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NEW GLOBAL CONVERGENCE OF NONMONOTONE LINE SEARCH ALGORITHM

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Abstract: In this paper, a class of nonmonotone line search, study the convergence properties of such an algorithm for general nonconvex function, and proved its global convergence. More conjugate gradient algorithm is used the Wolfe rule nonmonotone line search. The global convergence results are proved.

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1. Introduction

Our problem is to minimize a function of n variables

$$\min \{f(x) : x \in \mathbb{R}^n\} \tag{1.1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is smooth and its gradient $g(x)$ is available. Conjugate gradient method for solving (1.1) are iterative methods of the form

$$x_{k+1} = x_k + \alpha_k d_k \tag{1.2}$$

where $\alpha_k > 0$ is a steplength, d_k is asearch direction. Normally the search direction at the first iteration is the steepest descent direction, namely $d_1 = -g_1$.

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The other search direction can be defined recursively:

$$d_k = -g_k + \beta_k d_{k-1}, \ k \geq 1 \tag{1.3}$$

where $g_k = \nabla f(x_k)$,α_k is a stepe-size obtained by some line search, and β_k is a scalar There are many ways to select β_k, and some well-known formulas are given by

$$\beta_k^{PRP} = \frac{g_k^T y_k}{\|g_{k-1}\|^2}$$

$$\beta_k^{FR} = \frac{\|g_k\|^2}{\|g_{k-1}\|^2}$$

$$\beta_k^{CD} = \frac{\|g_k\|^2}{-d_{k-1}^T g_{k-1}}$$

$$\beta_k^{DY} = \frac{\|g_k\|^2}{d_{k-1}^T y_{k-1}}$$

$$\beta_k^{HS} = \frac{g_k^T (g_k - g_{k-1})}{d_{k-1}^T (g_k - g_{k-1})}$$

$$\beta_k^N = \frac{g_k^T y_{k-1}}{d_{k-1}^T g_{k-1}} - 2 g_k^T d_{k-1} \frac{\|y_{k-1}\|^2}{(d_{k-1}^T g_{k-1})^2} \tag{*}$$

respectively, where $y_{k-1} = g_k - g_{k-1}$and $\|\|$means the Euclidean norm. The technique of nonmonotone line search was proposed first in[1] and has received many successful applications or extensions in both unconstrained optimization and constrained optimization, A large portion of optimization methods require monotonicity of the objective values to guarantee their global convergence.This target is usually achieved by a suitable line technique even when the initial point is far away from the optimum. Among the most popular line search techniquesare are the Armijo rule, the Goldstein rule and the Wolfe rule (see[4, 5, 13]).In particular, enforcing monotonicity may considerably reduce the rate of convergence, when the iteration is trapped near a narrow curved valley, which can result in very short steps or zigzagging. Therefore, it might be advantageous to allow the iterative sequence to occasionally generate points with nonmonotone objective values, while retaining global convergence of the minimization algorithm.Several numerical tests show that the nonmonotone line search technique for unconstrained optimization and constrained optimization is efficient and competitive[7, 12, 14]. Note that the famous watchdog technique for constrained optimization proposed in [2] can also be viewed as strategy of the nonmonotone type. The forcing function introduced in [11] is an important class of functions which can be used to measure sufficiency of descent and prove convergence.In [11] a detailed steplength analysis with forcing function is given.Han and Liu [9] used the idea of forcing function and proposed a general line search rule. We combine forcing functions with the nonmonotone line search technique and give a general line search rule, called the nonmonotone F-rule, for unconstrained minimization problems. We show that some common nonmonotone line search rules such as the nonmonotone such as the nonmonotone Armijo line search rule, the nonmonotone Goldstein line search rule, and the Wolf line search rule are special cases of the nonmonotone F-rule. Finally, we prove the global convergence of the resulted nonmonotone descent methods under mild conditions. The remainder of this paper is organized as follows . In Section 2 we describe our nonmonotone F-rule and show that the aforementioned common nonmonotone line search rules, are particular cases of the nonmonotone F-rule.In Section 3, we establish the global convergence of nonmonotone descent methods for unconstrained optimization.Some conclusions are given in Section 4.

