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SOME RELATED FIXED POINTS THEOREMS FOR A PAIR OF MAPPINGS ON TWO METRIC SPACES

Karim Chaira[1], El-Miloudi Marhrani[2 §]

[1,2]Department of Mathematics and Computer Science
University Hassan II Mohammedia Casablanca
Faculty of Science Ben M'Sik
P.B. 7955, Sidi Othmane, Casablanca, MOROCCO

Abstract: In [4], B. Fisher presented some related fixed points theorems involving two mappings on metric spaces under some conditions of contractions. The main purpose of this paper is to give some generalizations of these results.

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1. Introduction

In 1981, B. Fisher presented the following related fixed point theorem

Theorem 1.1. *(B. Fisher) Let (X, d) and (Y, δ) two metric spaces. We assume that (X, d) is complete. Let $T : X \to Y$ and $S : Y \to X$ two mappings such that for all $(x, y) \in X \times Y$,*

$$\begin{cases} d(Sy, STx) \leq c.Max\{d(x, Sy); \delta(y, Tx); d(x, STx)\}, \\ \delta(Tx, TSy) \leq c.Max\{d(x, Sy); \delta(y, Tx); \delta(y, TSy)\}, \end{cases}$$

where $c \in [0, 1[$. Then there exists a unique pair $(x^*, y^*) \in X \times Y$ such that $Tx^* = y^*$ and $Sy^* = x^*$, $STx^* = x^*$ which gives $TSy^* = y^*$.

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§Correspondence author

Some generalizations of this result have been appeared in many different directions. For more details, we refer to [1], [5], [6] and [7].

The purpose of this paper, is to give some generalizations of this result by replacing the constant c using functions $\alpha, \beta, \gamma : [0, +\infty[\to [0, 1[$ such that $\limsup_{t \to t_0^+} (\alpha(t) + \beta(t) + \gamma(t)) < 1, \forall t_o \in [0, +\infty[$.

2. Main Results

Theorem 2.1. *Let (X, d) and (Y, δ) two metric spaces; we assume that (X, d) is complete. Let $T : X \to Y$ and $S : Y \to X$ two mappings such that, for all $(x, y) \in X \times Y$,*

$$\begin{cases} d(Sy, STx) \le \alpha(\delta(y, Tx)) Max\{d(x, Sy); \delta(y, Tx)\} + \beta(\delta(y, Tx)) d(x, STx), \\ \delta(Tx, TSy) \le \alpha(d(x, Sy)) Max\{d(x, Sy); \delta(y, Tx)\} + \beta(d(x, Sy)) \delta(y, TSy), \end{cases}$$

where α,β : $[0, +\infty[$ are two functions satisfying:

$$\limsup_{t \to t_0^+} (\alpha(t) + \beta(t)) < 1, \quad \text{for all } t_0 \in [0, +\infty[.$$

Then there exists a unique pair $(x^*, y^*) \in X \times Y$ such that $Tx^* = y^*$ and $Sy^* = x^*$; and then, $STx^* = x^*$ and $TSy^* = y^*$.

Proof. Let $x_0 \in X$; we construct two sequences $(x_n)_n$ and $(y_n)_n$ by

$$y_n = Tx_n \quad \text{and} \quad x_{n+1} = Sy_n, \quad \text{for all } n \in \mathbb{N}.$$

For all $n \in \mathbb{N}^*$, we obtain:

$$\begin{aligned} d(x_n, x_{n+1}) =& d(Sy_{n-1}, STx_n) \\ \le& \alpha(\delta(y_{n-1}, Tx_n) Max\{d(x_n, Sy_{n-1}); \delta(y_{n-1}, Tx_n)\} \\ &+ \beta(\delta(y_{n-1}, Tx_n)) d(x_n, STx_n), \end{aligned}$$

which leads to

$$d(x_n, x_{n+1}) \le \alpha(\delta(y_{n-1}, y_n)) \delta(y_{n-1}, y_n) + \beta(\delta(y_{n-1}, y_n)) d(x_n, x_{n+1})$$

since $d(x_n, Sy_{n-1}) = d(x_n, x_n) = 0$.

