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A NEW BLOCK NUMERICAL INTEGRATOR FOR THE SOLVING $y'' = f(x, y, y')$

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Abstract: In this paper, a new block numerical integrator for solving second order initial value problems (IVP) of ordinary differential equations (ODEs) is proposed. The method was derived using interpolation and collocation of power series approximate solution to generate a continuous hybrid linear multistep method which was later solved at independent grid point to give discrete hybrid block method. The analysis of the basic properties of the method shows that the scheme is consistent, convergent and zero stable. Numerical experimentations and comparative analysis with existing methods show that our scheme is efficient.

AMS Subject Classification: 65L05, 65L06, 65D30
Key Words: collocation, interpolation, Zero stable, consistent, convergent, efficient, hybrid block method

1. Introduction

Many problems emanating from physical, natural and social sciences just to mention a few could be modelled into ordinary differential equations. Many

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of these models do not always have exact solutions, thus numerical methods are often employed to solve them. In this paper we will be concerned with the numerical solution of second order initial value problem of ordinary differential equations of the form:

$$y^{''} = f(x, y, y^{'}),\ y(x_0) = y_0,\ y^{'}(x_0) = y_0^{'},\quad y\epsilon\mathbb{R} \tag{1}$$

where x_0 is the initial point and f is continuous within the interval of integration and satisfies the existence and uniqueness condition. Formerly equation(1) is solved by first reducing it to a system of first order ordinary differential equations, and then any method of solving first order ordinary differential equations would then be applied. The setbacks of this approach however were reported by Adesanya *et al.* [1,2] .

To avoid the set back pointed by the scholars mentioned above, the method of collocation and interpolation of the power series approximate solution to generate continuous linear multistep method for the direct solution of (1) had been adopted by many scholars among them are Awoyemi [3], Awoyemi and Kayode [4] to mention a few. Their approaches generate an implicit continuous linear multistep method in which separate predictors are required for its implementation; this method is called the predictor-corrector method. Some of the major setback of this method are that the cost of implementation is high and much time is wasted while trying to develop the method(Olabode [5], Adesanya *et al.* [6]

In order to address the setbacks of the predictor-corrector method, Zarina *et al.* [7], Badmus and Yahaya [8], Aladeselu [9], Jator [10], Jator and Li [11] and Awoyemi *et al.* [12] developed block method for solving higher order ordinary differential equation. The block method is capable of giving evaluations at different grids points without overlapping.. This method does not require the development of separate predictors and more over it is more accurate than other existing methods.

Adesanya *et al.*[6], Anake *et al* [13] developed one step method for the solution of higher order ordinary differential equations. They concluded that the lower the step length, the higher the accuracy of the developed scheme. In this paper, we developed one step method with eight off grid points for the solution of (1) implemented in block method. This method is an improvement on the work of Anake *et a*l. [13] where five off grid points with constant step size (h) was considered.

2. Derivation of the Method

We define the general power series approximate solution in the form:

$$y(x)=\sum_{j=0}^{r+s-1} a_j x^j, \tag{2}$$

where r and s are the numbers of interpolation and collocation points respectively. By substituting the second derivative of (2) into (1)gives

$$y''(x)=\sum_{j=2}^{r+s-1} j\,(j-1)\,a_j x^{j-2}. \tag{3}$$

Substituting (3) into (1) gives

$$f\left(x,y,y'\right)=\sum_{j=2}^{r+s-1} j\,(j-1)\,a_j x^{j-2}. \tag{4}$$

If we interpolate (2) at $x_{n+r}, r=\frac{3}{4},\frac{7}{8}$ and collocate (3) at $x_{n+s}, s=0,\frac{1}{4},\frac{1}{2},\frac{3}{4}1$ this will result into a system of non linear equation

$$AX=U \tag{5}$$

where

$$A=\begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \end{bmatrix}^T$$

$$U=\begin{bmatrix} y_{n+\frac{3}{4}} & y_{n+\frac{7}{8}} & f_n & f_{n+\frac{1}{4}} & f_{n+\frac{1}{2}} & f_{n+\frac{3}{4}} & f_{n+1} \end{bmatrix}^T,$$

