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FUZZY TOPOLOGICAL SUBSYSTEM ON A TM-ALGEBRA

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Abstract: Recently in 2010, Tamilarasi and Megalai introduced a new class of algebras known as TM-algebras. In this papaer, we discuss the notion of fuzzy topological sub systems on TM-algebras.

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1. Introduction

In 1966, Y.Imai and Iseki [3] introduced two new classes of algebras that arise from the study on classical and non-classical propositional logic. These algebras are known as BCK and BCI algebras. It is known that the notion of BCI-algebras is a generalization of BCK-algebras in such a way that the class of BCK algebras is a subclass of the class of BCI algebras [4]. Recently in 2010, Tamilarasi and Megalai introduced a new class of algebras, called TM-algebras [9]. In their paper they investigated the relationship between TM-algebras and other algebras. They claimed that the TM-algebra is a generalization of BCH/BCI/BCK and Q algebras. In [1] the authors, while studying L-fuzzy structures on TM-algebras brought out the fact that the TM-algebra is not a generalization of BCH/BCI/BCK algebras by giving counter examples.

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In 1965, L.A.Zadeh [11] introduced the notion of fuzzy sets, to evaluate the modern concept of uncertainty in real physical world. In the notion of fuzzy sets, the boundaries are not crisp or sharp but flexible. The study of fuzzy algebraic structures was initiated by A.Rosenfeld [8]. In 1991, X.Ougen [7] defined fuzzy subsets in BCK-algebras and investigated some properties. In 1993, Y.B.Jun [5] applied it to BCI-algebras.

The notion of a fuzzy set provides a natural framework for generalizing many of the concepts of general topology. The theory of fuzzy topological spaces is developed by Chang [2], Wong [10] , Lowen [6]. and others. In this paper, we discuss the notion of fuzzy topological subsystems on a TM-algebra and investigate some simple but elegant results.

2. Preliminaries

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1. For any non-empty set $X, \mu : X \to [0,1]$ is called a fuzzy set of X.

Definition 2.2. The union $A \cup B$, of two fuzzy sets A and B of a set X, is defined to be the fuzzy set

$$(A \cup B)(x) = max\,\{\mu_A(x), \mu_B(x)\}\ \ \forall\ x \in X.$$

Definition 2.3. The intersection $A \cap B$, of two fuzzy sets A and B of a set X, is defined to be the fuzzy set

$$(A \cap B)(x) = min\,\{\mu_A(x), \mu_B(x)\}\ \ \ \forall\ x \in X.$$

Definition 2.4. For any two fuzzy sets A and B of X $A \subset B$ if

$$A(x) \leq B(x)\ \forall\ x\ \in\ X.$$

Definition 2.5. Let A be a fuzzy set of X. Then the complement of A denoted by, A', is defined to be $A'(x) = 1 - A(x)\ \forall\ x\ \in\ X$.

Definition 2.6. A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions

1. $\phi, X \in T$

2. If $A, B \in T$ then $A \cap B \in T$

3. If $A_i \in T$ for each $i \in I$ then $\cup_I A_i \in T$ where I is an indexing set.

Remark 2.7. If X is a set with a fuzzy topology T then (X, T) is called a fuzzy topological space and any element in T is called a T-open fuzzy set in X.

Definition 2.8. Let f be a function from X to Y. Let σ be a fuzzy set in Y. The inverse image of σ under f is defined as $\sigma_{f^{-1}}(x) = \sigma(f(x)) \ \forall x \ \in X$. Let μ be a fuzzy set in X. The image of μ under f is defined as

$$\mu_f(y) = \begin{cases} sup_{z \in f^{-1}(x)} & \mu(z), \ f^{-1}(x) \ is \ not \ empty \\ 0 & otherwise \end{cases} \quad \forall y \ \in \ Y.$$

Definition 2.9.

A TM-Algebra $(X, *, 0)$ is a non-empty set X with a constant 0 and a binary operation $*$ satisfying the following axioms:

1. $X * 0 = X$
2. $(X * Y) * (X * Z) = Z * Y \ for \ all \ x, y, z \in X.$

3. Fuzzy Topological Subsystem on a TM-Algebra

Definition 3.1. Let $(X, *)$ be a TM-Algebra. Let (X, T) be a Fuzzy Topological System on X. Let A be a fuzzy set in X. Then the induced fuzzy topological system on X is the intersection of the fuzzy set A with T- open fuzzy sets of X. The induced fuzzy topological system on X is denoted by T_A. (A, T_A) is called a fuzzy topological subsystem of (X, T).

