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FEEDBACK NUMBERS OF FLOWER SNARK AND RELATED GRAPHS

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Abstract: A subset of vertices of a graph G is called a feedback vertex set of G if its removal results in an acyclic subgraph. The minimum cardinality of a feedback vertex set is called the feedback number. In this paper, we investigate the feedback number of flower snark and related graphs H_n. Let $f(H_n)$ denote the feedback number of H_n, we prove that

$$f(H_n) = n + 1 \ for \ n \geq 3.$$

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Key Words: feedback vertex set, feedback number, flower snark, acyclic subgraph, bipartite subgraph

1. Introduction

Let $G = (V, E)$ be a graph or digraph without multiple edges, with vertex set $V(G)$ and edge set $E(G)$. A subset $F \subset V(G)$ is called *a feedback vertex set* if the subgraph $G - F$ is acyclic, that is, if $G - F$ is a forest. The minimum cardinality of a feedback vertex set is called *the feedback number* (or *decycling number* proposed first by Beineke and Vandell [1]) of G. A feedback vertex set of this cardinality is called a *minimum feedback vertex set*.

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Apart from its graph-theoretical interest, the minimum feedback vertex set problem has important application to several fields. For example, the problems are in operating systems to resource allocation mechanisms that prevent deadlocks [2], in artificial intelligence to the constraint satisfaction problem and Bayesian inference, in synchronous distributed systems to the study of monopolies and in optical networks to converters placement problem(see [3] and [4]).

Determining the feedback number is quite difficult even for some elementary graphs. However, the problem has been studied for some special graphs and digraphs, such as hypercubes, meshes, toroids, butterflies, cube-connected cycles, directed split-stars (see [3]- [13]). In fact, the minimum feedback set problem is known to be NP-hard for general graphs [14] and the best known approximation algorithm is one with an approximation ratio two [5].

Many research of literatures have been studied about flower snark and its related graphs. For example, Zheng. [15] has been studied the crossing number of flower snark and its related graph; Xi. [16] has been studied super vertex-magic total labelings of flower snark and related graphs; Mo.hammad [17] and Tong. [18] have been studied labeling of flower snark and related graphs; In addition, the adjacent vertex distinguishing incidence coloring number of flower graphs has been studied in area of mathematics. But, there is little research done so far about the feedback number of flower snark and related graphs.

In this paper, we consider the feedback number of flower snark and related graphs H_n. Let $f(H_n)$ denote the feedback number of H_n, we proves that:

$$f(H_n) = n + 1, for\ n \geq 3.$$

2. Feedback Vertex Set of H_n

Let G_n be a simple nontrivial connected cubic graph with vertex set $V(G_n) = \{a_i, b_i, c_i, d_i : 0 \leq i \leq n-1\}$ and edge set

$$E(G_n) = \{a_i a_{i+1}, b_i b_{i+1}, c_i c_{i+1}, d_i a_i, d_i b_i, d_i c_i : 0 \leq i \leq n-1\},$$

where the vertex labels are read modulo n.

Let H_n be a graph obtained from G_n by replaceing the edges $b_{n-1}b_0$ and $c_{n-1}c_0$ with $b_{n-1}c_0$ and $c_{n-1}b_0$ respectively. For odd $n \geq 5$, H_n is called a Snark, namely Flower Snark [19] and [20]. G_n and $H_n (n = 3$ or even $n \geq 4)$ are called the related graphs of Flower Snark [21].

In this paper, we denote H_n as Flower Snark and its related graphs. By the definition of H_n, the set of $V(H_n)$ can be divided into three parts as follows:

$$\begin{cases} H_1 = \{a_i a_{i+1} : 0 \le i \le n-1\} \\ H_2 = \{b_j b_{j+1}, b_{n-1}c_0, c_k c_{k+1}, c_{n-1}b_0 : 0 \le j, k \le n-2\} \\ H_3 = \{d_i a_i, d_i b_i, d_i c_i : 0 \le i \le n-1\} \end{cases}$$

Obviously, H_1 is the only one cycle which contains all the vertices for $\{a_i\}$, H_2 is the only one cycle which contains all the vertices for $\{b_j\}$ and $\{c_k\}$. $H_1 \cap H_2 = \emptyset$ and $H_1 \cup H_2 \cup H_3 = H_n$.

Denote

$$\begin{cases} F_1 = \{a_{n-1}\} \\ F_2 = \{c_{n-1}\} \\ F_3 = \{d_i : 0 \le i \le n-2\} \end{cases}$$

and denote $F_n = F_1 \cup F_2 \cup F_3$, we have the following result.

Lemma 1. *F_n is a feedback vertex set of H_n for $n \ge 3$.*

Proof. Since H_1 is a cycle, then we only delete any one vertex, say a_{n-1}, to obtain an acyclic subgraph of H_1 denoted by $G[N_1]$, which implies that $G[N_1] = G[H_1 \backslash F_1]$.

Similarly, in H_2 we only delete any one vertex, say c_{n-1}, to obtain an acyclic subgraph denoted by $G[N_2]$, which implies that $G[N_2] = G[H_2 \backslash F_2]$.

Since $H_1 \cap H_2 = \emptyset$, then $G[N_1] \cap G[N_2] = \emptyset$, that is , $G[N_1] \cup G[N_2]$ is an acyclic graph.

