minimal semiopen} \\ & \nearrow & \\ \text{fuzzy } (r,s)\text{-minimal open} & & \not\downarrow \not\uparrow \\ & \searrow & \\ & & \text{fuzzy } (r,s)\text{-minimal preopen} \end{array}$$

Example 3.5. Let $X = I$. For $0 < n < 1$, consider the following fuzzy sets

$$\mu_n(x) = \begin{cases} \frac{x}{n} & \text{if} \quad 0 < x \leq n \\ \frac{1-x}{1-n} & \text{if} \quad n < x \leq 1; \end{cases}$$

$$\lambda(x) = \begin{cases} x + \frac{1}{2} & \text{if} \quad 0 \leq x \leq \frac{1}{2} \\ -x + \frac{3}{2} & \text{if} \quad \frac{1}{2} < x \leq 1. \end{cases}$$

$$\sigma(x) = \begin{cases} -\frac{x}{2} + \frac{1}{4} & \text{if} \quad 0 \leq x \leq \frac{1}{2} \\ \frac{x}{2} - \frac{1}{4} & \text{if} \quad \frac{1}{2} < x \leq 1. \end{cases}$$

(1) Define $\mathcal{M}, \mathcal{M}^* : I^X \to I$ on X by

$$\mathcal{M}(\mu) = \begin{cases} n & \text{if} \quad \mu = \mu_n \\ \max(\{\alpha, 1-\alpha\}) & \text{if} \quad \mu = \underline{\alpha} \\ \frac{7}{8} & \text{if} \quad \mu = \sigma \\ 0 & \text{otherwise}, \end{cases}$$

$$\mathcal{M}^*(\mu) = \begin{cases} 1-n & \text{if} \quad \mu = \mu_n \\ \min(\{1-\alpha, \alpha\}) & \text{if} \quad \mu = \underline{\alpha} \\ 0 & \text{if} \quad \mu = \sigma \\ 1 & \text{otherwise}. \end{cases}$$

From $\mathcal{C}_m(\mathcal{I}_m(\lambda, \frac{3}{4}, \frac{1}{4}), \frac{3}{4}, \frac{1}{4}) = \underline{1} - \sigma \geq \lambda$, λ is fuzzy $(\frac{3}{4}, \frac{1}{4})$-minimal semiopen but it is not fuzzy $(\frac{3}{4}, \frac{1}{4})$-minimal preopen.

(2) Define $\mathcal{M}, \mathcal{M}^* : I^X \to I$ on X by

$$\mathcal{M}(\mu) = \begin{cases} n & \text{if} \quad \mu = \mu_n \\ \max(\{\alpha, 1-\alpha\}) & \text{if} \quad \mu = \underline{\alpha} \\ \frac{7}{8} & \text{if} \quad \mu = \underline{1} - \lambda \\ 0 & \text{otherwise}, \end{cases}$$

$$\mathcal{M}^*(\mu) = \begin{cases} 1-n & \text{if} \quad \mu = \mu_n \\ \min(\{1-\alpha, \alpha\}) & \text{if} \quad \mu = \underline{\alpha} \\ 0 & \text{if} \quad \mu = \underline{1} - \lambda \\ 1 & \text{otherwise}. \end{cases}$$

From $\mathcal{C}_m(\mathcal{I}_m(\sigma, \frac{3}{4}, \frac{1}{4}), \frac{3}{4}, \frac{1}{4}) = \underline{1} - \lambda \geq \sigma$ and $\mathcal{I}_m(\sigma, \frac{3}{4}, \frac{1}{4}) = \underline{0}$, σ is fuzzy $(\frac{3}{4}, \frac{1}{4})$-minimal preopen but it is not fuzzy $(\frac{3}{4}, \frac{1}{4})$-minimal semiopen.

Theorem 3.6. *Let $(X, \mathcal{M}, \mathcal{M}^*)$ be an (r, s)-FMS. Any union of fuzzy (r, s)-minimal semiopen sets is fuzzy (r, s)-minimal semiopen.*

Proof. Let λ_i be a fuzzy (r,s)-minimal semiopen set for $i \in J$. Then from $\lambda_i \leq \vee\lambda_i$,

$$\lambda_i \leq mC(mI(\lambda_i,r,s),r,s) \leq mC(mI(\vee\lambda_i,r,s),r,s).$$

This implies $\vee\lambda_i \leq mC(mI(\vee\lambda_i,r,s),r,s)$ and so $\vee\lambda_i$ is fuzzy (r,s)-minimal semiopen. □

From the next example, we know the fact that the intersection of two fuzzy (r,s)-minimal semiopen sets is not fuzzy (r,s)-minimal semiopen in general.

