International Journal of Pure and Applied Mathematics
Volume 94 No. 3 2014, 425-438
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v94i3.10

ijpam.eu

$(\tau_1, \tau_2)^*$-g^*r CONTINUOUS FUNCTIONS

R. Ravikumar[1 §], M. Kothandapani[2]

[1]Department of Mathematics
Pallavan College of Engineering,
Thimmasamudram, Kanchipuram, 631502, INDIA
[2]Department of Mathematics
University College of Engineering
(A Constituent College of Anna University, Chennai
Arni, 632 326, INDIA

Abstract: The aim of the present paper is to introduce the different types of $(\tau_1, \tau_2)^*$-generalized star regular continuous functions and study their basic properties in bitopological spaces.

AMS Subject Classification: $(\tau_1, \tau_2)^*$-g^*r continuous functions; $(\tau_1, \tau_2)^*$-g continuous functions; $(\tau_1, \tau_2)^*$-gs continuous functions; $(\tau_1, \tau_2)^*$-rg continuous functions; $(\tau_1, \tau_2)^*$-g^*r irresolute functions
Key Words: 54E55

1. Introduction

In 1963, the concept of bitopological space was introduced by Kelly [1]. Considerable effort had been expended in obtaining appropriate generalizations of standard topological properties to bitopological category. Kelly initiated the study of separation properties for bitopological spaces. This is the initiative of the study of bitopological spaces and generalized closed sets in bitopological spaces. There are several works dedicated to the investigation of pairs of topologies on the same set(i.e) bitopologies), most of them deal with the theory

Received: March 11, 2014

§Correspondence author

itself but very few with applications. Maheshwari and Prasad [2] introduced semi open sets in bitopological spaces in 1977 and further properties of this notion were studied by Bose [3] in 1981. Fukutake [4] defined one kind of semi open sets in bitopological spaces and studied their properties in 1989.

Fukutake also introduced generalized closed sets and pairwise generalized closure operator [5] in bitopological spaces in 1986. For any subset $A \subseteq X$, the intersection (resp. union) of all τ_i-closed sets containing A (resp. τ_i-open sets contained in A) is called the τ_i-closure (resp. τ_i-interior) of A, denoted by τ_i-cl(A) {resp. τ_i-int(A)}. He defined a set A of a bitopological space (X, τ_1, τ_2) to be $\tau_i\tau_j$-generalized closed set (briefly $\tau_i\tau_j$-g closed) if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_i-open in X, $i, j = 1, 2$ and $i \neq j$. Also, he defined a new closure operator and strongly pairwise $T_{\frac{1}{2}}$-space. Semi generalized closed sets and generalized semi closed sets are extended to bitopological settings by Khedr and Al-saadi. In 2002, Fukutake, Sundaram and Sheik John [6] introduced the concept of w-closed sets, w-open sets and w-continuous maps in bitopological spaces. The g^*-closed sets in bitopological spaces were introduced by Sheik and Sundaram [7] in 2004. Lellis Thivagar and Ravi [8] introduced a new type of generalized sets called $(1,2)^*$-semi generalized closed sets and a new class of generalized functions called $(1,2)^*$-semi generalized continuous maps in 2006. On the other hand, Kannan et. al. [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] introduced the various concepts on closed sets in a bitopological space.

In the present paper, we introduce the concepts of $(\tau_1, \tau_2)^*$-generalized star regular continuous functions, $(\tau_1\tau_2)^*$-g^*r homeomorphisms, $(\tau_1, \tau_2)^*$-GRO compact spaces and study their basic properties in bitopological spaces.

2. Preliminaries

Let (X, τ_1, τ_2) or simply X denote a bitopological space. A subset A of X is $\tau_1\tau_2$-open if $A = S \cup T$ where $S \in \tau_1$, $T \in \tau_2$ and A is $\tau_1\tau_2$-closed if A^C is $\tau_1\tau_2$-open. For any subset $A \subseteq X$, the intersection (resp. union) of all $\tau_1\tau_2$-closed sets containing A (resp. $\tau_1\tau_2$-open sets contained in A) is called the $\tau_1\tau_2$-closure (resp. $\tau_1\tau_2$-interior) of A, denoted by $\tau_1\tau_2$-cl(A) {resp. $\tau_1\tau_2$-int(A)}.

Definition 2.1. A set A of a bitopological space (X, τ_1, τ_2) is called

(a) $\tau_1\tau_2$-semi open [4] if $A \subseteq \tau_2$-$cl[\tau_1$-$int(A)]$.

(b) $\tau_1\tau_2$-semi closed [4] if $X - A$ is $\tau_1\tau_2$-semi open.

Equivalently, a set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-semi closed if τ_2-$int[\tau_1$-$cl(A)] \subseteq A$

(c) $(\tau_1, \tau_2)^*$-semi open [8] if $A \subseteq \tau_1\tau_2$-$cl[\tau_1\tau_2$-$int(A)]$.

