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ESTIMATES FROM BELOW OF BLOW-UP TIME IN A PARABOLIC SYSTEM WITH GRADIENT TERM

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Abstract: This paper deals with a nonlinear and weakly coupled parabolic system, containing gradient terms, under Dirichlet boundary conditions. The blow-up phenomena of its positive solutions are analyzed and, in particular, an estimate from below of blow-up time is obtained.

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1. Introduction

Qualitative properties as blow up, decay bounds and extinction in finite time for solutions to linear and nonlinear parabolic problems have received much attention in the recent literature. We refer to the papers [1], [4], [5] and the references therein.

We also mention the results of Galaktionov-Mitidieri-Pohoazev ([3], [13]) on blow-up and global existence for different classes of parabolic problems, including higher order parabolic equations.

The blow-up phenomena of solutions to various nonlinear problems, particularly for parabolic systems, have been investigated by different authors. For results in this area, the reader can reference the book [19] and the survey paper

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[1]. In the case of a single equation, we refer to [2], [15] and [20] for different results about blow-up phenomena. For other contributions in this field, see for elliptic equations [7] and [19]; for reaction diffusion equations [10] - [12]; for systems [6], [8] and [9]. We focus our attention on lower bounds for blow-up time of parabolic systems, which are of a great interest in several practical cases (see, for example, [19] and [20]).

We use ideas and techniques introduced by Payne-Philippin-Schaefer and Vernier-Piro ([14] and [16]).

Let us consider the following weakly coupled system

$$\begin{cases} u_t = \Delta u + v^p - |\nabla u|^q, & \mathbf{x} \in \Omega,\ t \in (0, t^\star), \\ v_t = \Delta v + u^p - |\nabla v|^q, & \mathbf{x} \in \Omega,\ t \in (0, t^\star), \\ u = 0 \ \text{ and } \ v = 0, & \mathbf{x} \in \partial\Omega,\ t \in (0, t^\star), \\ u = u_0(\mathbf{x}) \geq 0 \ \text{ and } \ v = v_0(\mathbf{x}) \geq 0, & \mathbf{x} \in \Omega, \end{cases} \tag{1}$$

where $t^\star$ is the blow-up time, Ω is a convex domain in $\mathbb{R}^3$ with the origin inside, whose boundary $\partial\Omega$ is sufficiently smooth, $p > q > 1$, and $u_0(\mathbf{x})$ and $v_0(\mathbf{x})$ are nonnegative functions in Ω, satisfying the compatibility conditions on $\partial\Omega$; it follows by the maximum principle that in the interval of existence the solution $(u(\mathbf{x}, t), v(\mathbf{x}, t))$ is nonnegative.

The paper is organized as follows. In Section 2, we derive a first order differential inequality in terms of an appropriate auxiliary function, by using the Talenti-Sobolev inequality present in [17] and [21], which is valid for nonnegative functions defined in a bounded domain $\Omega \subset \mathbb{R}^3$, vanishing on $\partial\Omega$. Then we obtain an explicit lower bound of $t^\star$ for a classical solution of system (1); this estimate depends on the geometry of Ω, and on the data p, q, $u_0(\mathbf{x})$ and $v_0(\mathbf{x})$. We remark that in a forthcoming paper a numerical resolution algorithm of system (1) will be presented, and some numerical examples will be analyzed to confirm our theoretical results.

2. Estimate from Below

The aim of this section is to obtain a lower bound of the blow-up time $t^\star$ of the solution of (1). To this end we introduce the auxiliary function

$$W(t) = \int_\Omega \left(u^{2p} + v^{2p}\right) d\mathbf{x}, \tag{2}$$

and we say that (u, v) blows up in $W-$norm if $\lim\limits_{t \to t^*} W(t) = +\infty$.

Theorem 1. *Let $W(t)$ be defined in (2) and (u, v) be a classical solution of (1) which becomes unbounded in W-norm at some finite time $t^\star$. Then an estimate from below for $t^\star$ is given by*

$$t^\star \geq \frac{1}{2\mathcal{A}W_0^2}, \tag{3}$$

where $W(0) = W_0 := \displaystyle\int_\Omega \left(u_0^{2p} + v_0^{2p}\right) d\mathbf{x}$ and $\mathcal{A}$ is a constant which depends on the data.