In this paper, a class of non-monotone line search (NLS), study the convergence properties of such an algorithm for general nonconvex function, and proved its global convergence.This article studies the global convergence of a class of nonmonotone line search of Wolf rule conjugate gradient algorithm to prove that the idea comes from the on the F-rule search techniques. We study the condition of global convergence of four conjugate gradient methods with nonmonotone line searchs. When the conditions is being increased, for nonconvex functions, we prove the global convergence of modified method.

In the convergence analysis and implementation of conjugate gradient method, the extended wolf rule nonmonotone line search, namely

$$f(x_k + \alpha_k d_k) \leq \max_{0 \leq j \leq m(k)} f(x_{k-j}) + \gamma_1 \alpha_k g_k^T d_k \tag{1.4}$$

$$\gamma_2 g_k^T d_k \leq g\left(x_{k+}\alpha_k d_k\right)^T d_k \tag{1.5}$$

which $0 < \gamma_1 \leq \gamma_2 \leq 1, \lambda_1 > 0$ and $M \ \epsilon N$,

$$0 \leq m\left(k\right) \leq \min\left\{m\left(k-1\right)+1, M\right\}, m\left(0\right)=0 \tag{1.6}$$

Next, we present the nonmonotone F- rule. We begin with two definitions the forcing function and the reverse modulus of continuity of gradient..

2. Technique of the Nonmonotone Line Search

First given the general assumption of this section:

Assumption 2.1.

(A_1) The level set $L_0 = \{x \setminus f(x) \leq f(x_0), x \,\epsilon R^n\}$ is bounded, where x_0 is the starting point

(A_2) f is strongly convex and differentiable in the level set L_0 and its gradient $g_k = \nabla f\,(x_k)$ lipschitz continuous . ie, there exist constants $L > 0$ making

$$\|g(x) - g(y)\| \leq L\,\|x - y\|\,, \forall x, y \in \mathcal{N} \tag{2,1}$$

Definition 2.2. The function $\sigma : [0, \infty) \to [0, \infty)$ is a forcing (F-function), if for any sequence $\{t_i\} \subset [0, \infty)$

$$\lim_{k\to\infty} \sigma(t_i) = 0 \text{ implies} \lim_{k\to\infty} t_i = 0 \tag{2.2}$$

Definition 2.3. Let

$$\eta = sup\,\{\|g(x) - g(y)\| \;\;/\; x, y \in L_0\} > 0.$$

Then the mapping $\delta : [0, \infty) \to [0, \infty)$ defined by

$$\delta(t) = \begin{cases} \inf \;\left\{\frac{\|x-y\|}{\|g(x)-g(y)\|} \geq t\right\}, t \in [0, \eta) \\ \lim\limits_{s\to\eta^-} \delta(s), \qquad\qquad\qquad t \in [0, \eta) \end{cases}$$

is the reverse modulus of continuity of gradient $g(x)$.

Now we give the nonmonotone F-rule for line searches as follows.

$$f(x_k + \alpha_k d_k) \leq \max_{0\leq j\leq m(k)} f(x_{k-j}) - \sigma(t_k) \tag{2.3}$$

where σ is a forcing function and $t_k = -\frac{g_k^T d_k}{\|d_k\|}$. Set $x_{k+1} = x_k + \alpha_k d_k$

Obviously, if $M = 0$, the above nonmonotone F-rule is just the rule of sufficient decrease in [11].

Note also that any nondecreasing function $\sigma : [0, \infty) \to [0, \infty)$ such that $\sigma(0) = 0$ and $\sigma(t_i) > 0$ for $t > 0$ is necessarily an F-function.Hence, the presented rule is quite general.

For convenience, in the following, let:

$$f(x_k + \alpha_k d_k) \leq \max_{0\leq j\leq m(k)} f(x_{k-j})$$

where

$$k - m(k) \leq l(k) \leq k$$

Using Definition 2.3, we have

$$\alpha_k \|d_k\| \geq \delta \left[(\gamma_2 - 1) \frac{g_k^T d_k}{\|d_k\|} \right]$$

which means

$$\alpha_k \|d_k\| \geq \delta \left[(1 - \gamma_2)(-\frac{g_k^T d_k}{\|d_k\|}) \right] \tag{2.4}$$

where $\delta(.)$ is the reverse modulus. So, it follows from (1.4) and (2.4) that