We have also

$$\delta(y_n, y_{n+1}) = \delta(Tx_n, TSy_n)$$

$$\begin{aligned}&\leq \alpha(d(x_n, Sy_n))Max\{d(x_n, Sy_n); \delta(y_n, Tx_n)\}\\ &\quad + \beta(d(x_n, Sy_n))\delta(y_n, TSy_n).\end{aligned}$$

Since $\delta(y_n, Tx_n) = \delta(y_n, y_n) = 0$, we obtain

$$\begin{aligned}\delta(y_n, y_{n+1}) &\leq \alpha(d(x_n, x_{n+1}))d(x_n, x_{n+1}) + \beta(d(x_n, x_{n+1}))\delta(y_n, y_{n+1}),\\ \delta(y_n, y_{n+1})) &\leq \frac{\alpha(d(x_n, x_{n+1}))}{1-\beta(d(x_n, x_{n+1}))}\frac{\alpha(\delta(y_{n-1}, y_n))}{1-\beta(\delta(y_{n-1}, y_n))}\delta(y_{n-1}, y_n),\end{aligned}$$

and

$$d(x_n, x_{n+1}) \leq \frac{\alpha(\delta(y_{n-1}, y_n))}{1-\beta(\delta(y_{n-1}, y_n))}\frac{\alpha(d(x_{n-1}, x_n))}{1-\beta(d(x_{n-1}, x_n))]}d(x_{n-1}, x_n).$$

By hypothesis, $0 \leq \frac{\alpha(t)}{1-\beta(t)} < 1, \forall t \in [0, +\infty[$; it follows that the sequences $(d(x_n, x_{n+1}))_{n\in\mathbb{N}}$ and $(\delta(y_n, y_{n+1}))_{n\in\mathbb{N}}$ are décreasing and consequently they are convergent. We put

$$\ell_1 = \lim_{n\to+\infty} d(x_n, x_{n+1}) \text{ and } \ell_2 = \lim_{n\to+\infty} \delta(y_n, y_{n+1}).$$

Since

$$\limsup_{t\to\ell_1^+}(\alpha(t)+\beta(t))<1 \text{ and } \limsup_{t\to\ell_2^+}(\alpha(t)+\beta(t))<1,$$

there exists $n_0 \in \mathbb{N}$ and $r \in [0, 1[$ such that for any integer $n \geq n_0$,

$$Max\{\alpha(d(x_n, x_{n-1})) + \beta(d(x_n, x_{n-1})), \alpha(\delta(y_n, y_{n-1})) + \beta(\delta(y_n, y_{n+1}))\} \leq r.$$

Therefore, $\alpha(d(x_n, x_{n-1})) \leq r - \beta(d(x_n, x_{n-1})) \leq r(1-\beta(d(x_n, x_{n-1})))$; and then

$$\frac{\alpha(d(x_n, x_{n-1}))}{1-\beta(d(x_n, x_{n-1}))} \leq r.$$

It follows that, for all integers $n \geq n_0$,

$$d(x_n, x_{n+1})) \leq r\frac{\alpha(\delta(y_{n-1}, y_n))}{1-\beta(\delta(y_{n-1}, y_n))}d(x_{n-1}, x_n) \leq rd(x_{n-1}, x_n).$$

We obtain a Cauchy sequence $(x_n)_{n\geq 0}$ in the complete metric space (X, d); then there exists $x^* \in X$ such that

$$\lim_{n\to+\infty} d(x_n, x^*) = 0.$$

The same computations leads to

$$\lim_{n\to+\infty}\delta(y_n, y_{n+1}) = 0.$$

If we set $y^* = Tx^*$, then we obtain

$$\begin{aligned}\delta(y_n, y^*) =&\delta(TSy_{n-1}, Tx^*)\\ &\leq\alpha(d(x^*,Sy_{n-1}))Max\{d(x^*,Sy_{n-1});\delta(y_{n-1}, Tx^*)\}\\ &\quad+\beta(d(x^*, Sy_{n-1}))\delta(y_{n-1}, TSy_{n-1}),\\ &\leq\alpha(d(x^*, x_n))Max\{d(x^*, x_n); \delta(y_{n-1}, Tx^*)\}\\ &\quad+\beta(d(x^*, x_n))\delta(y_{n-1}, y_n).\end{aligned}$$