Example 3.2. Consider the set $X = \{0, 1, 2, 3\}$ with the following cayley table

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then $(X, *)$ is a TM-algebra.
Let the fuzzy subsets $\mu_i : X \to [0, 1], i = 1, 2, 3, 4, 5, 6, 7, 8$ be given by

$$\mu_1(x) = \begin{cases} .7 & if \ x = 0 \\ .4 & if \ x = 1, 2 \\ .5 & if \ x = 3 \end{cases} \qquad \mu_2(x) = \begin{cases} .3 & if \ x = 0 \\ 0 & if \ x = 1, 2 \\ .2 & if \ x = 3 \end{cases}$$

$$\mu_3(x) = \begin{cases} .6 & if\ x = 0 \\ .2 & if\ x = 1,2 \\ .4 & if\ x = 3 \end{cases} \qquad \mu_4(x) = \begin{cases} .5 & if\ x = 0 \\ .1 & if\ x = 1,2 \\ .4 & if\ x = 3 \end{cases}$$

$$\mu_5(x) = \begin{cases} .8 & if\ x = 0 \\ .5 & if\ x = 1,2 \\ .7 & if\ x = 3 \end{cases} \qquad \mu_6(x) = \begin{cases} .7 & if\ x = 0 \\ .3 & if\ x = 1,2 \\ .4 & if\ x = 3 \end{cases}$$

$$\mu_7(x) = \begin{cases} .8 & if\ x = 0 \\ .5 & if\ x = 1,2 \\ .6 & if\ x = 3 \end{cases} \qquad \mu_8(x) = \begin{cases} .8 & if\ x = 0 \\ .3 & if\ x = 1,2 \\ .7 & if\ x = 3 \end{cases}$$

Then the collection $T = \{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6\}$ is a Fuzzy Topology on X. Hence (X, T) is a fuzzy topological system. Choose $A = \mu_7$. Then

$T_A = \{\phi, \mu_1, \mu_2, \mu_3, \mu_4, \mu_6, \mu_7\}$ and $A = (\mu_7, T_A)$ is fuzzy topological subsystem of(X, T).

Definition 3.3. Let $(X, *)$,$(Y, *)$ be two TM-Algebras. Let (X, T),(Y, U) be two fuzzy topological systems . A mapping f of (X, T) into (Y, U) is fuzzy continuous iff the inverse image of each U- open fuzzy set is an T- open fuzzy set.

Example 3.4. Consider the set $X = \{0, 1, 2, 3, 4\}$ with the following cayley table

∗	0	1	2	3	4
0	0	4	3	2	1
1	1	0	4	3	2
2	2	1	0	4	3
3	3	2	1	0	4
4	4	3	2	1	0

Then $(X, *)$ is a TM-algebra.
Let the fuzzy subsets $\mu_i : X \to [0, 1], i = 1, 2, 3, 4, 5, 6, 7, 8$ be given by

$$\mu_1(x) = \begin{cases} .7 & if\ x = 0 \\ .4 & if\ x = 1,2 \\ .5 & if\ x = 3,4 \end{cases} \qquad \mu_2(x) = \begin{cases} .3 & if\ x = 0 \\ 0 & if\ x = 1,2 \\ .2 & if\ x = 3,4 \end{cases}$$

$$\mu_3(x) = \begin{cases} .6 & if\ x = 0 \\ .2 & if\ x = 1,2 \\ .4 & if\ x = 3,4 \end{cases} \qquad \mu_4(x) = \begin{cases} .5 & if\ x = 0 \\ .1 & if\ x = 1,2 \\ .4 & if\ x = 3,4 \end{cases}$$

$$\mu_5(x) = \begin{cases} .8 & if\ x = 0 \\ .5 & if\ x = 1,2 \\ .7 & if\ x = 3,4 \end{cases} \qquad \mu_6(x) = \begin{cases} .7 & if\ x = 0 \\ .3 & if\ x = 1,2 \\ .4 & if\ x = 3,4 \end{cases}$$

$$\mu_7(x) = \begin{cases} .8 & if\ x = 0 \\ .5 & if\ x = 1,2 \\ .6 & if\ x = 3,4 \end{cases} \qquad \mu_8(x) = \begin{cases} .8 & if\ x = 0 \\ .3 & if\ x = 1,2 \\ .7 & if\ x = 3,4 \end{cases}$$