Proof. We compute

$$W'(t) = 2p \int_\Omega (u^{2p-1}u_t + v^{2p-1}v_t) d\mathbf{x} := U'(t) + V'(t).$$

For simplicity we calculate separately U' and analogously V'.

$$U' = 2p \int_\Omega u^{2p-1} u_t d\mathbf{x} \tag{4}$$

$$= 2p \int_\Omega u^{2p-1} \Delta u d\mathbf{x} + 2p \int_\Omega u^{2p-1} v^p d\mathbf{x} - 2p \int_\Omega u^{2p-1} |\nabla u|^q d\mathbf{x}.$$

On the other hand, by using the divergence theorem and the boundary conditions (2.3), we have

$$\int_\Omega u^{2p-1} \Delta u d\mathbf{x} = \int_\Omega div\big(u^{2p-1} \nabla u\big) d\mathbf{x} - (2p-1) \int_\Omega u^{2(p-1)} |\nabla u|^2 d\mathbf{x} \tag{5}$$

$$= \int_{\partial\Omega} u^{2p-1} u_n ds - (2p-1) \int_\Omega u^{2(p-1)} |\nabla u|^2 d\mathbf{x} = -(2p-1)J,$$

with

$$J = \int_\Omega u^{2(p-1)} |\nabla u|^2 d\mathbf{x}. \tag{6}$$

We also have

$$\int_\Omega u^{2p-1} v^p d\mathbf{x} \leq \Big(\frac{1}{\tau} \int_\Omega u^{\frac{3}{2}(2p-1)} d\mathbf{x}\Big)^{\frac{2}{3}} \Big(\tau^2 \int_\Omega v^{3p} d\mathbf{x}\Big)^{\frac{1}{3}} \tag{7}$$

$$\leq \frac{2}{3\tau} \int_\Omega u^{\frac{3}{2}(2p-1)} d\mathbf{x} + \frac{\tau^2}{3} \int_\Omega v^{3p} d\mathbf{x},$$

with $\tau > 0$ constant to be chosen later on.

To estimate the first term on the right hand in (7) we use the inequality (see Lemma A2 in [16]) with $\mu_1=\frac{3}{2\rho_0}$, $\mu_2=1+\frac{d}{\rho_0}$, $\rho_0=\min\limits_{\partial\Omega}(\mathbf{x}\cdot\boldsymbol{\nu})>0$ and $d=\max\limits_{\overline{\Omega}}|\mathbf{x}|$, being $\boldsymbol{\nu}$ the normal unit vector to $\partial\Omega$,

$$\int_\Omega \omega^{\frac{3}{2}n}d\mathbf{x}\le\Big\{\mu_1\int_\Omega\omega^n d\mathbf{x}+\frac{n}{2}\mu_2\int_\Omega\omega^{n-1}|\nabla\omega|d\mathbf{x}\Big\}^{\frac{3}{2}},\tag{8}$$

valid for any nonnegative C^1–function $\omega(\mathbf{x})$ defined in a bounded domain $\Omega\subset\mathbb{R}^3$ star-shaped and convex in two orthogonal directions, with $n\ge 1$. If $n=2p-1$ and $\omega=u$, we obtain

$$\int_\Omega u^{\frac{3}{2}(2p-1)}d\mathbf{x}\le\Big\{\mu_1\int_\Omega u^{2p-1}d\mathbf{x}+\frac{2p-1}{2}\mu_2\int_\Omega u^{2p-2}|\nabla u|d\mathbf{x}\Big\}^{\frac{3}{2}}.\tag{9}$$

By using the arithmetic inequality

$$(a+b)^{\frac{3}{2}}\le\sqrt{2}(a^{\frac{3}{2}}+b^{\frac{3}{2}})\quad a,b>0,\tag{10}$$

and Hölder and Schwarz inequalities respectively in first and second term in (9), we have

$$\int_\Omega u^{\frac{3}{2}(2p-1)}d\mathbf{x}\tag{11}$$
$$\le\sqrt{2}\;\mu_1^{\frac{3}{2}}\Big(\int_\Omega u^{2p-1}d\mathbf{x}\Big)^{\frac{3}{2}}+\sqrt{2}\Big(\frac{2p-1}{2}\Big)^{\frac{3}{2}}\mu_2^{\frac{3}{2}}\Big(\int_\Omega u^{2p-2}|\nabla u|d\mathbf{x}\Big)^{\frac{3}{2}}$$
$$\le\sqrt{2}\;\mu_1^{\frac{3}{2}}\Big(\int_\Omega u^{2p}d\mathbf{x}\Big)^{\frac{3}{4}\frac{2p-1}{p}}|\Omega|^{\frac{3}{4p}}+\sqrt{2}\Big(\frac{2p-1}{2}\Big)^{\frac{3}{2}}$$
$$\mu_2^{\frac{3}{2}}\Big[\Big(\int_\Omega u^{2p-2}d\mathbf{x}\Big)^{\frac{1}{2}}\Big(\int_\Omega u^{2p-2}|\nabla u|^2d\mathbf{x}\Big)^{\frac{1}{2}}\Big]^{\frac{3}{2}}.$$