Taking in mind

$$\begin{aligned}Max\{d(x^*, x_n); \delta(y_{n-1}, Tx^*)\} \leq& d(x^*, x_n) + \delta(y_{n-1}, Tx^*)\\ \leq& d(x^*, x_n) + \delta(y_{n-1}, y_n) + \delta(y_n, Tx^*)\end{aligned}$$

we obtain

$$[1 - \alpha(d(x^*, x_n))]\delta(y_n, Tx^*) \leq d(x^*, x_n) + \delta(y_{n-1}, y_n).$$

By hypothesis on $\alpha(t)$, there exists $r_0 \in [0, 1[$ such that $\limsup_{t\to0^+}\alpha(t) < r_0$; such that

$$(1 - r_0)\delta(y_n, y^*) \leq d(x^*, x_n) + \delta(y_{n-1}, y_n), \text{for large integers } n,$$

witch leads to $\lim_n \delta(y_n, y^*) = 0$.

Now, we prove that $Sy^* = x^*$. We have:

$$\begin{aligned}d(x_n, Sy^*) =&d(STx_{n-1}, Sy^*)\\ &\leq\alpha(\delta(y^*, Tx_{n-1}))Max\{\delta(y^*, Tx_{n-1}); d(x_{n-1}, Sy^*)\}\\ &\quad+\beta(\delta(y^*, Tx_{n-1})d(x_{n-1}, STx_{n-1})\\ &\leq\alpha(\delta(y^*, y_{n-1}))Max\{d(x_{n-1}, Sy^*); \delta(y^*, y_{n-1})\}\\ &\quad+\beta(\delta(y^*, y_{n-1}))d(x_{n-1}, x_n)\\ &\leq\alpha(\delta(y^*, y_{n-1}))\{d(x_{n-1}, x_n) + d(x_n, Sy^*) + \delta(y^*, y_{n-1})\}\\ &\quad+\beta(\delta(y^*, y_{n-1}))d(x_{n-1}, x_n).\end{aligned}$$

witch leads to:

$$(1 - \alpha(\delta(y^*, y_{n-1})))d(x_n, Sy^*) \leq(\alpha + \beta)(\delta(y^*, y_{n-1}))d(x_{n-1}, x_n)$$

$$+\alpha(\delta(y^*,y_{n-1}))\delta(y_{n-1},y^*)$$
$$\leq d(x_{n-1},x_n)+\delta(y_{n-1},y^*).$$

As before, $1-\alpha(\delta(y^*,y_{n-1})$ is bounded below, then

$$\lim_{n\to+\infty} d(x_n,Sy^*)=0,$$

which asserts that $Sy^*=x^*$.

It follows that $TSy^*=Tx^*=y^*$ and $STx^*=Sy^*=x^*$; so x^* et y^* are fixed point of the mappings ST and TS respectively.

Uniqueness of x^* and y^*. Assume that there exists $x\in X$ such that $x\neq x^*$ and $STx=x$; we can write

$$\begin{aligned}
d(x,x^*) =& d(STx,STx^*)\\
\leq& \alpha(\delta(Tx,Tx^*))Max\{\delta(Tx,Tx^*);d(x,STx^*)\}\\
&+\beta(\delta(Tx,Tx^*))d(x,STx)\\
\leq& \alpha(\delta(y^*,Tx))Max\{d(x,x^*);\delta(y^*,Tx)\}+\beta(\delta(y^*,Tx))d(x,x)\\
\leq& \alpha(\delta(y^*,Tx))Max\{d(x,x^*);\delta(y^*,Tx)\}.
\end{aligned}$$