and

$$X=\begin{bmatrix} 1 & x_{n+\frac{3}{4}} & x^2_{n+\frac{3}{4}} & x^3_{n+\frac{3}{4}} & x^4_{n+\frac{3}{4}} & x^5_{n+\frac{3}{4}} & x^6_{n+\frac{3}{4}} \\ 1 & x_{n+\frac{7}{8}} & x^2_{n+\frac{7}{8}} & x^3_{n+\frac{7}{8}} & x^4_{n+\frac{7}{8}} & x^5_{n+\frac{7}{8}} & x^6_{n+\frac{7}{8}} \\ 0 & 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 & 30x_n^4 \\ 0 & 0 & 2 & 6x_{n+\frac{1}{4}} & 12x^2_{n+\frac{1}{4}} & 20x^3_{n+\frac{1}{4}} & 30x^4_{n+\frac{1}{4}} \\ 0 & 0 & 2 & 6x_{n+\frac{1}{2}} & 12x^2_{n+\frac{1}{2}} & 20x^3_{n+\frac{1}{2}} & 30x^4_{n+\frac{1}{2}} \\ 0 & 0 & 2 & 6x_{n+\frac{3}{4}} & 12x^2_{n+\frac{3}{4}} & 20x^3_{n+\frac{3}{4}} & 30x^4_{n+\frac{3}{4}} \\ 0 & 0 & 2 & 6x_{n+1} & 12x^2_{n+1} & 20x^3_{n+1} & 30x^4_{n+1} \end{bmatrix}.$$

Equation (5) is solved for the unknown constants' $a_j's$ using matrix inversion technique and substituting back into (2) to obtain a continuous linear multistep method of the form

$$y(t) = \sigma_{\frac{3}{4}} y_{n+\frac{3}{4}} + \sigma_{\frac{7}{8}} y_{n+\frac{7}{8}} + h^2 \left[\sum_{j=0}^{1} \beta_j f_{n+j} + \beta_v \quad f_{n+v} \right] \quad v = \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \qquad (6)$$

where

$$y_{n+j} = y(x_n + jh),\ f_{n+j} = [(x_n + jh), y(x_n + jh), y'(x_n + jh)],\ t = \frac{x - x_n}{h}$$

$$\sigma_{\frac{3}{4}} = 7 - 8t$$

$$\sigma_{\frac{7}{8}} = 8t - 6$$

$$\beta_0 = \frac{1}{368640}(131072t^6 - 491520t^5 + 716800t^4 - 512000t^3 + 1843201t^2 - 30772t + 1911)$$

$$\beta_{\frac{1}{4}} = \frac{-1}{11520}(16384t^6 - 55296t^5 + 66560t^4 - 30720t^3 + 3732t - 693)$$

$$\beta_{\frac{1}{2}} = \frac{1}{61440}(131072t^6 - 3932216t^5 + 389120t^4 - 122880t^3 - 12956t + 8421)$$

$$\beta_{\frac{3}{4}} = \frac{-1}{46080}(65536t^6 - 172032t^5 + 143360t^4 - 40960t^3 + 9196t - 5817)$$

$$\beta_1 = \frac{-1}{368640}(131072t^6 - 294912t^5 + 225280t^4 - 61440t^3 + 1980t - 189)$$

Solving for the independent solution gives the continuous block method of the form

$$y_{n+j}^{(i)} = \sum_{i=0}^{1} \frac{(jh)^{(i)}}{j!} y_n^{(i)} + h^2 \left[\sum_{t=0}^{1} \sigma_j f_{n+t} + \sigma_v \quad f_{n+v} \right] \qquad v = \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \quad i = 0, 1$$

where

$$\sigma_0 = \frac{1}{90}(32t^6 - 120t^5 + 175t^4 - 125t^3 + 45t^2),$$

$$\sigma_{\frac{1}{4}} = -\frac{1}{45}(64t^6 - 216t^5 + 260t^4 - 120t^3),$$
$$\sigma_{\frac{1}{2}} = \frac{1}{15}(32t^6 - 96t^5 + 95t^4 - 30t^3),$$
$$\sigma_{\frac{3}{4}} = -\frac{1}{45}(64t^6 - 168t^5 + 140t^4 - 40t^3),$$
$$\sigma_1 = \frac{1}{90}(32t^6 - 72t^5 + 55t^4 - 15t^3)$$

evaluating (6) a $t = \frac{1}{8}(\frac{1}{8})1$ gives a discrete block formula of the form

$$\mathbf{A}^{(0)}\mathbf{Y}_m^{(i)} = \sum_i^1 e_i y_n^{(i)} + h^{2-i}\left[df(y_n) + bF(Y_m)\right] \tag{7}$$

where i is the power of the derivative.