Successively, in the last term of the right hand of (11) we use both Hölder and the arithmetic inequalities $a^rb^s\le ra+sb,\ r+s=1,\ a>0,\ b>0$ to get

$$\int_\Omega u^{\frac{3}{2}(2p-1)}d\mathbf{x}\le A_1U^{\frac{3}{4}(\frac{(2p-1)}{p})}+\frac{1}{4}A_2|\Omega|^{\frac{3}{p}}U^{\frac{3(p-1)}{p}}+\frac{3}{4}A_2J,\tag{12}$$

with

$$\begin{cases}A_1=\sqrt{2}\;\mu_1^{\frac{3}{2}}|\Omega|^{\frac{3}{4p}},\\ A_2=\sqrt{2}\Big(\frac{2p-1}{2}\Big)^{\frac{3}{2}}\;\mu_2^{\frac{3}{2}}.\end{cases}$$

Let us estimate now $\int_\Omega v^{3p} d\mathbf{x}$ in (7). By using (8) with $n = 2p$, $\omega = v$ we obtain

$$\int_\Omega v^{3p} d\mathbf{x} \leq \Big[\mu_1 \int_\Omega v^{2p} d\mathbf{x} + p\mu_2 \int_\Omega v^{2p-1} |\nabla v| d\mathbf{x}\Big]^{\frac{3}{2}} \tag{13}$$

$$\leq \sqrt{2}\ \mu_1^{\frac{3}{2}} V^{\frac{3}{2}} + \sqrt{2}\Big(p\mu_2\Big)^{\frac{3}{2}} \Big[\int_\Omega v^{2p-1}|\nabla v| d\mathbf{x}\Big]^{\frac{3}{2}}$$

$$\leq B_1 V^{\frac{3}{2}} + B_2 V^{\frac{3}{4}} I^{\frac{3}{4}} \leq B_1 V^{\frac{3}{2}} + \frac{1}{4\delta^3} B_2 V^3 + \frac{3}{4}\delta B_2 I,$$

being

$$\begin{cases} I = \int_\Omega v^{2(p-1)} |\nabla v|^2 d\mathbf{x}, \\ B_1 = \sqrt{2}\mu_1^{\frac{3}{2}}, \\ B_2 = \sqrt{2}\big(p\mu_2\big)^{\frac{3}{2}}. \end{cases}$$

Now we replace (12) and (13) in (7) to obtain

$$\int_\Omega u^{2p-1} v^p d\mathbf{x} \leq \frac{2}{3\tau}\big(A_1 U^{\frac{3}{4}(\frac{(2p-1)}{p})} + \frac{1}{4} A_2 |\Omega|^{\frac{3}{p}} U^{\frac{3(p-1)}{p}} + \frac{3}{4} A_2 J\big) \tag{14}$$

$$+\frac{\tau^2}{3}\big(B_1 V^{\frac{3}{2}} + \frac{1}{4\delta^3} B_2 V^3 + \frac{3}{4}\delta B_2 I\big).$$

In order to estimate the third term in (4), we observe that

$$u^{2p-1} |\nabla u|^q = \xi^q |\nabla u^{\frac{2p+q-1}{q}}|^q, \tag{15}$$

with $\xi = \frac{q}{2p+q-1}$, and by using (2.10) in [14] we obtain

$$\xi^q \int_\Omega |\nabla u^{\frac{2p+q-1}{q}}|^q d\mathbf{x} \tag{16}$$

$$\geq \xi^q \Big(\frac{4\lambda_1}{q^2}\Big)^{\frac{q}{2}} \int_\Omega u^{2p+q-1} d\mathbf{x} = k^q \int_\Omega u^{2p+q-1} d\mathbf{x},$$

with $k = \frac{2\xi\sqrt{\lambda_1}}{q}$ and λ_1 the first eigenvalue of the problem

$$\begin{cases} \Delta w + \lambda w = 0 \quad \mathbf{x} \in \Omega, \quad w > 0, \\ w = 0, \quad \mathbf{x} \in \partial\Omega. \end{cases} \tag{17}$$