$$f(x_k + \alpha_k d_k) \leq f(x_{l(k)}) + \gamma_1 \alpha_k g_k^T d_k$$

$$f(x_k + \alpha_k d_k) = f(x_{l(k)}) - \gamma_1 \alpha_k \|d_k\| (-\frac{g_k^T d_k}{\|d_k\|})$$

$$f(x_k + \alpha_k d_k) \leq f(x_{l(k)}) - \gamma_1 (-\frac{g_k^T d_k}{\|d_k\|}) \delta \left[(1 - \gamma_2)(-\frac{g_k^T d_k}{\|d_k\|}) \right]$$

$$f(x_k + \alpha_k d_k) \leq f(x_{l(k)}) - \sigma(-\frac{g_k^T d_k}{\|d_k\|}) \tag{2.5}$$

where $\sigma(t) = \gamma_1 t \ \delta \left[(1 - \gamma_2) t \right], t \geq 0$. Clearly, $\sigma(t)$ is a forcing function. This indicates that the rule (1.4) - (1.5) satisfies the nonmonotone F-rule (2.3)

Algorithm 2.4. Given $\rho > 0$, $\beta \in [0, 1]$, $\gamma_2 \in [0, 1]$, M non-negative integer, and to make the initial test step

$$r_k = \frac{\rho g_k^T d_k}{\|d_k\|^2}$$

Take $\alpha_k = \beta^{h(k)} r_k$, $h(k) = 0, 1, 2, ...,$ $h(k)$ is to make smallest non-negative integer set up by the following formula,

$$f(x_k + \alpha_k d_k) \leq \max_{0 \leq j \leq m(k)} f(x_{k-j}) - \sigma(t_k)$$

where $m(0) = 0$, $0 \leq m(k) \leq \min \{ m(k-1) + 1, M \}, k \geq 1$. Obviously, if the descent direction d_k is met $d_k^T g_k$, then, when sufficiently large $m(k)$, inequality (2.3) always, holds, thus satisfying α_k the condition of existence.

The above line search in each iteration, it is recommended that the initial test step r_k no longer remain the same, but can be adjusted automatically. On the value, change the initial test approach can get better results, calculte a larger step

size α_k, there by reducing the number of iterations.

Remarks 2.5

1- In order to be able to use a nonmonotonic line search the NLS calculate the step length facor α_k must be the search direction d_k is a descent direction. In the next

section,we will prove that this study of the conjugate gradient algorithm to keep the search direction d_k is down.

2- In order to able to calculate a larger step size α_k.

$$\sum_{k=0}^{\infty} \sigma(t_k) < \infty \tag{2.6}$$

3. Convergence Analysis

In this section we establish the global convergence properties of optimization methods with nonmonotone F-rule.Note that to establish our result, we need some additional mild conditions.

Lemma 3.1. Let the search direction d_k is a descent direction and step size factor α_k by the nonmonotone line search NLS by (1.2) of iteration $\{x_k\} \subset L_0$.

Proof. By the nonmonotone line search in the NLS (2.3) show

$$f(x_1) \leq f(x_0) - \sigma(t_0) < f(x_0)$$

$$f(x_2) \leq \max_{0 \leq j \leq m(1)} f(x_{1-j}) - \sigma(t_1) < f(x_0)$$

$$f(x_3) \leq \max_{0 \leq j \leq m(2)} f(x_{2-j}) - \sigma(t_2) < f(x_0)$$

$$\ldots\ldots$$

$$f(x_{k+1}) \leq \max_{0 \leq j \leq m(k)} f(x_{k-j}) - \sigma(t_k) < f(x_0)$$

$$\ldots\ldots$$

As a result $\{x_k\} \subset L_0$

Lemma 3.2 [8] . The step factor α_k NLS1 of nonmonotone line search, by (1.2), amendment to the definition of method to meet