$A^0 = 8 \times 8$ identical matrix.

$$\mathbf{Y}_m = \begin{bmatrix} y_{n+\frac{1}{8}} & y_{n+\frac{1}{4}} & y_{n+\frac{3}{8}} & y_{n+\frac{1}{2}} & y_{n+\frac{5}{8}} & y_{n+\frac{3}{4}} & y_{n+\frac{7}{8}} & y_{n+1} \end{bmatrix}^T$$
$$\mathbf{y}_n^{(i)} = \begin{bmatrix} y_{n-\frac{1}{8}} & y_{n-\frac{1}{4}} & y_{n-\frac{3}{8}} & y_{n-\frac{1}{2}} & y_{n-\frac{5}{8}} & y_{n-\frac{3}{4}} & y_{n-\frac{7}{8}} & y_n \end{bmatrix}^T$$
$$F(Y_m) = \begin{bmatrix} f_{n+\frac{1}{8}} & f_{n+\frac{1}{4}} & f_{n+\frac{3}{8}} & f_{n+\frac{1}{2}} & f_{n+\frac{5}{8}} & f_{n+\frac{3}{4}} & f_{n+\frac{7}{8}} & f_{n+1} \end{bmatrix}^T$$
$$f(y_n) = \begin{bmatrix} f_{n-\frac{1}{8}} & f_{n-\frac{1}{4}} & f_{n-\frac{1}{8}} & f_{n-\frac{1}{2}} & f_{n-\frac{5}{8}} & f_{n-\frac{3}{8}} & f_{n-\frac{7}{8}} & f_n \end{bmatrix}^T$$

When $i = 0$:

$$e_0 = \begin{bmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1 \end{bmatrix}, e_1 = \begin{bmatrix} 0&0&0&0&0&0&0&\frac{1}{8}\\ 0&0&0&0&0&0&0&\frac{1}{4}\\ 0&0&0&0&0&0&0&\frac{3}{8}\\ 0&0&0&0&0&0&0&\frac{1}{2}\\ 0&0&0&0&0&0&0&\frac{5}{8}\\ 0&0&0&0&0&0&0&\frac{3}{4}\\ 0&0&0&0&0&0&0&\frac{7}{8}\\ 0&0&0&0&0&0&0&1 \end{bmatrix},$$

$$d_0 = \begin{bmatrix} 0&0&0&0&0&0&0&\frac{4081}{737280}\\ 0&0&0&0&0&0&0&\frac{367}{23040}\\ 0&0&0&0&0&0&0&\frac{2181}{81920}\\ 0&0&0&0&0&0&0&\frac{53}{1440}\\ 0&0&0&0&0&0&0&\frac{6925}{147456}\\ 0&0&0&0&0&0&0&\frac{147}{2560}\\ 0&0&0&0&0&0&0&\frac{50029}{737280}\\ 0&0&0&0&0&0&0&\frac{847}{11520} \end{bmatrix},$$

$$b_0 = \begin{bmatrix} 0 & \frac{121}{30720} & 0 & \frac{-313}{122880} & 0 & \frac{5}{4608} & 0 & \frac{-49}{245761} \\ 0 & \frac{3}{128} & 0 & \frac{-47}{3840} & 0 & \frac{29}{5760} & 0 & \frac{-7}{7680} \\ 0 & \frac{297}{5120} & 0 & \frac{-891}{40960} & 0 & \frac{93}{10240} & 0 & \frac{-27}{16384} \\ 0 & \frac{1}{10} & 0 & \frac{-1}{48} & 0 & \frac{1}{90} & 0 & \frac{-1}{480} \\ 0 & \frac{875}{6144} & 0 & \frac{-125}{24576} & 0 & \frac{125}{9216} & 0 & \frac{-125}{49152} \\ 0 & \frac{117}{640} & 0 & \frac{27}{1280} & 0 & \frac{3}{128} & 0 & \frac{-9}{2560} \\ 0 & \frac{343}{1536} & 0 & \frac{5831}{122880} & 0 & \frac{4459}{92160} & 0 & \frac{-343}{81920} \\ 0 & \frac{1969}{23040} & 0 & \frac{-397}{7680} & 0 & \frac{499}{23040} & 0 & \frac{-91}{23040} \end{bmatrix}.$$

When $i = 1$

$$e_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad d_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{847}{11520} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{251}{2880} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{213}{2560} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{29}{360} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{95}{1152} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{27}{320} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1883}{23040} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{7}{90} \end{bmatrix},$$