(d) $(\tau_1, \tau_2)^*$-semi closed [8] if $X - A$ is $(\tau_1, \tau_2)^*$-semi open.

Equivalently, a set A of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)^*$-semi closed if $\tau_1\tau_2$-$int[\tau_1\tau_2$-$cl(A)] \subseteq A$

(e) $(\tau_1, \tau_2)^*$-generalized closed $\{(\tau_1, \tau_2)^*$-g closed $\}$ [20] if $\tau_1\tau_2$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$-open in X.

(f) $(\tau_1, \tau_2)^*$-generalized open $\{(\tau_1, \tau_2)^*$-g open $\}$ [20] if $X - A$ is $(\tau_1, \tau_2)^*$-g closed.

For any subset $A \subseteq X$, the intersection (resp. union) of all $(\tau_1, \tau_2)^*$-semi closed sets containing A (resp. $(\tau_1, \tau_2)^*$-semi open sets contained in A) is called the $(\tau_1, \tau_2)^*$-semi closure (resp. $(\tau_1, \tau_2)^*$-semi interior) of A, denoted by $(\tau_1, \tau_2)^*$-scl(A) $\{$resp. $(\tau_1, \tau_2)^*$-sint$(A)\}$.

Definition 2.2. A set A of a bitopological space (X, τ_1, τ_2) is called

(a) $(\tau_1, \tau_2)^*$-semi generalized closed $\{(\tau_1, \tau_2)^*$-sg closed$\}$ [8] if $(\tau_1, \tau_2)^*$-$scl(A)$ $\subseteq U$ whenever $A \subseteq U$ and U is $(\tau_1, \tau_2)^*$-semi open in X.

(b) $(\tau_1, \tau_2)^*$-semi generalized open $\{(\tau_1, \tau_2)^*$-sg open $\}$ [8] if $X - A$ is $(\tau_1, \tau_2)^*$-sg closed.

(c) $\tau_1\tau_2$-*regular closed* [8] if τ_1-cl$[\tau_2$-int$(A)] = A$.

(d) $\tau_1\tau_2$-*regular open* [8] if τ_1-int$[\tau_2$-cl$(A)] = A$.

(e) $(\tau_1, \tau_2)^*$-*regular closed* [8] if $\tau_1\tau_2$- cl$[\tau_1\tau_2$- int$(A)] = A$.

(f) $(\tau_1, \tau_2)^*$-regular open [8] if $X - A$ is $(\tau_1, \tau_2)^*$-regular closed.

The intersection (resp. union) of all $(\tau_1, \tau_2)^*$-regular closed sets containing A (resp. $(\tau_1, \tau_2)^*$-regular open sets contained in A) is called the $(\tau_1, \tau_2)^*$-regular closure (resp. $(\tau_1, \tau_2)^*$-regular interior) of A, denoted by $(\tau_1, \tau_2)^*$-rcl(A) $\{$resp. $(\tau_1, \tau_2)^*$-rint$(A)\}$. The set of all $(\tau_1, \tau_2)^*$-regular closed sets in X is denoted by $(\tau_1, \tau_2)^*$-R.C (X, τ_1, τ_2). The set of all $(\tau_1, \tau_2)^*$-regular open sets in X is denoted by $(\tau_1, \tau_2)^*$-R.O (X, τ_1, τ_2). A^C denotes the complement of A in X unless explicitly stated.

Definition 2.3. A set A of a bitopological space (X, τ_1, τ_2) is called

(a) $(\tau_1, \tau_2)^*$-regular generalized closed ($(\tau_1\tau_2)^*$-rg closed) [21] in X if $(\tau_1, \tau_2)^*$-cl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $(\tau_1, \tau_2)^*$-regular open in X.

(b) $(\tau_1, \tau_2)^*$-regular generalized open ($(\tau_1\tau_2)^*$-rg open) [21] in X if $F \subseteq (\tau_1, \tau_2)^*$-int$(A)$ whenever $F \subseteq A$ and F is $(\tau_1, \tau_2)^*$-regular closed in X.

(c) $(\tau_1, \tau_2)^*$-regular generalized star closed ($(\tau_1\tau_2)^*$-rg^* closed) [22] in X if $(\tau_1, \tau_2)^*$-rcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $(\tau_1, \tau_2)^*$-regular open in X.

(d) $(\tau_1, \tau_2)^*$-regular generalized star open ($(\tau_1\tau_2)^*$-rg^* open) [22] in X if $F \subseteq (\tau_1, \tau_2)^*$-rint$(A)$ whenever $F \subseteq A$ and F is $(\tau_1, \tau_2)^*$-regular closed in X.

(e) $(\tau_1, \tau_2)^*$-generalized star regular closed ($(\tau_1\tau_2)^*$-g^*r closed) [23] in X if $(\tau_1, \tau_2)^*$-rcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$-open in X.