$$g_k^T d_k \leq -\frac{7}{8}\left\|g_k\right\|^2, k = 0, 1, 2... \tag{3.1}$$

Proof. Since $d_0 = -g_0$, we have $g_0^T d_0 = -\left\|g_0\right\|^2$

$$g_k^T d_k = -\left\|g_k\right\|^2 + \beta_k g_k^T d_{K-1}$$

when $\beta_k = \beta_k^N$

$$g_k^T d_k = -\left\|g_k\right\|^2 + \beta_k^N g_k^T d_{k-1}$$

$$-g_k^T d_k = \left\|g_k\right\|^2 - g_k^T d_{k-1}\left(\frac{g_K^T y_{k-1}}{-g_{k-1}^T d_{k-1}} - 2 g_k^T d_{k-1}\frac{\left\|y_{k-1}\right\|^2}{(-g_{k-1}^T d_{k-1})^2}\right)$$

$$= \frac{\left\|g_k\right\|^2(-g_{k-1}^T d_{k-1})^2 - g_k^T d_{k-1} g_k^T y_{k-1}(-g_{k-1}^T d_{k-1}) + 2(g_k^T d_{k-1})^2\left\|y_{k-1}\right\|^2}{(-g_{k-1}^T d_{k-1})^2} \tag{3.2}$$

we apply the inequality

$$u^T v \leq \frac{1}{2}(\|u\|^2 + \|v\|^2)$$

to the second term in (3.2) with

$$u = \frac{1}{2} g_k(-g_{k-1}^T d_{k-1}),\ v = 2(g_k^T d_{k-1}) y_{k-1}$$

Therefore, we can get that (3.1) is true for all $k \in N$.

For simplicity, we introduce the notation

$$l(k) = \max\left\{ i \mid 0 \leq k - i \leq m(k), f(x_i) = \max_{0 \leq\ j\ \leq m\langle k \rangle} f(x_k - j)\right\} \tag{3.3}$$

Namely $l(k)$ is non-negative integer, and satisfy the following two formulas

$$k - m(k) \leq l(k) \leq k \tag{3.4}$$

$$f(x_{l(k)}) = \max_{0 \leq j \leq m(k)} f(x_{k-j}) \tag{3.5}$$

So nonmonotone line search NLS1 (1.4) can be rewritten as

$$f(x_k + \alpha_k d_k) \leq f(x_{l(k)}) - \sigma(t_k) \tag{3.6}$$

Lemma 3.3. Under the conditions of the assumptions (A_1) , the sequence $f(x_{l(k)})$ is decreases monotonically.

Proof. By (2.3), knowledge of all k

$$f(x_{k+1}) \leq f(x_{l(k)})$$

have been established. Nonmonotine line search NLS, $0 \leq m\,(k) \leq m\,(k-1) + 1$,therefore:

$$\begin{aligned} f(x_{l(k)}) &= \max_{0 \leq j \leq m(k)} f(x_{k-j}) \leq \max_{0 \leq j \leq m(k-1)+1} f(x_{k-j}) \\ &= \max \left\{ \max_{0 \leq j \leq m(k-1)} f(x_{k-1-j}),\ f(x_k) \right\} \\ &= \max \left\{ f(x_{l(k-1)}),\ f(x_k) \right\} \\ &= f(x_{l(k-1)}) \end{aligned}$$

Lemma 3.4. **A**ssuming (A_1) holds, then the $f(x_{l(k)})$ converges i.e $\lim_{k\to\infty} f(x_{l(k)})$ exists.

and

$$\lim_{k\to\infty} \sigma(t_{l(k)-1}) = 0 \tag{3.7}$$

Proof. Known $f(x)$ is bounded below . Since $f(x_k) \leq f(x_0)$,$\forall k$, $\{x_k\} \subset L_0$ (see proo Lemma 3.1) and $\left\{f(x_{l(k)})\right\}$ decreases monotonically (see proo Lemma 3.3) , then

$\lim_{k\to\infty} f(x_{l(k)})$ exists. By (3.6)

$$f(x_{l(k)}) \leq f(x_{l(l(k)-1)}) - \sigma(t_{l(k)-1})$$

On both sides of order $k \to \infty$ that it. So

$$\lim_{k\to\infty} \sigma(t_{l(k)-1}) = 0$$

Theorem 3.5. Let function $f : \mathbb{R}^n \to \mathbb{R}$ satisfy Assumption 2.1. Let the sequence $\{x_k\}$ be defined by (1.2)

where the steplength α_k is defined by the nonmonotone F-rule (2.3). If the direction d_k satisfies