$$b_1 = \begin{bmatrix} 0 & \frac{1969}{23040} & 0 & \frac{-397}{7680} & 0 & \frac{499}{23040} & 0 & \frac{-91}{23040} \\ 0 & \frac{323}{1440} & 0 & \frac{-11}{120} & 0 & \frac{53}{1440} & 0 & \frac{-19}{2880} \\ 0 & \frac{813}{2560} & 0 & \frac{-117}{2560} & 0 & \frac{63}{2560} & 0 & \frac{-3}{640} \\ 0 & \frac{31}{90} & 0 & \frac{1}{15} & 0 & \frac{1}{90} & 0 & \frac{-1}{360} \\ 0 & \frac{1525}{4608} & 0 & \frac{275}{1536} & 0 & \frac{175}{4608} & 0 & \frac{-25}{4608} \\ 0 & \frac{51}{160} & 0 & \frac{9}{40} & 0 & \frac{21}{160} & 0 & \frac{-3}{320} \\ 0 & \frac{7963}{23040} & 0 & \frac{1421}{7680} & 0 & \frac{6223}{23040} & 0 & \frac{49}{11520} \\ 0 & \frac{16}{45} & 0 & \frac{2}{15} & 0 & \frac{16}{45} & 0 & \frac{7}{90} \end{bmatrix}.$$

3. Implementation of the Method

Writing equation [8] in the generalised form

$$Y_m = Ey_n + h^{\mu} DF(y_n) + h^{\mu} BF(Y_m) \tag{8}$$

where $Y_m = [y_{n+1}, y_{n+2,\ldots,} y_{n+k}]^T$, μ is the order of the differential equation, k is the steplength, E, D and B are matrices. We then propose a prediction equation in the form

$$Y_m^{(0)} = Ey_n + \sum_{\lambda=0}^{2} h^{\mu+\lambda} F^{\lambda}(y_n) \tag{9}$$

where $F^{(\lambda)}\ (y_n) = \frac{\delta\lambda}{\delta x^\lambda} f(x, y, y')_{y_n}$. Substituting (9) into (8) gives

$$Y_m = Ey_n + h^\mu DF(y_n) + h^\mu BF(y_m^{(0)}) \tag{10}$$

Equation (10) is our non self starting block method since the prediction equation is not gotten directly from the block formula (Adesanya *et al.*[3])

4. Basic Properties of the Developed Method

4.1. Order of the block

Let the linear operator l $L\{y(x) : h\}$ on (7) as

$$L\{y(x) : h\} = \mathbf{A}^0 y_m^{(i)} - \sum_{i=0}^{1-i} h^i e_i y_n^{(i)} - h^{2-i}\ [df(y_n) + bF(y_m)] \tag{11}$$

Expanding y_{n+j} and f_{n+j} in Taylor series and comparing the coefficients of h gives

$L\{y(x) : h\} = C_0 y(x) + C_1 y'(x) + ... + C_p h^p y^p(x) + C_{p+1} h^{p+1} y^{p+1}(x) + C_{p+2} h^{p+2} y^{p+2}(x) + ...$

Definition 1. *The linear operator* L *and associated block method are said to be of order* p *if* $C_0 = C_1 = ... = C_p = C_{p+1} = 0\ C_{p+2} \neq 0. C_{p+2}$ *is called the error constant and implies that the truncation error is given by* $t_{n+k} = C_{p+2} h^{p+2} y^{p+2}(x) + O\left(h^{p+3}\right)$. *Comparing the coefficient of h , the order of the method is five with error constant*

$$c_0 = c_1 = c_2 = c_3 = c_4 = c_5 = 0 = c_6 = 0,$$

$$c_7 = \left[\frac{311}{2113929216}, \frac{107}{165150720}, \frac{1377}{1174405120}, \frac{1}{645120}, \frac{4075}{2113929216}, \frac{9}{3670016}, \frac{4459}{1509949440}, \frac{1}{322560}\right]^T.$$

4.2. Consistency

A method is said to be consistent, if it has order greater than one. From the above analysis, it is obvious that our method is consistent.

4.3. Zero Stability

A block method is said to be zero stable as $h \to 0$ the r_j, $j = 1(1)\ k$ of the first characteristics polynomial $\rho(r) = 0$ that is $\left|\left[\sum A^0 R^{k-1}\right]\right| \leq 1$, for those root with $|R| = 1$ must be simple

For our method

$$\rho(r) = \left| r \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right| = 0,$$

$$r^7(r-1) = 0,$$

$$r_1 = r_2 = r_3 = r_1 = r_1 = r_1 = r_1 = 0$$

or

$$r_8 = 1.$$

Hence it is evident that our method is zero stable

4.4. Convergence

Definition 2. *The necessary and sufficient conditions for a linear multistep method to be convergent is that it must be consistent and zero stable. Hence our method is convergent.*