(f) $(\tau_1, \tau_2)^*$-generalized star regular open ($(\tau_1\tau_2)^*$-g^*r open) [23] in X if $F \subseteq (\tau_1, \tau_2)^*$-rint$(A)$ whenever $F \subseteq A$ and F is $\tau_1\tau_2$-closed in X.

Definition 2.4. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called

(a) $(\tau_1, \tau_2)^*$-g continuous [8] in X if the inverse image of each $\sigma_1\sigma_2$-closed set is $(\tau_1, \tau_2)^*$-g closed in X.

(b) $(\tau_1, \tau_2)^*$-gs continuous [24] in X if the inverse image of each $\sigma_1\sigma_2$-closed set is $(\tau_1, \tau_2)^*$-gs closed in X.

(c) $(\tau_1, \tau_2)^*$-rg continuous [21] in X if the inverse image of each $\sigma_1\sigma_2$-closed set is $(\tau_1, \tau_2)^*$-rg closed in X.

(d) almost $(\tau_1, \tau_2)^*$-g continuous [21] in X if the inverse image of each $(\sigma_1, \sigma_2)^*$-regular closed set is $(\tau_1, \tau_2)^*$-g closed in X.

(e) almost $(\tau_1, \tau_2)^*$-gs continuous in X if the inverse image of each $(\sigma_1, \sigma_2)^*$-regular closed set is $(\tau_1, \tau_2)^*$-gs closed in X.

(f) almost $(\tau_1, \tau_2)^*$-rg continuous [21] in X if the inverse image of each $(\sigma_1, \sigma_2)^*$-regular closed set is $(\tau_1, \tau_2)^*$-rg closed in X.

Definition 2.5. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called

(a) $(\tau_1, \tau_2)^*$-g irresolute [8] in X if the inverse image of each $(\sigma_1, \sigma_2)^*$-g closed set is $(\tau_1, \tau_2)^*$-g closed in X.

(b) $(\tau_1, \tau_2)^*$-gs irresolute [24] in X if the inverse image of each $(\sigma_1, \sigma_2)^*$-gs closed set is $(\tau_1, \tau_2)^*$-gs closed in X.

(c) $(\tau_1, \tau_2)^*$-rg irresolute [21] in X if the inverse image of each $(\sigma_1, \sigma_2)^*$-rg closed set is $(\tau_1, \tau_2)^*$-rg closed in X.

3. $(\tau_1, \tau_2)^*$-g^*r Continuous Functions

Definition 3.1. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called

(a) $(\tau_1, \tau_2)^*$-generalized star regular continuous ($(\tau_1, \tau_2)^*$-g^*r continuous) in X if the inverse image of each $\sigma_1\sigma_2$-closed set is $(\tau_1, \tau_2)^*$-generalized star regular closed in X.

(b) $(\tau_1, \tau_2)^*$-g^*r irresolute in X if the inverse image of each $(\sigma_1, \sigma_2)^*$-generalized star regular closed set is $(\tau_1, \tau_2)^*$-generalized star regular closed in X.

Example 3.2. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{b, c\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{b\}\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function. Then, f is $(\tau_1, \tau_2)^*$-g^*r continuous and $(\tau_1, \tau_2)^*$-g^*r irresolute.

Theorem 3.3. Every $(\tau_1, \tau_2)^*$-g^*r continuous function is $(\tau_1, \tau_2)^*$-g continuous.

Proof. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be $(\tau_1, \tau_2)^*$-g^*r continuous and F be $\sigma_1\sigma_2$-closed in Y. Since $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be $(\tau_1, \tau_2)^*$-g^*r continuous, $f^{-1}(F)$ is $(\tau_1, \tau_2)^*$-g^*r closed in X. Since every $(\tau_1, \tau_2)^*$-g^*r closed set is $(\tau_1, \tau_2)^*$-g closed, $f^{-1}(F)$ is $(\tau_1, \tau_2)^*$-g closed in X. Therefore, f is $(\tau_1, \tau_2)^*$-g continuous. □

Corollary 3.4. Every $(\tau_1, \tau_2)^*$-g^*r continuous function is $(\tau_1, \tau_2)^*$-gs continuous {resp. $(\tau_1, \tau_2)^*$-rg continuous, almost $(\tau_1, \tau_2)^*$-g continuous, almost $(\tau_1, \tau_2)^*$-gs continuous, almost $(\tau_1, \tau_2)^*$-rg continuous}.

Remark 3.5. The converse of the above theorem is not true in general. The following example supports our claim

Example 3.6. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, c\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function. Then, f is $(\tau_1, \tau_2)^*$-g continuous, $(\tau_1, \tau_2)^*$-gs continuous, $(\tau_1, \tau_2)^*$-rg continuous, almost $(\tau_1, \tau_2)^*$-g continuous, almost

$(\tau_1, \tau_2)^*$-gs continuous, almost $(\tau_1, \tau_2)^*$-rg continuous but not a $(\tau_1, \tau_2)^*$-g^*r continuous function.