$$-\frac{g_k^T d_k}{\|d_k\|} \geq \sigma(\|g_k\|), k = 0, 1, ..., \tag{3.8}$$

and

$$\|d_k\| \leq m_1 \|g_k\| \tag{3.9}$$

where $\sigma(\cdot)$ is a forcing function and $m_1 > 0$. Then the sequence $\{x_k\} \subset L_0$ and

$$\lim_{k\to\infty} \|g_k\| = 0$$

Proof. Which means from Definition 2.2 that

$$\lim_{k\to\infty} t_{l(k)-1} = -\frac{g_{l(k)-1}^T d_{l(k)-1}}{\left\|d_{l(k)-1}\right\|} = 0 \tag{3.10}$$

Using condition (3.8), we deduce

$$\lim_{k\to\infty} \sigma(\left\|g_{l(k)-1}\right\|) = 0$$

which implies

$$\lim_{k\to\infty} \left\|g_{l(k)-1}\right\| = 0$$

from Definition 2.2. Then it follows from (3.9) that

$$\lim_{k\to\infty} \left\|d_{l(k)-1}\right\| = 0 \tag{3.11}$$

Let

$$l_1(k) = l(k + M + 2) \tag{3.12}$$

We prove by induction that for any given $j \geq 1$

$$\lim_{k\to\infty} \left\|d_{l_1(k)-j}\right\| = 0 \tag{3.13}$$

and

$$\lim_{k\to\infty} f(x_{l_1(k)-j}) = \lim_{k\to\infty} f(x_{l(k)}) \tag{3.14}$$

If $j = 1$, since $\{l_1(k)\} \subset \{l(k)\}$, (3.13) and (3.14) follow from (3.11). Assume that (3.13) and (3.14) hold for a given j. We consider the case of $j + 1$. Since

$$f(x_{l_1(k)-j}) \leq f(x_{l(l_1(k)-(j+1))}) - \sigma(\gamma_{l_1(k)-(j+1)})$$

using the same argument for deriving (3.11), we deduce

$$\lim_{k\to\infty} \left\| d_{l_1(k)-(j+1)} \right\| = 0 \tag{3.15}$$

Know (3.13) was established. But

$$x_{l_1(k)-j} - x_{l_1(k)-(j+1)} = \alpha_{l_1(k)-(j+1)} d_{l_1(k)-(j+1)}$$

Noting that $L_0 = \{x \setminus f(x) \leq f(x_0), x \,\epsilon R^n\}$ is bounded, $x_{k+1} = x_k + \alpha_k d_k$ $\in L_0$,for all k and that α_k stay bounded, we have

$$\lim_{k\to\infty} \left\| x_{l_1(k)-j} - x_{l_1(k)-(j+1)} \right\| = 0$$

$f(x)$ was uniformly continuous on the level L_0, which

$$\lim_{k\to\infty} f(x_{l_1(k)-(j+1)}) = \lim_{k\to\infty} f(x_{l_1(k)-j})$$

$$= \lim_{k\to\infty} f(x_{l_1(k)}) = \lim_{k\to\infty} f(x_{l(k)}) \tag{3.16}$$

This shows that the arbitrary$j \geq 1$. (3.16) have also set up
By L_1definition and (3.4) are available

$$L_1(k) = L(k + M + 2) \leq k + M + 2$$

namely

$$L_1(k) - k - 1 \leq M + 1 \tag{3.17}$$

Thus, for any k, do deformed

$$x_{k+1} = x_{L_1(k)} - \sum_{j=1}^{L(k)-k-1} (x_{l_1(k)-j+1} - x_{l_1(k)-j}) \tag{3.18}$$

,

$$= x_{L_1(k)} - \sum_{j=1}^{L(k)-k-1} \alpha_{l_1(k)-j} d_{l_1(k)-j}$$

where $x_{L_1(k)}$ transposition, and noting (3.17), was

$$\left\|x_{k+1}-x_{L_1(k)}\right\| = \left\|-\sum\lim its_{j=1}^{L(k)-k-1}\alpha_{l_1(k)-i}d_{l_1(k)-i}\right\| \leq \sum_{j=1}^{M+1}\left\|\alpha_{l_1(k)-i}d_{l_1(k)-i}\right\| \tag{3.19}$$

On both sides of order $k\to\infty$, by (3.17),

$$\lim_{k\to\infty}\left\|x_{k+1}-x_{L_1(k)}\right\| = 0 \tag{3.20}$$

.