4.5. Stability Region

The method (6) is said to be absolutely stable if for a given h, all roots z_s ofthe characteristics polynomial $\pi(z,h) = \rho(z) + h^2\sigma(z) = 0$, satisfies $|z_s| < 1$, $s = 1, 2, ..., n$. *where* $h = -\lambda^2 h^2$ and $\lambda = \frac{\partial f}{\partial y}$. The boundary locus method is adopted to determine the region of absolute stability. Substituting the test equation $y'' = -\lambda^2 y$, $y' = \lambda y$ into the polynomial gives the stability region as shown in fig. 1

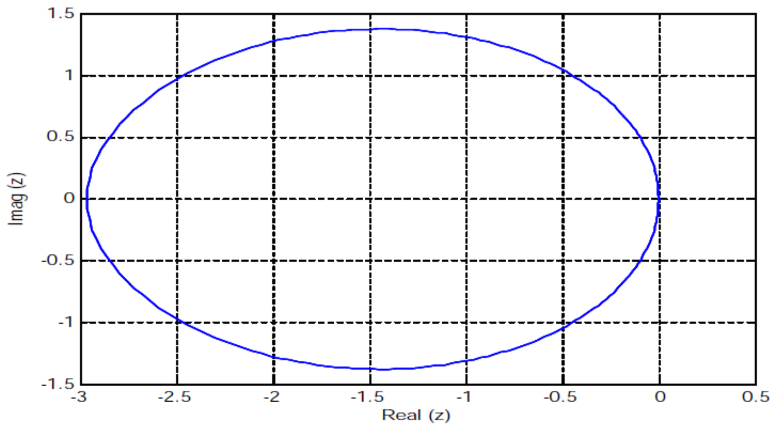

Figure 1: Stability region of the method

5. Numerical Experiments

5.1. Numerical Examples

Problem 1. $y'' - x(y')^2 = 0,$

$y(0) = 1, \ y'(0) = \frac{1}{2}, h = 0.05.$

Exact Solution: $y(x) = 1 + \frac{1}{2}\log(\frac{2+x}{2-x})$

Source: Awoyemi[14]

Problem 2. $y'' - \frac{(y')^2}{2y} + 2y = 0$

$y(\frac{\pi}{6}) = \frac{1}{4}, \ y'(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} \quad h = 0.05$

Exact Solution: $y(x) = (\sin x)^2$

Source: Awoyemi[14]

Error = |Exact result − Computed result|

NM= Error in New Method

Table 1: Comparison of absolute errors for Problem I

X	Error in NM	Error in [10]	Error in [14]
0.1	2.997602(−14)	0.716290(−11)	2.607525(−10)
0.2	2.280398(−13)	0.150910(−10)	1.981670(−09)
0.3	1.283640(−12)	0.452860(−10)	6.507412(−09)
0.4	4.687806(−12)	1.080840(−10)	1.559238(−08)
0.5	1.328893(−11)	1.781860(−10)	3.150448(−08)
0.6	3.242140(−11)	4.443440(−10)	5.637458(−08)
0.7	7.217982(−11)	7.444600(−10)	9.616405(−08)
0.8	1.521361(−10)	1.500980(−09)	1.568680(−07)
0.9	3.115295(−10)	3.757970(−09)	2.486977(−07)
1.0	6.324443(−10)	4.741080(−09)	3.879839(−07)

Table 2: Comparison of absolute errors for Problem 2

X	Error in NM	Error in [10]	Error in [14]
1.1048	1.673621(−09)	2.804740(−10)	4.692146(−07)
1.2048	1.940750(−09)	2.795040(−10)	4.080287(−07)
1.3048	2.126002(−09)	2.149060(−10)	2.228974(−07)
1.4048	2.205066(−09)	0.549750(−10)	8.128713(−07)
1.5048	2.203835(−09)	1.154550(−10)	5.244722(−07)
1.6048	2.142969(−09)	4.482520(−10)	1.089744(−06)
1.7048	2.041706(−09)	7.796920(−09)	1.753725(−06)
1.8048	1.940879(−09)	1.184050(−09)	2.481481(−06)
1.9048	1.858869(−09)	1.631810(−09)	3.228416(−06)
2.0048	1.811858(−09)	2.056760(−09)	3.943015(−06)

5.2. Discussion of Result

We have considered two numerical examples to test the efficiency of our developed scheme. Jator [10] and Awoyemi [14] solved Problem I and II where they developed methods of order six implemented in block method and predictor corrector method with step size $h = \frac{1}{320}$, $h = 0.05$ respectively. Despite the low order of our method and the high step size (h) used in solving problems I and II, our method gave better approximation as shown in Tables I and II.

6. Conclusion

A new approach for the derivation of block numerical integrator for direct solution of second order ordinary differential equations has been presented in this paper.It was tested on some initial value problems, It shows that our method perform better in terms of accuracy than the existing methods we compared with.

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