Since $\alpha(\delta(y^*,Tx))<1$, we obtain

$$d(x,x^*)\leq\delta(y^*,Tx).$$

And then

$$d(x,x^*)\leq\alpha(\delta(y^*,Tx))\delta(y^*,Tx).$$

On the other hand, we have

$$\begin{aligned}
\delta(y^*,Tx) =& \delta(TSy^*,Tx)\\
\leq& \alpha(d(x,Sy^*))Max\{\delta(Tx,y^*);d(x,Sy^*)\}+\beta(d(x,Sy^*))\delta(y^*,TSy^*)\\
\leq& \alpha(d(x,x^*))Max\{\delta(Tx,y^*);d(x,x^*)\}+\beta(d(x,x^*))\delta(y^*,y^*)\\
\leq& \alpha(d(x,x^*))Max\{\delta(Tx,y^*);d(x,x^*)\}.
\end{aligned}$$

It follows that

$$\delta(y^*,Tx))\leq\alpha(d(x,x^*))\delta(Tx,y^*),$$

and then $y^*=Tx$; which leads to $x=x^*$ and $y=y^*$.

Example 2.2. Let $X = [0,3]$ et $Y = [\frac{1}{2}, 2]$; we consider the following distances

$$d(x,x') = |x - x'| \quad \text{and} \quad \delta(y,y') = \frac{3}{2}|y - y'|,$$

$\forall (x,x') \in X^2$, $\forall (y,y') \in Y^2$ and the mapping defined by $T : X \longrightarrow Y$ and $S : Y \longrightarrow X$ with

$$T(x) = \frac{x+1}{2} \quad \text{and} \quad S(y) = y, \forall (x,y) \in X \times Y.$$

We define two functions $\alpha(t)$ and $\beta(t)$ on $[0,+\infty[$ by

$$\alpha(t) = \begin{cases} \dfrac{3}{4} & \text{if} \quad t \in [0,+\infty[\setminus \left\{\frac{5}{2}\right\}, \\ \dfrac{57}{80} & \text{if} \quad t = \frac{5}{2}, \end{cases}$$

and

$$\beta(t) = \begin{cases} \dfrac{1}{8} & \text{if} \quad t \in [0,+\infty[\setminus \left\{\frac{5}{2}\right\}, \\ \dfrac{1}{4} & \text{if} \quad t = \frac{5}{2}. \end{cases}$$

Note that $\{(x,y) \in X \times Y \; ; \; |x - y| = \frac{5}{2}\} = \{(3, \frac{1}{2})\}$.

First case. $(x,y) \in X \times Y \setminus \left\{(3, \frac{1}{2})\right\}$. We have

$$\delta(Tx, TSy) = \frac{3}{2}|Tx - TSy| = \frac{3}{4}|x - y|, \quad d(x, Sy) = |x - y|,$$

$$\delta(y, Tx) = \frac{3}{2}|y - \frac{x+1}{2}| \quad \text{and} \quad \delta(y, TSy) = \frac{3}{4}|y - 1|.$$

As $|x - y| \neq \frac{5}{2}$, we have $\alpha(d(x,Sy)) = \frac{3}{4}$; and consequently:

$$\delta(Tx, TSy) = \frac{3}{4}|x - y| = \frac{3}{4}d(x, Sy),$$

$$\delta(Tx, TSy) \leq \alpha(d(x,Sy)) Max\{d(x,Sy); \delta(y,Tx)\} + \beta(d(x,Sy)).\delta(y,TSy),$$

Similarly, we have

$$d(Sy, STx) = |Sy - STx| = |y - \frac{x+1}{2}|, \quad \delta(y, Tx) = \frac{3}{2}|y - \frac{x+1}{2}|,$$

$$d(x, Sy) = |x - y| \quad \textit{and} \quad d(x, STx) = \frac{|x-1|}{2}.$$

Since $\frac{3}{2}.|y - \frac{x+1}{2}| \neq \frac{5}{2}$, we obtain

$$\alpha(\delta(y, Tx)) = \frac{3}{4};$$

and consequently:

$$d(Sy, STx) = |y - \frac{x+1}{2}| = \frac{2}{3}(\frac{3}{2}|y - \frac{x+1}{2}|) = \frac{2}{3}\delta(y, Tx) \leq \frac{3}{4}\delta(y, Tx),$$

$$d(Sy, STx) \leq \alpha(\delta(y, Tx))Max\{d(x, Sy); \delta(y, Tx)\} + \beta(\delta(y, Tx)).d(x, STx).$$