Example 3.7. As in Examle 3.2, let us consider a fuzzy $(\frac{3}{4},\frac{1}{4})$-minimal structure $(\mathcal{M}_{\frac{3}{4}},\mathcal{M}^*_{\frac{1}{4}})$. Consider two fuzzy sets λ and γ defined as the following:

$$\lambda = \mu_{\frac{1}{4}} \qquad \gamma = \mu_{\frac{3}{4}}$$

Then obviously λ and γ are fuzzy $(\frac{3}{4},\frac{1}{4})$-minimal semiopen. But since $mI(\lambda \wedge \gamma,\frac{3}{4},\frac{1}{4}) = \underline{0}$, $\lambda \wedge \gamma$ can not be fuzzy $(\frac{3}{4},\frac{1}{4})$-minimal semiopen.

Definition 3.8. Let $(X,\mathcal{M},\mathcal{M}^*)$ be an (r,s)-FMS. For $\lambda \in I^X$, $msC(\lambda,r,s)$ and $msI(\lambda,r,s)$ are defined as the following:

$$msC(\lambda,r,s) = \wedge\{\gamma \in I^X : \lambda \leq \gamma,\ \ \gamma \text{ is fuzzy } (r,s)\text{-minimal semiclosed}\}$$

$$msI(\lambda,r,s) = \vee\{\mu \in I^X : \mu \leq \lambda,\ \ \mu \text{ is fuzzy } (r,s)\text{-minimal semiopen }\}.$$

Theorem 3.9. *Let $(X,\mathcal{M},\mathcal{M}^*)$ be an (r,s)-FMS and $\lambda \in I^X$. Then:*

(1) $msI(\lambda,r,s) \leq \lambda$.

(2) If $\lambda \leq \mu$, then $msI(\lambda,r,s) \leq msI(\mu,r,s)$.

(3) λ is (r,s)-minimal semiopen iff $msI(\lambda,r,s) = \lambda$.

(4) $msI(smI(\lambda,r,s),r,s) = msI(\lambda,r,s)$.

(5) $msC(\underline{1}-\lambda,r,s) = \underline{1}-msI(\lambda,r,s)$ and $msI(\underline{1}-\lambda,r,s) = \underline{1}-msC(\lambda,r,s)$.

Proof. (1), (2), (3) and (4) are obviously obtained from Theorem 3.6.

(5) For $\lambda \in I^X$,

$$\begin{aligned}\underline{1} - msI(\lambda,r,s) &= \underline{1} - \vee\{\mu \in I^X : \mu \leq \lambda,\\ &\qquad \mu \text{ is fuzzy } (r,s)\text{-minimal semiopen}\}\\ &= \wedge\{\underline{1} - \mu : \mu \leq \lambda,\\ &\qquad \mu \text{ is fuzzy } (r,s)\text{-minimal semiopen}\}\\ &= \wedge\{\underline{1} - \mu : \underline{1} - \lambda \leq \underline{1} - \mu,\\ &\qquad \mu \text{ is fuzzy } (r,s)\text{-minimal semiopen}\}\\ &= msC(\underline{1} - \lambda,r,s).\end{aligned}$$

Similarly, we have $msI(\underline{1} - \lambda, r, s) = \underline{1} - msC(\lambda, r, s)$. □

Theorem 3.10. *Let $(X, \mathcal{M}, \mathcal{M}^*)$ be an (r, s)-FMS and $\lambda \in I^X$. Then:*

(1) $\lambda \leq msC(\lambda, r, s)$.

(2) If $\lambda \leq \mu$, then $msC(\lambda, r, s) \leq msC(\mu, r, s)$.

(3) μ is (r, s)-minimal semiclosed iff $msC(\mu, r, s) = F$.

(4) $msC(msC(\lambda, r, s), r, s) = msC(\lambda, r, s)$.

Proof. It is similar to the proof of Theorem 3.9. □

Definition 3.11. Let $(X, \mathcal{M}, \mathcal{M}^*)$ and $(Y, \mathcal{N}, \mathcal{N}^*)$ be (r, s)-FMS's. Then a mapping $f : X \to Y$ is said to be *fuzzy (r, s)-M semicontinuous* if for each fuzzy (r, s)-minimal open set $\lambda \in I^Y$, $f^{-1}(\lambda)$ is a fuzzy (r, s)-minimal semiopen set.

Every fuzzy (r, s)-M continuous mapping is fuzzy (r, s)-M semicontinuous but the converse is not true in general.

