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ON THE DIOPHANTINE EQUATION $483^x + 485^y = z^2$

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Abstract: In this paper, we show that the Diophantine equation $483^x + 485^y = z^2$ has a unique non-negative integer solution where x, y and z are non-negative integers. The solution (x, y, z) is $(1, 0, 22)$.

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1. Introduction

In [9], Sroysang showed that $(1, 0, 2)$ is a unique non-negative integer solution (x, y, z) for the Diophantine equation $3^x + 5^y = z^2$ where x, y and z are non-negative integers. In [10], he showed that the Diophantine equation $5^x + 7^y = z^2$ has no non-negative integer solution where x, y and z are non-negative integers. In [13], he showed that the Diophantine equation $47^x + 49^y = z^2$ has no non-negative integer solution where x, y and z are non-negative integers. In [14], he showed that the Diophantine equation $89^x + 91^y = z^2$ has no non-negative integer solution where x, y and z are non-negative integers. Many equations of type $a^x + b^y = z^2$ were solved [1, 2, 3, 4, 6, 7, 8, 11, 12]. In this paper, $(1, 0, 22)$ is a unique non-negative integer solution (x, y, z) for the Diophantine equation $483^x + 485^y = z^2$ where x, y and z are non-negative integers.

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2. Preliminaries

Proposition 2.1. [5] (**Catalan's conjecture**) $(3,2,2,3)$ *is a unique solution* (a,b,x,y) *for the Diophantine equation* $a^x - b^y = 1$ *where* a,b,x *and* y *are integers such that* $\min\{a,b,x,y\} > 1$.

Lemma 2.2. $(1,22)$ *is a unique solution* (x,z) *for the Diophantine equation* $483^x + 1 = z^2$ *where* x *and* z *are non-negative integers.*

Proof. Let x and z be non-negative integers such that $483^x + 1 = z^2$. If $x = 0$, then $z^2 = 2$ which is impossible. Thus, $x \geq 1$. Note that $z^2 = 483^y + 1 \geq 483^1 + 1 = 484$. It follows that $z \geq 22$. Now, we consider on the equation $z^2 - 483^x = 1$. By Proposition 2.1, we obtain that $x = 1$. Hence, $z^2 = 484$ and then $z = 22$. □

Lemma 2.3. *The Diophantine equation* $1+485^y = z^2$ *has no non-negative integer solution where* y *and* z *are non-negative integers.*

Proof. Suppose that there are non-negative integers y and z such that $1 + 485^y = z^2$. If $y = 0$, then $z^2 = 2$ which is impossible. Thus, $y \geq 1$. Note that $z^2 = 1 + 485^y \geq 1 + 485^1 = 486$. It follows that $z \geq 23$. Now, we consider on the equation $z^2 - 485^y = 1$. By Proposition 2.1, we obtain that $y = 1$. Hence, $z^2 = 486$. This is a contradiction. □

3. Main Results

Theorem 3.1. $(1,0,22)$ *is a unique non-negative integer solution* (x,y,z) *for the Diophantine equation* $483^x+485^y = z^2$ *where* x,y *and* z *are non-negative integers.*

Proof. Let x,y and z be non-negative integers such that $483^x + 485^y = z^2$. Since z is even, we have $z^2 \equiv 0 \pmod 4$. Thus, $483^x \equiv 3 \pmod 4$. By Lemma 2.3, we obtain that $x \geq 1$. Then x is odd. Now, we will divide the number y into two cases.

Case $y = 0$. By Lemma 2.2, it follows that $x = 1$ and $z = 22$.

Case $y \geq 1$. Note that $483^y \equiv 2 \pmod 5$ or $483^y \equiv 3 \pmod 5$. Since $485^y \equiv 0 \pmod 5$, we have $z^2 \equiv 2 \pmod 5$ or $z^2 \equiv 3 \pmod 5$. Thus, z is odd. This is a contradiction.

Hence, the solution (x,y,z) is $(1,0,22)$. □

Corollary 3.2. *The Diophantine equation* $483^x + 485^y = w^4$ *has no non-negative integer solution where* x, y *and* w *are non-negative integers.*

Proof. Suppose that there are non-negative integers x, y and w such that $483^x + 485^y = w^4$. Let $z = w^2$. We obtain that $483^x + 485^y = z^2$. By Theorem 3.1, we have $(x, y, z) = (1, 0, 22)$. It follows that $w^2 = z = 22$. This is a contradiction. □

Corollary 3.3. $(1, 0, 2)$ *is a unique non-negative integer solution* (x, y, z) *for the Diophantine equation* $483^x + 485^y = 121u^2$ *where* x, y *and* u *are non-negative integers.*

Proof. Let x, y and u be non-negative integers such that $483^x + 485^y = 4u^4$. Let $z = 11u$. This implies that $483^x + 485^y = z^2$. By Theorem 3.1, we have $(x, y, z) = (1, 0, 22)$. Hence, $u = 2$. □

Corollary 3.4. $(1, 0, 11)$ *is a unique non-negative integer solution* (x, y, z) *for the Diophantine equation* $483^x + 485^y = 4v^4$ *where* x, y *and* v *are non-negative integers.*

Proof. Let x, y and v be non-negative integers such that $483^x + 485^y = 4v^4$. Let $z = 2v^2$. This implies that $483^x + 485^y = z^2$. By Theorem 3.1, we have $(x, y, z) = (1, 0, 22)$. Hence, $v = 11$. □

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