Thus, by the uniform continuity of $f(x)$, $\lim_{k\to\infty} f(x_{l_1(k)}) = \lim_{k\to\infty} f(x)$, then from (3.19), we can see

$$\lim_{k\to\infty} f(x_{l(k)}) = \lim_{k\to\infty} f(x_K) \tag{3.21}$$

So, for

$$f(x_{_{k+1}}) \leq f(x_{l(k)}) - \sigma(\frac{-g_k^T d_k}{\|d_k\|})$$

taking limits for $k\to\infty$, we get

$$\lim_{k\to\infty}\sigma(-\frac{g_k^T d_k}{\|d_k\|}) = 0 \tag{3.22}$$

which means

$$\lim_{k\to\infty}\|g_k\| = 0$$

Corollary 3.6. Let function $f:\mathbb{R}^n\to\mathbb{R}$ satisfy Assumption 2.1. Let the sequence $\{x_k\}$ be defined by (1.2) where the steplength α_k is defined by the nonmonotone F-rule
(2.3) with F-function $\sigma(t) = (\frac{m_2}{m_1})\, t$, and the direction d_k satisfies

$$g_k^T d_k \leq m_2\|g_k\|^2 \tag{3.23}$$

and

$$\|d_k\| \leq m_1\|g_k\| \tag{3.24}$$

where m_1, $m_2>0$. Then the sequence $\{x_k\}\subset L_0$ and

$$\lim_{k\to\infty} \|g_k\| = 0$$

Proof. (This follows directly from [3] and **Theorem 3.5**)

Corollary 3.7. Under the conditions of the assumptions 2.1. Consider any iterative method (1.2), where d_k satisfies (3.23) – (3.24) and α_k is obtained by the nonmonotone line search (1.4) – (1.5)

Then, there exists a constant m_3 such that

$$\|g_{k+1}\| \leq m_3 \|g_k\|, \text{for all } k \tag{3.25}$$

Further, we have that

$$\lim_{k\to\infty} \|g_k\| = 0 \tag{3.26}$$

Proof. Noting that $\alpha_k \leq \lambda_1$, by this, (1.2), and (3.24) we have that

$$\|x_{k+1} - x_K\| = \|\alpha_k d_k\| \leq \alpha_k \|d_k\|$$

$$\leq \lambda_1 m_1 \|g_k\| \tag{3.27}$$

and from the lipschitz continuity (2.1) and (3.27), we can get that

$$\|g_{k+1} - g_K\| \leq L \|x_{k+1} - x_k\| \leq L\lambda_1 m_1 \|g_k\|$$

$$\|g_{k+1}\| \leq (L\lambda_1 m_1 + 1) \|g_k\|$$

Thus, (3.25) holds with

$$m_3 = L\lambda_1 m_1 + 1$$

In addition, it follows by (2.6), (3.23), (3.24) and (3.25) ,that (3.26) holds

4. Conclusions

The program prepared by the Matlab6.5 in general on a PC. Test functions from [10], indicated in brackets after the function name is the number of variables. NLS is a line search method proposed in this paper, by GLL on Grippo- Lampariello-Lucidi from non-monotonic line search. n_i represents the number of iterations, n_f the number of times that the function value, gradient calculation is the number $n_i + 1$. We were calculated for different values of M, when $M = 0$, ie, monotone line search.To the pros and cons of the algorithm, the parameters are uniform taken as, $\sigma = 1, \beta = 0.2$, $\gamma_2 = 0.9, \varepsilon = 10^{-6}$.Our conjugate gradient method is divided into two kinds of numerical experiment for a class of initial testing step according to this formula to the case of correction, the other is the case of an initial test step length fixed for a. From the results of the comparison, the proposed line search termination criterion has the following advantages:

1 -Monotone line search ($M = 0$), or non-monotone line search ($M > 0$), the number of iterations of the NLS method, the function value calculation times are reduced.

2-Usually better than the initial test step fixed the case when the initial testing step according to this formula be amended.

3-Non-monotone strategy is effective for most of the functions, especially high-dimensional, or initial testing step fixed the situation

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