Example 3.12. Let $X = I$. For $0 < n < 1$, consider the following fuzzy sets

$$\mu_n(x) = \begin{cases} \frac{x}{n} & \text{if } \ 0 < x \leq n \\ \frac{1-x}{1-n} & \text{if } \ n < x \leq 1; \end{cases}$$

$$\sigma(x) = \begin{cases} -\frac{x}{2} + \frac{1}{4} & \text{if } \ 0 \leq x \leq \frac{1}{2} \\ \frac{x}{2} - \frac{1}{4} & \text{if } \ \frac{1}{2} < x \leq 1. \end{cases}$$

Define $\mathcal{M}, \mathcal{M}^* : I^X \to I$ on X by

$$\mathcal{M}(\mu) = \begin{cases} n & \text{if } \ \mu = \mu_n \\ \max(\{\alpha, 1 - \alpha\}) & \text{if } \ \mu = \underline{\alpha} \\ 0 & \text{otherwise,} \end{cases}$$

$$\mathcal{M}^*(\mu) = \begin{cases} 1 - n & \text{if } \ \mu = \mu_n \\ \min(\{1 - \alpha, \alpha\}) & \text{if } \ \mu = \underline{\alpha} \\ 1 & \text{otherwise.} \end{cases}$$

Define $\mathcal{N}, \mathcal{N}^* : I^X \to I$ on X by

$$\mathcal{N}(\mu) = \begin{cases} n & \text{if } \ \mu = \mu_n \\ \max(\{\alpha, 1 - \alpha\}) & \text{if } \ \mu = \underline{\alpha} \\ \frac{7}{8} & \text{if } \ \mu = \sigma \\ 0 & \text{otherwise,} \end{cases}$$

$$\mathcal{N}^*(\mu)=\begin{cases} 1-n & \text{if} \quad \mu=\mu_n \\ \min(\{1-\alpha,\alpha\}) & \text{if} \quad \mu=\underline{\alpha} \\ 0 & \text{if} \quad \mu=\sigma \\ 1 & \text{otherwise.} \end{cases}$$

Note $\sigma \in \mathcal{N}_{\frac{3}{4},\frac{1}{4}}$, $\sigma \notin \mathcal{M}_{\frac{3}{4},\frac{1}{4}}$ and σ is fuzzy (r,s)-minimal semiopen in $(X,\mathcal{M},\mathcal{M}^*)$. Finally we can say the identity mapping $f:(X,\mathcal{M},\mathcal{M}^*)\to(Y,\mathcal{N},\mathcal{N}^*)$ is fuzzy (r,s)-M semicontinuous but not fuzzy (r,s)-M continuous.

Theorem 3.13. *Let $f:X\to Y$ be a mapping on (r,s)-FMS's $(X,\mathcal{M},\mathcal{M}^*)$ and $(Y,\mathcal{N},\mathcal{N}^*)$. Then the following statements are equivalent:*

(1) f is fuzzy (r,s)-M semicontinuous.

(2) $f^{-1}(\gamma)$ is a fuzzy (r,s)-minimal semiclosed set for each fuzzy (r,s)-minimal closed set $\gamma\in I^Y$.

(3) $f(msC(\lambda,r,s))\leq mC(f(\lambda),r,s)$ for $\lambda\in I^X$.

(4) $msC(f^{-1}(\mu),r,s)\leq f^{-1}(mC(\mu,r,s))$ for $\mu\in I^Y$.

(5) $f^{-1}(mI(\mu,r,s))\leq msI(f^{-1}(\mu),r,s)$ for $\mu\in I^Y$.

Proof. (1) $\Rightarrow$ (2) It is obvious.

(2) $\Rightarrow$ (3) For $\lambda\in I^X$,

$$\begin{aligned} f^{-1}(mC(f(\lambda),r,s)) &= f^{-1}(\wedge\{\gamma\in I^Y : f(\lambda)\leq\gamma \text{ and} \\ &\qquad \gamma \text{ is fuzzy } (r,s)\text{-minimal closed}\}) \\ &= \wedge\{f^{-1}(\gamma)\in I^X : \lambda\leq f^{-1}(\gamma) \text{ and} \\ &\qquad f^{-1}(\gamma) \text{ is fuzzy } (r,s)\text{-minimal semiclosed}\} \\ &\geq \wedge\{\mu\in I^X : \lambda\leq\mu \text{ and} \\ &\qquad \mu \text{ is fuzzy } (r,s)\text{-minimal semiclosed}\} \\ &= msC(\lambda,r,s) \end{aligned}$$

Hence $f(msC(\lambda,r,s))\leq mC(f(\lambda),r,s)$.

(3) $\Rightarrow$ (4) For $\mu\in I^Y$,

$$f(msC(f^{-1}(\mu),r,s))\leq mC(f(f^{-1}(\mu)),r,s)\leq mC(\mu,r,s).$$

Thus we have $msC(f^{-1}(\mu),r,s)\leq f^{-1}(mC(\mu,r,s))$.