Theorem 3.7. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-g^*r irresolute and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*$-g^*r continuous, then $gof : (X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*$-g^*r continuous.

Proof. Let F be $\mu_1\mu_2$-closed set in Z. Since $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*$-g^*r continuous, $g^{-1}(F)$ is $(\sigma_1, \sigma_2)^*$-g^*r closed in Y. Since $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-g^*r irresolute, $(gof)^{-1}(F) = f^{-1}[g^{-1}(F)]$ is $(\tau_1, \tau_2)^*$-g^*r closed in X. Hence $gof : (X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*$-g^*r continuous. □

Theorem 3.8. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-g^*r continuous and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*$-continuous, then $gof : (X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*$-g^*r continuous.

Proof. Let F be $\mu_1\mu_2$-closed set in Z. Since $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*$-continuous, $g^{-1}(F)$ is $\sigma_1\sigma_2$-closed in Y. Since $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-g^*r irresolute, $(gof)^{-1}(F) = f^{-1}[g^{-1}(F)]$ is $(\tau_1, \tau_2)^*$-g^*r closed in X. Hence $gof : (X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*$-g^*r continuous. □

Remark 3.9. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-g^*r continuous and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*$-g^*r continuous, then $gof : (X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ need not be $(\tau_1, \tau_2)^*$-g^*r continuous. The following example supports our claim.

Example 3.10. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{b, c\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{b, c\}\}$, $Z = \{a, b, c\}$, $\mu_1 = \{\phi, Z, \{a\}\}$, $\mu_2 = \{\phi, Z, \{a, b\}\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ be the function defined by $g(a) = b$, $g(b) = c$, $g(c) = a$. Then f and g are $(\tau_1, \tau_2)^*$-g^*r continuous and $\sigma_1, \sigma_2)^*$-g^*r continuous functions, but gof is not $(\tau_1, \tau_2)^*$-g^*r continuous.

Theorem 3.11. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-g^*r irresolute and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*$-g^*r irresolute, then $gof : (X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*$-g^*r irresolute.

Proof. Let F be $(\mu_1, \mu_2)^*$-g^*r closed set in Z. Since $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*$-g^*r irresolute, $g^{-1}(F)$ is $(\sigma_1, \sigma_2)^*$-g^*r closed in Y. Since $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-g^*r irresolute, $(gof)^{-1}(F) = f^{-1}[g^{-1}(F)]$ is $(\tau_1, \tau_2)^*$-g^*r closed in X. Hence $gof : (X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*$-g^*r irresolute. □

Definition 3.12. Recall that a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called

(a) $(\tau_1, \tau_2)^*$-g closed [24] in X if the image of each $\tau_1\tau_2$-closed set is $(\sigma_1, \sigma_2)^*$-g closed in Y.

(b) $(\tau_1, \tau_2)^*$-gs closed [24] in X if the image of each $\tau_1\tau_2$-closed set is $(\sigma_1, \sigma_2)^*$-gs closed in Y.

(c) $(\tau_1, \tau_2)^*$-rg closed [21] in X if the image of each $\tau_1\tau_2$-closed set is $(\sigma_1, \sigma_2)^*$-rg closed in Y.

(d) almost $(\tau_1, \tau_2)^*$-g closed [21] in X if the image of each $(\tau_1, \tau_2)^*$-regular closed set is $(\sigma_1, \sigma_2)^*$-g closed in Y.

(e) almost $(\tau_1, \tau_2)^*$-gs closed in X if the image of each $(\tau_1, \tau_2)^*$-regular closed set is $(\sigma_1, \sigma_2)^*$-gs closed in Y.

(f) almost $(\tau_1, \tau_2)^*$-rg closed [21] in X if the image of each $(\tau_1, \tau_2)^*$-regular closed set is $(\sigma_1, \sigma_2)^*$-rg closed in Y.

Definition 3.13. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $(\tau_1, \tau_2)^*$-g^*r closed in X if the image of each $\tau_1\tau_2$-closed set is $(\sigma_1, \sigma_2)^*$-generalized star regular closed in Y.

Example 3.14. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{b\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}, \{b, c\}\}$, $\sigma_2 = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function. Then, f is a $(\tau_1, \tau_2)^*$-g^*r closed function.

Theorem 3.15. Every $(\tau_1, \tau_2)^*$-g^*r closed function is $(\tau_1, \tau_2)^*$-g closed.

