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MAPPING ON GENERALIZED VAGUE SOFT EXPERT SET

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Abstract: We introduce the mapping on generalized vague soft expert set and its operations are studied. The basic operations of mapping on generalized vague soft expert set theory are defined.

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1. Introduction

The notion of mapping on soft classes are introduced by Kharal and Ahmad [1]. One of the most important new mathematical tools is soft set theory defined by Molodtsov [2]. This is extended to fuzzy soft sets [3], [4], [5], [6], [7] and then to vague soft sets [8], [9], [10], [11], [12] followed by interval valued vague soft sets [13], [14], [15]. Fuzzy sets were applied to genetic algorithms [16], [17] and into multi Q-fuzzy [18]. In this paper we introduce the notion of mapping on generalized vague soft expert classes and study the properties of generalized vague soft expert images and generalized vague soft expert inverse images of generalized vague soft expert sets. Finally, we give some examples of mapping on generalized vague soft expert classes.

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2. Mapping on Generalized Vague Soft Expert Set

In this paper, we introduce the notion of a mapping on generalized vague soft expert classes. Generalized vague soft expert classes are collections of generalized vague soft expert sets. We also define and study the properties of generalized vague soft expert images and generalized vague soft expert inverse images of generalized vague soft expert sets, and support them with example and theorems.

Definition 2.1. Let U be a universe and E be a set of parameters, X a set of expert (agents) and $O = \{1 = agree, 0 = disagree\}$ a set of opinions. Let $Z = E \times X \times O$. Then the collection of all generalized vague soft expert sets over U with parameters from Z is called a generalized vague soft expert class and is denoted as $\widetilde{(U,Z)}$.

Definition 2.2. Let $\widetilde{(U,Z)}$ and $\widetilde{(Y,\acute{Z})}$ be a generalized vague soft expert classes. Let $r : U \to Y$ and $s : Z \to \acute{Z}$ be mappings. Then a mapping $f : \widetilde{(U,Z)} \to \widetilde{(Y,\acute{Z})}$ is defined as follows:

For a generalized vague soft expert set $(\tilde{F}_\mu, A) \in \widetilde{(U,Z)}$, $f(\tilde{F}_\mu, A) \in \widetilde{(Y,\acute{Z})}$ is a generalized vague soft expert set obtained as follows:

$$f\left(\tilde{F}_\mu, A\right)(\beta)(y) = \begin{cases} \left(\left\{\bigvee_{x\in r^{-1}(y_i), i=1,2\ldots,n} \left(\bigvee_\alpha F(\alpha)\right)\right\}, \bigvee_\alpha \mu(\alpha)\right) & , if\, r^{-1}(y_i) \; and \\ & s^{-1}(\beta) \cap A \neq \emptyset \\ (\phi, 0) & , otherwise \end{cases}$$

for $\beta \in s(Z) \subseteq \acute{Z}$, $y_i \in Y$ and $\forall \alpha \in s^{-1}(\beta) \cap A$, $f(\tilde{F}_\mu, A)$ is called a generalized vague soft expert image of the expert vague soft set $(\tilde{F}_(\mu), A)$.

Definition 2.3. Let $\widetilde{(U,Z)}$ and $\widetilde{(Y,\acute{Z})}$ be a generalized vague soft expert classes. Let $f^{-1} : \widetilde{(Y,\acute{Z})} \to \widetilde{(U,Z)}$, $r : U \to Y$ and $s : Z \to \acute{Z}$ be mappings. For a generalized vague soft expert set $(\tilde{G}_\delta, B) \in \widetilde{(Y,\acute{Z})}$ where $B \subseteq Z'$, $f^{-1}(\tilde{G}_\delta, B) \in \widetilde{(U,Z)}$ is a generalized vague soft expert set obtained as follows:

$$f^{-1}(\tilde{G}_\delta, A)(\alpha)(u) = \begin{cases} \left(\left\{G(s(\alpha))(r(u_j)), \delta(s(\alpha))\right\}\right) & , if \alpha \in s^{-1}(B) \\ (\phi, 0) & , otherwise \end{cases}$$

for $\alpha \in s^{-1}(B) \subseteq Z$, and $u_j \in U$, $j = \{1, 2, ..., m\}$. $f^{-1}(\tilde{G}_\delta, B)$ is called a generalized vague soft expert inverse image of the generalized vague soft expert set $(\tilde{G}_\delta, B)$.