Second case. $(x, y) = (3, \frac{1}{2})\}$; and then $|x - y| = \frac{5}{2}$.

$$\delta(Tx, TSy) = \frac{3}{4}|x - y| = \frac{57}{80}|x - y| + \frac{3}{4}.\frac{1}{20}.\frac{5}{2};$$

and

$$\delta(y, TSy) = \frac{3}{2}|\frac{y-1}{2}| = \frac{3}{8}.$$

So, we have

$$\delta(Tx, TSy) = \frac{57}{80}|x - y| + \frac{1}{4}.\frac{3}{8} = \frac{57}{80}d(x, Sy) + \frac{1}{4}\delta(y, TSy);$$

with $\alpha(d(x, Sy)) = \alpha(\frac{5}{2}) = \frac{57}{80}$ and $\beta(d(x, Sy)) = \beta(\frac{5}{2}) = \frac{1}{4}$. Thus,

$$\delta(Tx, TSy) \leq \alpha(d(x, Sy))Max\{d(x, Sy); \delta(y, Tx)\} + \beta(d(x, Sy))\delta(y, TSy)$$

Since, $\frac{3}{2}|y - \frac{x+1}{2}| \neq \frac{5}{2}$, we have $\alpha(\delta(y, Tx)) = \frac{3}{4}$ and $\beta(\delta(y, Tx)) = \frac{1}{8}$. Consequently,

$$d(Sy, STx) = \frac{2}{3}\delta(y, Tx) \leq \frac{3}{4}\delta(y, Tx) \leq \alpha(\delta(y, Tx))\delta(y, Tx)$$

witch leads to:

$$d(Sy, STx) \leq \alpha(\delta(y, Tx))Max\{d(x, Sy); \delta(y, Tx)\} + \beta(\delta(y, Tx))d(x, STx).$$

Note that 1 is the unique fixed point of the mappings S and T.

Corollary 2.3. *Let (X, d) and (Y, δ) two metric spaces; we assume that (X, d) is complete. Let $T : X \to Y$ and $S : Y \to X$ two mappings such that for all $(x, y) \in X \times Y$,*

$$d(Sy, STx) \leq \alpha(\delta(y, Tx))Max\{d(x, Sy); \delta(y, Tx)\},$$

$$\delta(Tx, TSy) \leq \alpha(d(x, Sy))Max\{d(x, Sy); \delta(y, Tx)\},$$

where $\alpha : [0, +\infty[\to [0, 1[$ *is a function satisfying* $\limsup_{t \to t_0^+} \alpha(t) < 1$, *for all* $t_0 \in [0, +\infty[$.

Then there exists a unique pair $(x^*, y^*) \in X \times Y$ *such that* $Tx^* = y^*$ *and* $Sy^* = x^*$.

Corollary 2.4. *Let* (X, d) *and* (Y, δ) *two metric spaces; we assume that* (X, d) *is complete. Let* $T : X \to Y$ *and* $S : Y \to X$ *two mappings such that for all* $(x, y) \in X \times Y$,

$$d(Sy, STx) \leq \beta(\delta(y, Tx))d(x, STx),$$
$$\delta(Tx, TSy) \leq \beta(d(x, Sy))\delta(y, TSy),$$

where $\beta : [0, +\infty[\to [0, 1[$ *function satisfying* $\limsup_{t \to t_0^+} \beta(t) < 1$, *for all* $t_0 \in [0, +\infty[$.

Then, there exists a unique pair $(x^*, y^*) \in X \times Y$ *such that* $Tx^* = y^*$ *and* $Sy^* = x^*$.

Corollary 2.5. *Let* (X, d) *and* (Y, δ) *two metric spaces; we assume that* (X, d) *is complete. Let* $T : X \to Y$ *and* $S : Y \to X$ *two mappings such that for all* $(x, y) \in X \times Y$,

$$d(Sy, STx) \leq c.Max\{d(x, Sy); \delta(y, Tx); d(x, STx)\},$$
$$\delta(Tx, TSy) \leq c.Max\{d(x, Sy); \delta(y, Tx); \delta(y, TSy)\},$$

where $c \in [0, \frac{1}{2}[$.