Proof. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be $(\tau_1, \tau_2)^*$-g^*r closed and F be $\tau_1\tau_2$-closed in X. Since $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-g^*r closed, $f(F)$ is $(\sigma_1, \sigma_2)^*$-g^*r closed in Y. Since every $(\sigma_1, \sigma_2)^*$-g^*r closed set is $(\sigma_1, \sigma_2)^*$-g closed, $f(F)$ is $(\sigma_1, \sigma_2)^*$-g closed in Y. Therefore, f is $(\tau_1, \tau_2)^*$-g closed. $\square$

Corollary 3.16. Every $(\tau_1, \tau_2)^*$-g^*r closed function is $(\tau_1, \tau_2)^*$-gs closed $\{$resp. $(\tau_1, \tau_2)^*$-rg closed, almost $(\tau_1, \tau_2)^*$-g closed, almost $(\tau_1, \tau_2)^*$-gs closed, almost $(\tau_1, \tau_2)^*$-rg closed$\}$.

Remark 3.17. The converse of the above theorem is not true in general. The following example supports our claim

Example 3.18. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{a, b\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}, \{a, b\}\}$, $\sigma_2 = \{\phi, Y, \{a, c\}\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function. Then, f is $(\tau_1, \tau_2)^*$-g closed, $(\tau_1, \tau_2)^*$-gs closed, $(\tau_1, \tau_2)^*$-rg closed, almost $(\tau_1, \tau_2)^*$-g closed, almost $(\tau_1, \tau_2)^*$-gs closed, almost $(\tau_1, \tau_2)^*$-rg closed but not a $(\tau_1, \tau_2)^*$-g^*r closed function.

Theorem 3.19. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-closed and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*$-g^*r closed, then $gof : (X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*$-g^*r closed.

Proof. Let F be $\tau_1\tau_2$-closed set in X. Since $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-closed, $f(F)$ is $\sigma_1\sigma_2$-closed in Y. Since $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-g^*r closed, $(gof)(F) = g[f(F)]$ is $(\mu_1, \mu_2)^*$-g^*r closed in Z. Hence $gof : (X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*$-g^*r closed. $\square$

Corollary 3.20. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-closed and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*$-g^*r closed, then $gof : (X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ is $(\tau_1, \tau_2)^*$-rg closed {resp. $(\tau_1, \tau_2)^*$-gs closed, almost $(\tau_1, \tau_2)^*$-g closed, almost $(\tau_1, \tau_2)^*$-rg closed, almost $(\tau_1, \tau_2)^*$-gs closed}.

Definition 3.21. Recall that a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called

(a) $(\tau_1, \tau_2)^*$-g open [24] in X if the image of each $\tau_1\tau_2$-open set is $(\sigma_1, \sigma_2)^*$-g open in Y.

(b) $(\tau_1, \tau_2)^*$-gs open [24] in X if the image of each $\tau_1\tau_2$-open set is $(\sigma_1, \sigma_2)^*$-gs open in Y.

(c) $(\tau_1, \tau_2)^*$-rg open in X if the image of each $\tau_1\tau_2$-open set is $(\sigma_1, \sigma_2)^*$-rg open in Y.

(d) almost $(\tau_1, \tau_2)^*$-g open in X if the image of each $(\tau_1, \tau_2)^*$-regular open set is $(\sigma_1, \sigma_2)^*$-g open in Y.

(e) almost $(\tau_1, \tau_2)^*$-gs open in X if the image of each $(\tau_1, \tau_2)^*$-regular open set is $(\sigma_1, \sigma_2)^*$-gs open in Y.

(f) almost $(\tau_1, \tau_2)^*$-rg open in X if the image of each $(\tau_1, \tau_2)^*$-regular open set is $(\sigma_1, \sigma_2)^*$-rg open in Y.

Definition 3.22. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $(\tau_1, \tau_2)^*$-g^*r open in X if and only if image of each $\tau_1\tau_2$-open set is $(\sigma_1, \sigma_2)^*$-g^*r open in Y.

Example 3.23. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{b\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}, \{b, c\}\}$, $\sigma_2 = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function. Then, f is a $(\tau_1, \tau_2)^*$-g^*r open function.

Theorem 3.24. Every $(\tau_1, \tau_2)^*$-g^*r open function is $(\tau_1, \tau_2)^*$-g open.

Proof. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be $(\tau_1, \tau_2)^*$-g^*r open and F be $\tau_1\tau_2$-open in X. Since $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-g^*r open, $f(F)$ is $(\sigma_1, \sigma_2)^*$-g^*r open in Y. Since every $(\sigma_1, \sigma_2)^*$-g^*r open set is $(\sigma_1, \sigma_2)^*$-g open, $f(F)$ is $(\sigma_1, \sigma_2)^*$-g open in Y. Therefore, f is $(\tau_1, \tau_2)^*$-g open. □

Corollary 3.25. Every $(\tau_1, \tau_2)^*$-g^*r open function is $(\tau_1, \tau_2)^*$-rg open {resp.
$(\tau_1, \tau_2)^*$-gs open, almost $(\tau_1, \tau_2)^*$-g open, almost $(\tau_1, \tau_2)^*$-rg open, almost $(\tau_1, \tau_2)^*$-gs open}.