Example 2.1. Let $U=\{u_1,u_2,u_3\}$, $Y=\{y_1,y_2,y_3\}$ and let $A\subseteq Z=\{(e_1,p,1),(e_2,p,0)\,,\,(e_3,p,1)\}$,
and $A'\subseteq Z'=\{(e_1',p',1),(e_2',p',0)\,,\,(e_1',q',1)\}$.

Suppose that $\widetilde{(U,A)}$ and $\widetilde{(Y,A')}$ are generalized vague soft expert classes. Define $r:U\to Y$ and $s:A\to A'$ as follows:
$r(u_1)=y_1, r(u_2)=y_3, r(u_3)=y_1$,
$s\,(e_1,p,1)=(e_2',p',0)\,, s\,(e_2,p,0)=(e_1',p',1)\,, s\,(e_3,p,1)=(e_1',p',1)$.

Let $(\tilde F_\mu,A)$ and $(\tilde G_\delta,B)$ be two generalized vague soft expert sets over U and Y respectively such that

$$\begin{aligned}\left(\tilde F_\mu,A\right)=\Bigg\{&\left((e_1,p,1)\,,\left\{\tfrac{u_1}{\langle 0.8,0.8\rangle},\tfrac{u_2}{\langle 0.5,0.7\rangle},\tfrac{u_3}{\langle 0.4,0.5\rangle}\right\},0.5\right),\\ &\left((e_3,p,1)\,,\left\{\tfrac{u_1}{\langle 0.6,0.7\rangle},\tfrac{u_2}{\langle 0.7,0.8\rangle},\tfrac{u_3}{\langle 0.9,0.9\rangle}\right\},0.6\right),\\ &\left((e_2,p,0)\,,\left\{\tfrac{u_1}{\langle 0.7,0.8\rangle},\tfrac{u_2}{\langle 0.1,0.6\rangle},\tfrac{u_3}{\langle 0.2,0.5\rangle}\right\},0.7\right)\Bigg\},\end{aligned}$$

$$\begin{aligned}\left(\tilde G_\delta,A'\right)=\Bigg\{&\left((e_1',p',1)\,,\left\{\tfrac{y_1}{\langle 0.7,0.7\rangle},\tfrac{y_2}{\langle 0.4,0.6\rangle},\tfrac{y_3}{\langle 0.3,0.4\rangle}\right\},0.4\right),\\ &\left((e_1',q',1)\,,\left\{\tfrac{y_1}{\langle 0.5,0.6\rangle},\tfrac{y_2}{\langle 0.6,0.7\rangle},\tfrac{y_3}{\langle 0.8,0.8\rangle}\right\},0.5\right),\\ &\left((e_2',p',0)\,,\left\{\tfrac{y_1}{\langle 0.6,0.7\rangle},\tfrac{y_2}{\langle 0,0.5\rangle},\tfrac{y_3}{\langle 0.1,0.4\rangle}\right\},0.6\right)\Bigg\}.\end{aligned}$$

Then we define the mapping from $f:\widetilde{(U,Z)}\to\widetilde{(Y,\acute Z)}$ as follows:
For generalized vague soft expert set $\left(\tilde F_\mu,A\right)$ in $\widetilde{(X,Z)}$, $f\left((\tilde F_\mu,A),K\right)$ is a generalized vague soft expert set in $\widetilde{\left(Y,\acute E\right)}$ where
$K=s(A)=\{(e_1',p',1),(e_2',p',0)\,,\,(e_1',q',1)\}$ and is obtained as follows:

$$f\left(\tilde F_\mu,A\right)(e_1',p',1)=\left(\left\{\bigvee_{u\in r^{-1}(y_i),i=\{1,2,3\}}\left(\bigvee_\alpha F(\alpha)\right)(x)\right\},\bigvee_\alpha\mu(\alpha)\right)$$