(4) $\Rightarrow$ (5) For $\mu \in I^Y$,

$$\begin{aligned} f^{-1}(mI(\mu, r, s)) &= f^{-1}(\underline{1} - mC(\underline{1} - \mu, r, s)) \\ &= \underline{1} - f^{-1}(mC(\underline{1} - \mu, r, s)) \\ &\leq \underline{1} - msC(f^{-1}(\underline{1} - \mu), r, s) \\ &= msI(f^{-1}(\mu), r, s). \end{aligned}$$

Hence $f^{-1}(mI(\mu, r, s)) \leq msI(f^{-1}(\mu), r, s)$.

(5) $\Rightarrow$ (1) Let λ be any fuzzy (r, s)-minimal open set. Then from (5), it follows $f^{-1}(\lambda) = f^{-1}(mI(\lambda, r, s)) \leq msI(f^{-1}(\lambda), r, s)$ and $f^{-1}(\lambda) = msI(f^{-1}(\lambda), r, s)$. This implies $f^{-1}(\lambda)$ is a fuzzy (r, s)-minimal semiopen set. Hence f is fuzzy (r, s)-M semicontinuous. □

Theorem 3.14. *Let $(X, \mathcal{M}, \mathcal{M}^*)$ be an (r, s)-FMS and $\lambda \in I^X$. Then:*

(1) $mI(mC(\lambda, r, s), r, s) \leq mI(mC(msC(\lambda, r, s), r, s), r, s) \leq msC(\lambda, r, s)$.

(2) $msI(\lambda, r, s) \leq mC(mI(msI(\lambda, r, s), r, s), r, s) \leq mC(mI(\lambda, r, s), r, s)$.

Proof. (1) Since $msC(\lambda, r, s)$ is fuzzy (r, s)-minimal semiclosed, it is obtained from Lemma 3.3 and Theorem 3.10.

(2) Obvious. □

Theorem 3.15. *Let $f : X \to Y$ be a mapping on (r, s)-FMS's $(X, \mathcal{M}, \mathcal{M}^*)$ and $(Y, \mathcal{N}, \mathcal{N}^*)$. Then the following statements are equivalent:*

(1) f is fuzzy (r, s)-M semicontinuous.

(2) $f^{-1}(\mu) \leq mC(mI(f^{-1}(\mu), r, s), r, s)$ for each fuzzy (r, s)-minimal open set μ in Y.

(3) $mI(mC(f^{-1}(\gamma), r, s), r, s) \leq f^{-1}(\gamma)$ for each fuzzy (r, s)-minimal closed set γ in Y.

(4) $f(mI(mC(\lambda, r, s), r, s)) \leq mC(f(\lambda), r, s)$ for $\lambda \in I^X$.

(5) $mI(mC(f^{-1}(\mu), r, s), r, s) \leq f^{-1}(mC(\mu, r, s))$ for $\mu \in I^Y$.

(6) $f^{-1}(mI(\mu, r, s)) \leq mC(mI(f^{-1}(\mu), r, s), r, s)$ for $\mu \in I^Y$.

Proof. (1) $\Leftrightarrow$ (2) It is easily obtained from concepts of fuzzy (r, s)-M semicontinuity and fuzzy (r, s)-minimal semiopen sets.

(1) $\Leftrightarrow$ (3) Obvious.

(1) $\Rightarrow$ (4) For $\lambda \in I^X$, we have

$$mI(mC(\lambda,r,s),r,s) \le msC(\lambda,r,s) \le f^{-1}(f(msC(\lambda,r,s))) \le f^{-1}(mC(f(\lambda),r,s)).$$

So $f(mI(mC(\lambda,r,s),r,s)) \le mC(f(\lambda),r,s)$.

(4) $\Rightarrow$ (5) Obvious.

(5) $\Rightarrow$ (6) For $\mu \in I^Y$, from hypothesis,

$$\begin{aligned} f^{-1}(mI(\mu,r,s)) &= f^{-1}(\underline{1} - mC(\underline{1} - \mu,r,s)) \\ &= \underline{1} - (f^{-1}(mC(\underline{1} - \mu,r,s))) \\ &\le \underline{1} - mI(mc(f^{-1}(\underline{1} - \mu),r,s),r,s) \\ &= mC(mI(f^{-1}(\mu),r,s),r,s). \end{aligned}$$

So we have (6).

(6) $\Rightarrow$ (1) For $\mu \in I^Y$, let μ be a fuzzy (r,s)-minimal open set. Then since $\mu = mI(\mu,r,s)$, by hypothesis

$$f^{-1}(\mu) = f^{-1}(mI(\mu,r,s)) \le mC(mI(f^{-1}(\mu),r,s),r,s)$$

and so $f^{-1}(\mu)$ is fuzzy (r,s)-minimal semiopen. Hence f is fuzzy (r,s)-M semicontinuous. $\square$

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