Remark 3.26. The converse of the above theorem is not true in general. The following example supports our claim

Example 3.27. In Example 3.18, f is $(\tau_1, \tau_2)^*$-g open, $(\tau_1, \tau_2)^*$-gs open, $(\tau_1, \tau_2)^*$-rg open, almost $(\tau_1, \tau_2)^*$-g open, almost $(\tau_1, \tau_2)^*$-gs open, almost $(\tau_1, \tau_2)^*$-rg open but not a $(\tau_1, \tau_2)^*$-g^*r-open function.

4. $(\tau_1, \tau_2)^*$-g^*r Homeomorphisms

Definition 4.1. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $(\tau_1, \tau_2)^*$-generalized star regular homeomorphism ($(\tau_1, \tau_2)^*$-g^*r homeomorphism) if and only if both f is $(\tau_1, \tau_2)^*$-g^*r continuous and $(\tau_1, \tau_2)^*$-g^*r open.

Example 4.2. In Example 3.10, f is $(\tau_1, \tau_2)^*$-g^*r homeomorphism.

Definition 4.3. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $(\tau_1, \tau_2)^*$-g^*rc homeomorphism) if and only if f is $(\tau_1, \tau_2)^*$-g^*r irresolute and f^{-1} is $(\sigma_1, \sigma_2)^*$-g^*r irresolute.

Example 4.4. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{a, b\}\}$, $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{b\}\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function. Then, f is a $(\tau_1, \tau_2)^*$-g^*rc homeomorphism.

Theorem 4.5. For any bijection $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following statements are equivalent.

(a) $f^{-1} : (Y, \sigma_1, \sigma_2) \to (X, \tau_1, \tau_2)$ is $(\sigma_1, \sigma_2)^*$-g^*r continuous.

(b) f is $(\tau_1, \tau_2)^*$-g^*r open.

(c) f is $(\tau_1, \tau_2)^*$-g^*r closed.

Proof. $(a) \Rightarrow (b)$: Let F be $\tau_1\tau_2$-open in X. Then $X - F$ is $\tau_1\tau_2$-closed in X. Since f^{-1} is $(\sigma_1, \sigma_2)^*$-g^*r continuous, $\left[f^{-1}\right]^{-1}(X - F) = f(X - F) = Y - f(F)$ is $(\sigma_1, \sigma_2)^*$-g^*r closed in Y. Then $f(F)$ is $(\sigma_1, \sigma_2)^*$-g^*r open in Y. Hence f is $(\tau_1, \tau_2)^*$-g^*r open.

$(b) \Rightarrow (c)$: Let F be $\tau_1\tau_2$-closed in X. Then $X - F$ is $\tau_1\tau_2$-open in X. Since f is $(\tau_1, \tau_2)^*$-g^*r open, $f(X - F) = Y - f(F)$ is $(\sigma_1, \sigma_2)^*$-g^*r open in Y. Then $f(F)$ is $(\sigma_1, \sigma_2)^*$-g^*r closed in Y. Hence f is $(\tau_1, \tau_2)^*$-g^*r closed.

$(c) \Rightarrow (a)$: Let F be $\tau_1\tau_2$-closed in X. Since f is $(\tau_1, \tau_2)^*$-g^*r closed, $\left[f^{-1}\right]^{-1}(F) = f(F)$ is $(\sigma_1, \sigma_2)^*$-g^*r closed in Y. Hence f is $(\tau_1, \tau_2)^*$-g^*r continuous. □

Theorem 4.6. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a bijective and $(\tau_1, \tau_2)^*$-g^*r continuous function. Then the following statements are equivalent.

(a) f is $(\tau_1, \tau_2)^*$-g^*r open.

(b) f is $(\tau_1, \tau_2)^*$-g^*r homeomorphism.

(c) f is $(\tau_1, \tau_2)^*$-g^*r closed.

Proof. $(a) \Rightarrow (b)$: obvious from the definitions.

$(b) \Rightarrow (c)$: Let F be $\tau_1\tau_2$-closed in X. Then $X - F$ is $\tau_1\tau_2$-open in X. Since f is $(\tau_1, \tau_2)^*$-g^*r homeomorphism, f is $(\tau_1, \tau_2)^*$-g^*r open. Hence by above theorem, f is $(\tau_1, \tau_2)^*$-g^*r closed.

$(c) \Rightarrow (a)$: Follows from above Theorem. □

Remark 4.7. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-g^*r homeomorphism and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ is $(\sigma_1, \sigma_2)^*$-g^*r homeomorphism, then $gof : (X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ need not be a $(\tau_1, \tau_2)^*$-g^*r homeomorphism.

Example 4.8. In Example 3.10, f and g are $(\tau_1, \tau_2)^*$-g^*r homeomorphism and $\sigma_1, \sigma_2)^*$-g^*r homeomorphism, but gof is not $(\tau_1, \tau_2)^*$-g^*r homeomorphism.