Now for $\left\{\bigvee_{u\in r^{-1}(y_i),i=\{1,2,3\}}\left(\bigvee_\alpha F(\alpha)\right)(u)\right\}$ and for $i=1$, we have

$$\bigvee_{u\in\{u_1,u_3\}}\left(\bigvee_{\alpha\in\{(e_2,p,0)\}}F(\alpha)\right)(u)=\bigvee_{u\in\{u_1,u_3\}}F\,(e_2,p,0)\,(u)$$

$$= \left\{ \tfrac{u_1}{\langle 0.7,0.8\rangle}, \tfrac{u_2}{\langle 0.1,0.6\rangle}, \tfrac{u_3}{\langle 0.2,0.5\rangle} \right\}(u)$$
$$= \langle 0.7, 0.8\rangle \cup \langle 0.2, 0.5\rangle = \langle 0.7, 0.5\rangle$$

For $i = 2$,

$$\bigvee_{u\in\{x_2\}} \left(\bigvee_{\alpha\in(e_2,p,0)} F(\alpha) \right)(u) = 0 \text{ since } r^{-1}(y_2) = \phi.$$

For $i = 3$,

$$\bigvee_{u\in\{u_2\}} \left(\bigvee_{\alpha\in\{e_3\}} F(\alpha) \right)(u) = (F(e_1))(u_2)$$
$$= \left(\left\{ \tfrac{u_1}{\langle 0.7,0.8\rangle}, \tfrac{u_2}{\langle 0.1,0.6\rangle}, \tfrac{u_3}{\langle 0.2,0.5\rangle} \right\} \right)(u_2)$$
$$= \langle 0.1, 0.6\rangle .$$

Now for $\bigvee \mu(\alpha)$, we have

$$\bigvee_{e_3} \mu(e_3) = \mu(e_3) = 0.7.$$

Then

$$f\left(\tilde{F}_\mu, A\right)(e_2', p', 0) = \left\{ (e_2', p', 0), \left(\left\{ \tfrac{y_1}{\langle 0.7,0.5\rangle}, \tfrac{y_2}{\langle 0,0\rangle}, \tfrac{y_3}{\langle 0.1,0.6\rangle} \right\}, 0.7 \right) \right\}$$

By similar calculations, consequently, we get

$$\begin{aligned}\left(f\left(\tilde{F}_\mu, A\right), K\right) = \Bigg\{ & (e_1', p', 1), \left(\left\{ \frac{y_1}{\langle 0.7, 0.5\rangle}, \frac{y_2}{\langle 0, 0\rangle}, \frac{y_3}{\langle 0.1, 0.6\rangle} \right\}, 0.7 \right) \\ & (e_2', p', 0), \left(\left\{ \frac{y_1}{\langle 0.8, 0.8\rangle}, \frac{y_2}{\langle 0, 0\rangle}, \frac{y_3}{\langle 0.5, 0.7\rangle} \right\}, 0.5 \right), \\ & (e_1', q', 1), \left(\left\{ \frac{y_1}{\langle 0.9, 0.7\rangle}, \frac{y_2}{\langle 0, 0\rangle}, \frac{y_3}{\langle 0.7, 0.8\rangle} \right\}, 0.6 \right) \Bigg\}.\end{aligned}$$

Next, for the generalized vague soft expert inverse images, we have the following: For a generalized vague soft expert set $(\tilde{G}_\delta, A')$ in $\widetilde{(Y, \acute{Z})}$, $(f^{-1}(\tilde{G}_\delta, A'), D)$ is a generalized vague soft expert set in $\widetilde{(U, Z)}$ where $D = s^{-1}(A') = \{(e_1, p, 1), (e_2, p, 0), (e_3, p, 1)\}$ and obtained as follows:

$$f^{-1}\left(\tilde{G}_\delta, B\right)((e_1, p, 1))(u_1) = G(e_1', p, 1)(r(u_1))$$
$$= G((e_1', p', 1))(y_1)$$
$$= \left\{ \tfrac{y_1}{\langle 0.6,0.7\rangle}, \tfrac{y_2}{\langle 0,0.5\rangle}, \tfrac{y_3}{\langle 0.1,0.4\rangle} \right\}(y_1,) = \langle 0.6, 0.7\rangle.$$