Hölder inequality allows us to write

$$\int_\Omega u^{2p} d\mathbf{x} \leq \Big(\int_\Omega u^{2p+q-1} d\mathbf{x}\Big)^{\frac{2p}{2p+q-1}} |\Omega|^{\frac{q-1}{2p+q-1}},$$

i.e.,

$$\int_\Omega u^{2p+q-1}d\mathbf{x} \geq |\Omega|^{\frac{1-q}{2p}}\Big(\int_\Omega u^{2p}d\mathbf{x}\Big)^{\frac{2p+q-1}{2p}} = |\Omega|^{\frac{1-q}{2p}}U^{\frac{2p+q-1}{2p}}. \tag{18}$$

Replacing (18) in (16) we get

$$\xi^q\int_\Omega|\nabla u^{\frac{2p+q-1}{q}}|^q d\mathbf{x} \geq k^q|\Omega|^{\frac{1-q}{2p}}U^{\frac{2p+q-1}{2p}}. \tag{19}$$

Finally we substitute (5), (14) and (19) in (4) and obtain

$$\begin{aligned} U' \leq{}& -2p(2p-1)J + \frac{4p}{3\tau}\Big\{A_1U^{\frac{3}{4}(\frac{2p-1}{p})} + \frac{1}{4}A_2|\Omega|^{\frac{3}{p}}U^{\frac{3(p-1)}{p}} + \frac{3}{4}A_2J\Big\} \\ &+\frac{2p}{3}\tau^2\Big[B_1V^{\frac{3}{2}} + \frac{1}{4\delta^3}B_2V^3 + \frac{3}{4}\delta B_2 I\Big] - 2pk^q|\Omega|^{\frac{1-q}{2p}}U^{\frac{2p+q-1}{2p}}. \end{aligned} \tag{20}$$

By using Young inequality in the second term in (20) we have

$$\begin{aligned} U' \leq{}& -2p(2p-1)J \\ &+\frac{4p}{3\tau}\Big\{A_1\frac{c}{2}U^{\frac{2p+q-1}{2p}} + A_1\frac{1}{2c}U^{\frac{4p-q-2}{2p}} + \frac{1}{4}A_2|\Omega|^{\frac{3}{p}}U^{\frac{3(p-1)}{p}} + \frac{3}{4}A_2J\Big\} \\ &+\frac{2p}{3}\tau^2\Big[B_1V^{\frac{3}{2}} + \frac{1}{4\delta^3}B_2V^3 + \frac{3}{4}\delta B_2 I\Big] - 2pk^q|\Omega|^{\frac{1-q}{2p}}U^{\frac{2p+q-1}{2p}} \\ \leq{}& \bar{A}_1W^{\frac{4p-q-2}{2p}} + \bar{A}_2W^{\frac{2p+q-1}{2p}} + \bar{A}_3W^{\frac{3(p-1)}{p}} + \bar{A}_4W^{\frac{3}{2}} + \bar{A}_5W^3 + \bar{A}_6J + \bar{A}_7I, \end{aligned} \tag{21}$$

with $c>0$ a suitable constant such that $\bar{A}_2 \geq 0$, and

$$\begin{cases} \bar{A}_1 = \frac{2p}{3\tau}\frac{A_1}{c}, \quad \bar{A}_2 = \frac{2p}{3\tau}A_1c - 2pk^q|\Omega|^{\frac{1-q}{2p}}, \\ \bar{A}_3 = \frac{p}{3\tau}A_2|\Omega|^{\frac{3}{p}}, \quad \bar{A}_4 = \frac{2p}{3}\tau^2B_1, \\ \bar{A}_5 = \frac{p}{6\delta^3}\tau^2B_2, \quad \bar{A}_6 = \frac{p}{\tau}A_2 - 2p(2p-1), \quad \bar{A}_7 = \frac{p}{2}\tau^2\delta B_2. \end{cases}$$

Analogously

$$\begin{aligned} V' \leq{}& \bar{B}_1W^{\frac{4p-q-2}{2p}} + \bar{B}_2W^{\frac{2p+q-1}{2p}} + \bar{B}_3W^{\frac{3(p-1)}{p}} + \bar{B}_4W^{\frac{3}{2}} \\ &+\bar{B}_5W^3 + \bar{B}_6J + \bar{B}_7I \end{aligned} \tag{22}$$

with

$$\begin{cases} \bar{B}_1 = \frac{2p}{3\sigma}\frac{A_1}{\bar{c}}, \quad \bar{B}_2 = \frac{2p}{3\sigma}A_1\bar{c} - 2p\Big(\frac{2\xi\sqrt{\lambda_1}}{q}\Big)^q|\Omega|^{\frac{1-q}{2p}}, \\ \bar{B}_3 = \frac{p}{3\gamma^3}A_2|\Omega|^{\frac{3}{p}}, \quad \bar{B}_4 = \frac{2p}{3}\sigma^2B_1, \\ \bar{B}_5 = \frac{p\sigma^2}{6\gamma^3}B_2, \quad \bar{B}_6 = \frac{\sigma^2}{2}\gamma pB_2, \quad \bar{B}_7 = \frac{\gamma p}{6}A2 - 2p(2p-1). \end{cases}$$