5. $(\tau_1, \tau_2)^*$-G^*RO Compact Space

Definition 5.1. A subset B of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)^*$-G^*RO compact relative to X if for every cover $\{A_i : i \in \Lambda\}$ of B by $(\tau_1, \tau_2)^*$-g^*r open subsets of (X, τ_1, τ_2), i.e. $B \subseteq \bigcup_{i\in\Lambda} A_i$ where A_i, $i \in \Lambda$ are $(\tau_1, \tau_2)^*$-g^*r open sets of (X, τ_1, τ_2), there exists a finite subset Λ_0 of Λ such that $B \subseteq \bigcup_{i\in\Lambda_0} A_i$.

Definition 5.2. A bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)^*$-G^*RO compact if for every cover $\{A_i : i \in \Lambda\}$ of X by $(\tau_1, \tau_2)^*$-g^*r open subsets of (X, τ_1, τ_2), i.e. $X \subseteq \bigcup_{i\in\Lambda} A_i$ where A_i, $i \in \Lambda$ are $(\tau_1, \tau_2)^*$-g^*r open sets of (X, τ_1, τ_2), there exists a finite subset Λ_0 of Λ such that $X \subseteq \bigcup_{i\in\Lambda_0} A_i$.

Theorem 5.3. Every $(\tau_1, \tau_2)^*$-GO compact space {resp. $(\tau_1, \tau_2)^*$-GSO compact space, $(\tau_1, \tau_2)^*$-RGO compact space} is $(\tau_1, \tau_2)^*$-G^*RO compact space.

Proposition 5.4. A $(\tau_1, \tau_2)^*$-g^*r closed subset of a $(\tau_1, \tau_2)^*$-G^*RO compact space (X, τ_1, τ_2) is $(\tau_1, \tau_2)^*$-G^*RO compact relative to X.

Proof. Let (X, τ_1, τ_2) be a $(\tau_1, \tau_2)^*$-G^*RO compact space and Y be a $(\tau_1, \tau_2)^*$-g^*r closed subset of X. Let $\zeta = \{A_i : i \in \Lambda\}$ be a $(\tau_1, \tau_2)^*$-g^*r open cover of Y. Since Y is $(\tau_1, \tau_2)^*$-g^*r closed subset, Y^C is $(\tau_1, \tau_2)^*$-g^*r open. Also $\zeta \cup Y^C = Y^C \cup \left(\bigcup_{i\in\Lambda} A_i\right)$ be a $(\tau_1, \tau_2)^*$-g^*r open cover of X. Since X is $(\tau_1, \tau_2)^*$-G^*RO compact, $X = Y^C \cup \left(\bigcup_{i\in\Lambda_0} A_i\right)$ where Λ_0 is a finite subset Λ. Hence $Y = \bigcup_{i\in\Lambda_0} A_i$. Therefore, Y is $(\tau_1, \tau_2)^*$-G^*RO compact. □

Proposition 5.5. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(\tau_1, \tau_2)^*$-g^*r continuous and a subset B of X is $(\tau_1, \tau_2)^*$-G^*RO compact relative to X, then $f(B)$ is $(\sigma_1, \sigma_2)^*$-compact in Y.

Proof. Let $\{A_i : i \in \Lambda\}$ be any collection of $\sigma_1\sigma_2$-open subsets of Y such that $f(B) \subseteq \bigcup_{i\in\Lambda} A_i$. Then $B \subseteq \bigcup_{i\in\Lambda} f^{-1}(A_i)$ holds and there exists a finite subset Λ_0 of Λ such that $B \subseteq \bigcup_{i\in\Lambda_0} f^{-1}(A_i)$. Therefore, we have $f(B) \subseteq \bigcup_{i\in\Lambda_0} A_i$ which shows that $f(B)$ is $(\sigma_1, \sigma_2)^*$-compact in Y. □

Proposition 5.6. If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is resp.$(\tau_1, \tau_2)^*$-g^*r irresolute and a subset B of X is $(\tau_1, \tau_2)^*$-G^*RO compact relative to X, then $f(B)$ is $(\tau_1, \tau_2)^*$-G^*RO compact relative to Y.

Proof. Let $\{A_i : i \in \Lambda\}$ be any collection of $(\sigma_1, \sigma_2)^*$-g^*r open subsets of Y such that $f(B) \subseteq \bigcup_{i\in\Lambda} A_i$. Then $B \subseteq \bigcup_{i\in\Lambda} f^{-1}(A_i)$ holds and there exists a finite subset Λ_0 of Λ such that $B \subseteq \bigcup_{i\in\Lambda_0} f^{-1}(A_i)$. Therefore, we have $f(B) \subseteq \bigcup_{i\in\Lambda_0} A_i$ which shows that $f(B)$ is $(\sigma_1, \sigma_2)^*$-G^*RO compact in Y. □

References

[1] J. C. Kelly, Bitopological spaces, *Proc. London Math. Society*, **13**(1963), 71–89.