Then there exists a unique pair $(x^*, y^*) \in X \times Y$ *such that* $Tx^* = y^*$ *and* $Sy^* = x^*$.

Corollary 2.6. *Let* (X, d) *a complete metric space and* $T : X \to X$ *a mapping such that for all* $(x, y) \in X^2$,

$$d(Tx, T^2y) \leq \alpha(d(x, Ty))Max\{d(x, Ty); d(y, Tx)\} + \beta(d(x, Ty))d(y, T^2y),$$

where $\alpha, \beta : [0, +\infty[\to [0, 1[$ *with* $\limsup_{t \to t_0^+} \alpha(t) + \beta(t) < 1$, *for all* $t_0 \geq 0$. *Then there exists a unique element* $x^* \in X$ *such that* $Tx^* = x^*$.

Theorem 2.7. *Let* (X, d) *and* (Y, δ) *two metric spaces, we assume that* (X, d) *is complete. Let* $T : X \to Y$ *and* $S : Y \to X$ *be two mappings such that for all* $(x, y) \in X \times Y$,

$$d(Sy, STx) \leq \alpha(\delta(y, Tx))\delta(y, Tx) + \beta(\delta(y, Tx))d(x, Sy)$$

$$\begin{aligned} &+\gamma(\delta(y,Tx))d(x,ST(x)),\\ \delta(Tx,TSy) \leq &\alpha(d(x,Sy))d(x,Sy)+\beta(d(x,Sy))\delta(y,Tx)\\ &+\gamma(d(x,Sy))\delta(y,TSy),\end{aligned}$$

where $\alpha,\beta,\gamma:[0,+\infty[\to[0,1[$ *are functions such that*

$$\limsup_{t\to t_0^+}(\alpha(t)+\beta(t)+\gamma(t))<1, \quad \textit{for all } t_0\in[0,+\infty[.$$

Then there exists a unique pair $(x^*,y^*)\in X\times Y$ *such that* $Tx^*=y^*$ *and* $Sy^*=x^*$.

Proof. Let $\alpha_0(t)=\alpha(t)+\beta(t)$; we have

$$\limsup_{t\to t_0}(\alpha_0(t)+\gamma(t))<1, \quad \text{for all } t_0\geq 0,$$

and

$$\begin{aligned} d(Sy,STx) \leq &\alpha(\delta(y,Tx))\delta(y,Tx)\}+\beta(\delta(y,Tx))d(x,Sy)\\ &+\gamma(\delta(y,Tx))d(x,ST(x))\\ \leq &\alpha_0(\delta(y,Tx))Max\{\delta(y,Tx);d(x,Sy)\}\\ &+\gamma(\delta(y,Tx))d(x,ST(x)),\end{aligned}$$

for all $(x,y)\in X\times Y$.

Similarly, we obtain

$$\begin{aligned} \delta(Tx,TSy) \leq &\alpha(d(x,Sy))d(x,Sy)+\beta(d(x,Sy))\delta(y,Tx)\}\\ &+\gamma(d(x,Sy))\delta(y,TSy)\\ \leq &(\alpha_0(d(x,Sy)))Max\{d(x,Sy);\delta(y,Tx)\}\\ &+\gamma(d(x,Sy))\delta(y,TSy).\end{aligned}$$

So there exists a unique pair $(x^*,y^*)\in X\times Y$ such that $Tx^*=y^*$ et $Sy^*=x^*$. Moreover, $STx^*=x^*$ and $TSy^*=y^*$.

Corollary 2.8. *Let* (X,d) *be a complete metric space and* $T:X\to X$ *a mapping such that, for all* $(x,y)\in X^2$,

$$d(Tx,T^2y)\leq\alpha(d(x,Ty))d(x,Ty),$$

where $\alpha:[0,+\infty[\to[0,1[$ *is a function satisfying* $\limsup_{t\to t_0^+}\alpha(t)<1$, *for all* $t_0\in[0,+\infty[$. *Then there exists a unique element* $x^*\in X$ *such that* $Tx^*=x^*$.

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