Due to (20) and (22), we can write

$$W'(t) \leq (\bar{A}_1 + \bar{B}_1)W^{\frac{4p-q-2}{2p}} + (\bar{A}_2 + \bar{B}_2)W^{\frac{2p+q-1}{2p}} + (\bar{A}_3 + \bar{B}_3)W^{\frac{3(p-1)}{p}} \tag{23}$$
$$+(\bar{A}_4 + \bar{B}_4)W^{\frac{3}{2}} + (\bar{A}_5 + \bar{B}_5)W^3 + (\bar{A}_6 + \bar{B}_6)J + (\bar{A}_7 + \bar{B}_7)I.$$

Now, by choosing in (23) $\tau,\ \sigma,\ \delta,$ and γ such that

$$\begin{cases} \bar{A}_6 + \bar{B}_6 = 0, \\ \bar{A}_7 + \bar{B}_7 = 0, \end{cases}$$

we have

$$W'(t) \leq \xi_1 W^{\frac{4p-q-2}{2p}} + \xi_2 W^{\frac{2p+q-1}{2p}} + \xi_3 W^{\frac{3(p-1)}{p}} + \xi_4 W^{\frac{3}{2}} + \xi_5 W^3, \tag{24}$$

where $\xi_i, \quad i = 1, .., 5$ are computable coefficients.

Our aim is to derive an inequality containing only W^3 and then to get an explicit lower bound of the blow-up at time $t^\star$. To this end we use the inequality $W(t) \geq W_0$, i.e. $\dfrac{W(t)}{W_0} \geq 1$, giustified as follows. If $W(t)$ is non decreasing, then $W(t) \geq W_0$ for $\mathrm{t} \in [0, \mathrm{t}^\star)$. If $W(t)$ is non increasing (or possibly with some kind of oscillations), since we have assumed $W(t)$ blowing up at time $t^\star$, then there exists a time t_1 where $W(t_1) = W_0$. As a consequence, $W(t) \geq W_0, \quad t \in [t_1, t^\star)$ so that $\left(\dfrac{W(t)}{W_0}\right)^r \leq \left(\dfrac{W(t)}{W_0}\right)^3$, for some power $r < 3$. . It follows that

$$\begin{cases} W^{\frac{4p-q-2}{2p}} \leq W_0^{\frac{4p-q-2}{2p}-3} W^3, \\ W^{\frac{2p+q-1}{2p}} \leq W_0^{\frac{2p+q-1}{2p}-3} W^3, \\ W^{\frac{3(p-1)}{p}} \leq W_0^{\frac{3(p-1)}{p}-3} W^3, \\ W^{\frac{3}{2}} \leq W_0^{-\frac{3}{2}} W^3, \end{cases}$$

where by the hypotheses all the powers of W in (24) are less than equal 3. By replacing the previous inequalities in (24) we get

$$W'(t) \leq \mathcal{A} W^3(t), \quad \forall t \in [t_1, t^\star) \tag{25}$$

with $\mathcal{A} := \xi_1 W_0^{\frac{4p-q-2}{2p}-3} + \xi_2 W_0^{\frac{2p+q-1}{2p}-3} + \xi_3 W_0^{\frac{3(p-1)}{p}-3} + \xi_4 W_0^{-\frac{3}{2}} + \xi_5$. Integrating (25) we have

$$\frac{1}{2W_0^2} \leq \int_{t_1}^{t^\star} \mathcal{A} d\tau \leq \int_0^{t^\star} \mathcal{A} d\tau = \mathcal{A} t^\star,$$

that leads to the lower bound (3). $\square$

Remark. We remark that if in the system (1) under investigation, we replace the Dirichlet boundary condition with the Neumann or Robin boundary conditions, the result of Theorem 1 can be extended. In fact in deriving the lower bound for the blow-up time of the solution, we use a Sobolev type inequality valid for any boundary condition. To this end, one can follow the computations in [12], where the authors obtain a lower bound for $t^\star$ in the case of a single equation and under Dirichlet, Neumann and Robin boundary conditions.

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