For $j = 2$, we have $f^{-1}(G, B)((e_1, p, 1))(u_2) = G((e_1', p', 1))(y_3)$

$$= \left\{ \frac{y_1}{\langle 0.6, 0.7\rangle}, \frac{y_2}{\langle 0, 0.5\rangle}, \frac{y_3}{\langle 0.1, 0.4\rangle} \right\} (y_3) = \langle 0.1, 0.4\rangle .$$

For $j = 3$, we have $f^{-1}(G, B)((e_1, p, 1))(x_3) = G(e_2)(y_1)$

$$= \left\{ \frac{y_1}{\langle 0.6, 0.7\rangle}, \frac{y_2}{\langle 0, 0.5\rangle}, \frac{y_3}{\langle 0.1, 0.4\rangle} \right\} (y_1) = \langle 0.6, 0.7\rangle .$$

Now for $\delta(s(\alpha))$, we have $s(e_3^{'}) = 0.6$. Then
$f^{-1}\left(\tilde{G}_\delta, B\right) = \left((e_1, p, 1), \left(\left\{ \frac{y_1}{\langle 0.6, 0.7\rangle}, \frac{y_2}{\langle 0.1, 0.4\rangle}, \frac{y_3}{\langle 0.6, 0.7\rangle} \right\}, 0.6\right)\right)$

By similar calculations, consequently, we get

$$\begin{aligned}\left(f^{-1}\left(\tilde{G}_\delta, A\right), D\right) = \Big\{ &(e_1, p, 1), \left(\left\{ \frac{u_1}{\langle 0.5, 0.6\rangle}, \frac{u_2}{\langle 0.8, 0.8\rangle}, \frac{u_3}{\langle 0.5, 0.6\rangle} \right\}, 0.5\right), \\ &(e_2, p, 0), \left(\left\{ \frac{u_1}{\langle 0.6, 0.7\rangle}, \frac{u_2}{\langle 0.1, 0.4\rangle}, \frac{u_3}{\langle 0.6, 0.7\rangle} \right\}, 0.6\right), \\ &(e_3, p, 1), \left(\left\{ \frac{u_1}{\langle 0.7, 0.7\rangle}, \frac{u_2}{\langle 0.3, 0.4\rangle}, \frac{u_3}{\langle 0.7, 0.7\rangle} \right\}, 0.4\right) \Big\}.\end{aligned}$$

Definition 2.4. Let $f : \widetilde{(U, X)} \to \widetilde{\left(Y, \acute{X}\right)}$ be a mapping and $\left(\tilde{F}_\mu, A\right)$ and $\left(\tilde{G}_\mu, B\right)$ a generalized vague soft expert sets in $\widetilde{(X, E)}$. Then for $\beta \in \acute{X}$, the union and intersection of generalized vague soft expert images $\left(\tilde{F}_\mu, A\right)$ and $\left(\tilde{G}_\delta, B\right)$ are defined follows:

$$\left(f\left(\tilde{F}_\mu, A\right) \widetilde{\bigvee} f\left(\tilde{G}_\delta, B\right)\right)(\beta)(y) = f\left(\tilde{F}_\mu, A\right)(\beta)(y) \bigvee f\left(\tilde{G}_\delta, B\right)(\beta)(y).$$

$$\left(f\left(\tilde{F}_\delta, A\right) \widetilde{\bigwedge} f\left(\tilde{G}_\delta, B\right)\right)(\beta)(y) = f\left(\tilde{F}_\mu, A\right)(\beta)(y) \bigwedge f\left(\tilde{G}_\delta, B\right)(\beta)(y).$$