We denote $G[H_3 \backslash F_3]$ as $G[N_3]$ which include only one edge $d_{n-1}b_{n-1}$. Obviously, $V(G[N_3]) \cap V(G[N_2]) = \{b_{n-1}\}$, $V(G[N_3]) \cap V(G[N_1]) = \emptyset$. Its relationships are shown as Figure.1.

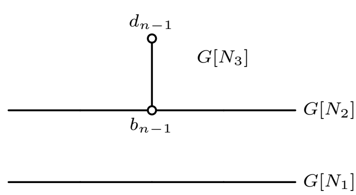

Figure 1: The relationship of $G[N_1], G[N_2]$ and $G[N_3]$.

It is a simple matter to verify that $G[H_n \backslash (F_1 \cup F_2 \cup F_3)]$ is an acyclic graph. By the definition of feedback vertex set, F_n is a feedback vertex set of H_n. □

3. Feedback Number of H_n

Lemma 2. *For feedback vertex set F_n in a graph $G = (V, E)$ with maximum degree Δ, it holds that*

$$F_n \geq \lceil \frac{|E| - |V| + 1}{\Delta - 1} \rceil$$

Lemma 3. *The feedback vertex set in H_n is of size at least:*

$$f(H_n) \geq n + 1, \;\; for\ n \geq 3.$$

Proof. Noting that H_n $(n \geq 3)$ has $4n$ vertices and $6n$ edges,the maximum degree is 3, by Lemma 2, we immediately obtain the lower bound

$$f(H_n) \geq \lceil \frac{6n - 4n + 1}{3 - 1} \rceil = \lceil \frac{2n + 1}{2} \rceil = n + 1.$$

□

Theorem 1. *Feedback vertex number of flower snark and related graphs H_n:*

$$f(H_n) = n + 1, \;\; for\ n \geq 3.$$

Proof. For $n \geq 3$, by definition, it is easy to find that $|F_1| = |F_2| = 1$, $|F_3| = n - 1$.

Since F_n is a feedback vertex set, then we have $f(H_n) \leq |F_n| = 1{+}1{+}n{-}1 = n + 1$. In addition, $f(H_n) \geq n + 1$, we conclude that

$$f(H_n) = n + 1, for\ n \geq 3.$$

The theorem holds. □

In Figure 2, we show the feedback vertex sets of several H_n graphs with small n, where the vertices of feedback vertex sets are in red, the acyclic graph are in blue.

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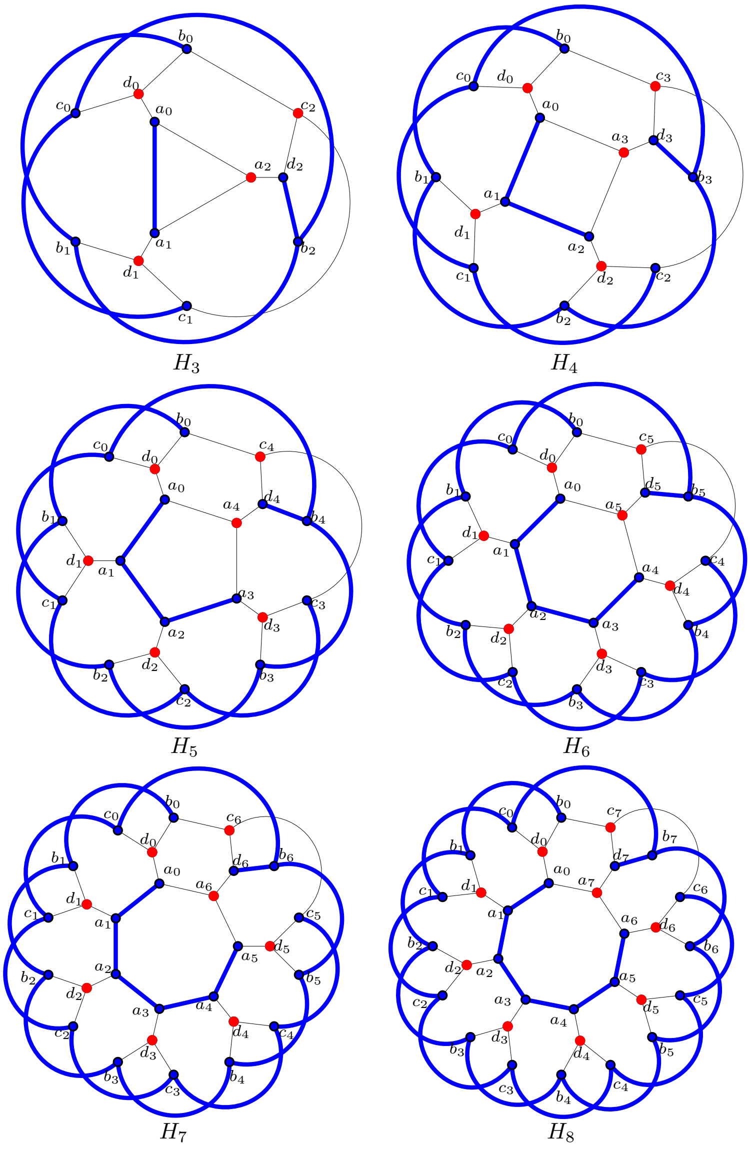

Figure 2: Flower snark and its related graph of H_n

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