[2] S.N. Maheshwari and R. Prasad, Semiopen sets and semi continuous functions in bitopological spaces, *Math. Note*, **26** (1977/78), 29–37.

[3] S. Bose, Semi open sets, semi continuity and semi open mappings in bitopological spaces, *Bull. Cal. Math. Soc.*, **73** (1981), 237–246.

[4] T. Fukutake, Semi open sets on bitopological spaces, *Bull. Fukuoka Uni. Education*, **38**(3)(1989), 1–7.

[5] T. Fukutake, On generalized closed sets in bitopological spaces, *Bull. Fukuoka Univ. Ed. Part* III, **35** (1986), 19–28.

[6] T. Fukutake, P. Sundaram and M. Sheik John, w-closed sets, w-open sets and w-ontinuity in bitopological spaces, *Bull. Fukuoka Univ. Ed. Part* III, **51** (2002), 1–9.

[7] M. Sheik John and P. Sundaram, g^*-closed sets in Bitopological Spaces, *Indian J. Pure Appl. Math.*, **35**(1) (2004), 71–80.

[8] M. Lellis Thivagar and O. Ravi, A Bitopological $(1, 2)^*$ Semi generalised Continuous Maps, *Bull. Malays. Math. Sci. Soc.*, **(2)** 29(1)(2006), 79–88.

[9] K. Chandrasekhara Rao and K. Kannan, Semi star generalized closed sets and semi star generalized open sets in bitopological spaces, *Varahimir Journal of Mathematics*, **5 (2)**(2005), 473–485.

[10] K. Chandrasekhara Rao, K. Kannan and D. Narasimhan, Characterizations of $\tau_1\tau_2$-s^*g closed sets, *Acta Ciencia Indica*, **XXXIII M, (3)**,(2007), 807–810.

[11] K. Kannan, D. Narasimhan and K. Chandrasekhara Rao, On Semi Star Generalized Closed Sets in Bitopological Spaces, *Bol. Soc. Paranaense de Mat.* **(3s.) v.28 (1)** (2010), 29–40.

[12] K. Chandrasekhara Rao and K. Kannan, Regular generalized star closed sets, *Thai Journal of Mathematics*, **4(2)** (2006) , 341–349.

[13] K. Chandrasekhara Rao and K. Kannan, Generalized star regular closed sets, *Antarctica J. Math.*, **4 (2)** (2007), 139–146.

[14] K. Chandrasekhara Rao , K. Kannan and D. Narasimhan, Generalized star star regular closed sets, *Natl. Acad. Sci. Lett.*, **Vol.31, No. 1 & 2** , (2008), 31–33.

[15] K. Kannan, D. Narasimhan, K. Chandrasekhara Rao and R. Ravikumar, Regular generalized star star closed sets, *World Academy of Science, Engineering and Technology I: Mathematical and Computational Sciences*, **5(2)** 2011, 67–69.

[16] Kannan K (2012) $\tau_1\tau_2$-Semi Star Star Generalized Closed Sets. International Journal of Pure and Applied Mathematics 76(2):277–294.

[17] Kannan K, Chandrasekhara Rao (2013) $\tau_1\tau_2$-Q^* closed sets. Thai Journal of Mathematics 11(2):439–445.

[18] K. Kannan, A note on some generalized closed sets in bitopological spaces associated to digraphs *Journal of Applied Mathematics*, **Volume 2012**, Article ID 508580, 5 pages.

[19] K. Kannan, D. Narasimhan, K. Chandrasekhara Rao and M. Sundararaman, $(\tau_1, \tau_2)^*$-Semi Star Generalized Closed Sets in Bitopological Spaces, *Journal of Advanced Research in Pure Mathematics*, **Vol. 2, Issue. 3**, (2010), 34–47.

[20] M. Lellis Thivagar and Nirmala Mariappan A note on $(1, 2)^*$-strongly genelized semi-preclosed sets, *Proceedings of the International conference on Mathematice and computer science*,(2010), 422–425.

[21] K. Kayathri, O. Ravi, M. L. Thivagar, M. Joseph Israel, Mildly $(1,2)^*$-Normal spaces and some bitopological functions, Mathematica Bohemica, **135(1)** (2010), 1–13.

[22] R. Ravikumar and M.Kothandapani $(\tau_1,\tau_2)^*$-Regular Generalized Star Closed Sets in Bitopological Spaces, *Journal of Advanced Research in Pure Mathematics*, **Vol. 3, Issue. 3**, (2011), 140–150.

[23] R. Ravikumar and M.Kothandapani $(\tau_1,\tau_2)^*$-Generalized Star Regular Closed Sets, *preprint*,

[24] O. Ravi, S. Pious Missier and T. Salai Parkunan, On Bitopological (1, 2)*-Generalized Homeomorphisms, *Int. J. Contemp. Math. Sciences*, **5(11)** 2010, 543–557.