Definition 2.5. Let $f : \widetilde{(U, X)} \to \widetilde{\left(Y, \acute{X}\right)}$ be a mapping and $\left(\tilde{F}_\mu, A\right)$ and $\left(\tilde{G}_\delta, B\right)$ a generalized vague soft expert sets in $\widetilde{(X, E)}$. Then for $\beta \in \acute{X}$, the union and intersection of generalized vague soft expert images $\left(\tilde{F}_\mu, A\right)$ and $\left(\tilde{G}_\delta, B\right)$ are defined follows:

$$\left(f^{-1}\left(\tilde{F}_\mu, A\right) \widetilde{\bigvee} f^{-1}\left(\tilde{G}_\delta, B\right)\right)(\alpha)(u) = f^{-1}\left(\tilde{F}_\mu, A\right)(\alpha)(u) \bigvee f^{-1}\left(\tilde{G}_\delta, B\right)(\alpha)(u).$$

$$\left(f^{-1}\left(\tilde{F}_\mu, A\right) \widetilde{\bigwedge} f^{-1}\left(\tilde{G}_\delta, B\right)\right)(\alpha)(u) = f^{-1}\left(\tilde{F}_\mu, A\right)(\alpha)(u) \bigwedge f^{-1}\left(\tilde{G}_\delta, B\right)(\alpha)(u).$$

Proposition 2.1. *Let* $f : \widetilde{(U, X)} \to \widetilde{\left(Y, \acute{X}\right)}$ *be a mapping. Then for generalized vague soft expert sets* $\left(\tilde{F}_\mu, A\right)$ *and* $\left(\tilde{G}_\delta, B\right)$ *in the generalized vague soft expert class* $\widetilde{(U, X)}$*,*

1. $f(\phi) = \phi$.
2. $f(X) \subseteq Y$.
3. $f\left(\left(\tilde{F}_\mu, A\right) \tilde{\bigvee} \left(\tilde{G}_\delta, B\right)\right) = f\left(\tilde{F}_\mu, A\right) \tilde{\bigvee} f\left(\tilde{G}_\delta, B\right)$.
4. $f\left(\left(\tilde{F}_\mu, A\right) \tilde{\bigwedge} f\left(\tilde{G}_\delta, B\right)\right) \subseteq f\left(\tilde{F}_\mu, A\right) \tilde{\bigwedge} f\left(\tilde{G}_\delta, B\right)$.
5. *If* $\left(\tilde{F}_\mu, A\right) \subseteq \left(\tilde{G}_\delta, B\right), then f\left(\tilde{F}_\mu, A\right) \subseteq f\left(\tilde{G}_\delta, B\right)$.

Proof. The proof is straightforward.

Proposition 2.2. *Let* $f : \widetilde{(U, X)} \to \widetilde{(Y, \acute{X})}$ *be a mapping. Then for generalized vague soft expert sets* $\left(\tilde{F}_\mu, A\right)$, $\left(\tilde{G}_\delta, B\right)$ *in the generalized vague soft expert class* $\left(U, \acute{X}\right)$*, we have:*

1. $f^{-1}(\emptyset) = \phi$.
2. $f^{-1}(Y) = X$.
3. $f^{-1}\left(\left(\tilde{f}_\mu, A\right) \tilde{\bigvee} \left(\tilde{G}_\delta, B\right)\right) = f^{-1}\left(\tilde{F}_\mu, A\right) \tilde{\bigvee} f^{-1}\left(\tilde{G}_\delta, B\right)$.
4. $f^{-1}\left(\left(\tilde{f}_\mu, A\right) \tilde{\bigwedge} \left(\tilde{G}_\delta, B\right)\right) = f^{-1}\left(\tilde{F}_\mu, A\right) \tilde{\bigwedge} f\left(\tilde{G}_\delta, B\right)$.
5. *If* $\left(\tilde{f}_\mu, A\right) \subseteq \left(\tilde{G}_\delta, B\right), then f^{-1}\left(\tilde{f}_\mu, A\right) \subseteq f^{-1}\left(\tilde{G}_\delta, B\right)$.

Proof. The proof is straightforward.

3. Conclusion

In this paper, we studied a mapping on generalized vague soft expert classes and its properties. We give some illustrative examples of mapping on generalized vague soft expert set.

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