Then the collection $T = \{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6\}$ is a Fuzzy Topology on X. Hence (X, T) is a fuzzy topological system .
Consider the set $Y = \{0, a, b, c\}$ with the following cayley table

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then $(Y, *)$ is a TM-algebra.
Let the fuzzy subsets $\sigma_i : Y \to [0, 1], i = 1, 2, 3, 4$ be given by

$$\sigma_1(y) = \begin{cases} .8 & if\ y = 0 \\ .7 & if\ y = b \\ .5 & if\ y = a,c \end{cases} \qquad \sigma_2(y) = \begin{cases} .5 & if\ y = 0 \\ .4 & if\ y = b \\ .1 & if\ y = a,c \end{cases}$$

$$\sigma_3(y) = \begin{cases} .6 & if\ y = 0 \\ .4 & if\ y = b \\ .2 & if\ y = a,c \end{cases} \qquad \sigma_4(y) = \begin{cases} .7 & if\ y = 0 \\ .4 & if\ y = b \\ .3 & if\ y = a,c \end{cases}$$

Then the collection $U = \{\phi, Y, \sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ is a Fuzzy Topology on Y.

Hence (Y, U) is a fuzzy topological system. Let f:$X \to Y$ be the function given by, $f(0) = 0, f(1) = a, f(2) = c, f(3) = b, f(4) = b$

$\sigma_{f^{-1}}$(x)=σ(f(x)) for all x in X for any fuzzy set σ in Y.

$$(\sigma_1)_{f^{-1}}(x) = \mu_5(x) \quad (\sigma_2)_{f^{-1}}(x) = \mu_4(x) \quad (\sigma_3)_{f^{-1}}(x) = \mu_3(x)$$

$$(\sigma_4)_{f^{-1}}(x) = \mu_6(x), x \in X$$

Hence the inverse image of each U-open fuzzy set is T-open and hence the function f is F-continuous.

Definition 3.5. Let $(X, *)$,$(Y, *)$ be two TM-Algebras. Let (X, T),(Y, U) be two fuzzy topological systems . A mapping f of (X, T) into (Y, U) is fuzzy open iff the image of each T- open fuzzy set is U- open fuzzy set.

Example 3.6. Consider the TM-algebra $(X, *)$ as in the example 3.4. Let the fuzzy subsets $\mu_i : X \to [0, 1], i = 1, 2, 3, 4$ be given by

$$\mu_1(x) = \begin{cases} .7 & if\ x = 0 \\ .4 & if\ x = 1, 2 \\ .5 & if\ x = 3, 4 \end{cases} \qquad \mu_2(x) = \begin{cases} .3 & if\ x = 0 \\ 0 & if\ x = 1, 2 \\ .2 & if\ x = 3, 4 \end{cases}$$

$$\mu_3(x) = \begin{cases} .6 & if\ x = 0 \\ .2 & if\ x = 1, 2 \\ .4 & if\ x = 3, 4 \end{cases} \qquad \mu_4(x) = \begin{cases} .5 & if\ x = 0 \\ .1 & if\ x = 1, 2 \\ .4 & if\ x = 3, 4 \end{cases}$$

Then the collection $T = \{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4\}$ is a Fuzzy Topology on X. Hence (X, T) is a fuzzy topological system .
Consider the TM-algebra $(Y, *)$ as in the example 3.4 Let the fuzzy subsets $\sigma_i : Y \to [0, 1], i = 1, 2, 3, 4, 5, 6$ be given by

$$\sigma_1(y) = \begin{cases} .8 & if\ y = 0 \\ .7 & if\ y = b \\ .5 & if\ y = a, c \end{cases} \qquad \sigma_2(y) = \begin{cases} .5 & if\ y = 0 \\ .4 & if\ y = b \\ .1 & if\ y = a, c \end{cases}$$

$$\sigma_3(y) = \begin{cases} .6 & if\ y = 0 \\ .4 & if\ y = b \\ .2 & if\ y = a, c \end{cases} \qquad \sigma_4(y) = \begin{cases} .7 & if\ y = 0 \\ .4 & if\ y = b \\ .3 & if\ y = a, c \end{cases}$$

$$\sigma_5(y) = \begin{cases} .7 & if\ y = 0 \\ .5 & if\ y = b \\ .4 & if\ y = a, c \end{cases} \qquad \sigma_6(y) = \begin{cases} .3 & if\ y = 0 \\ .2 & if\ y = b \\ 0 & if\ y = a, c \end{cases}$$

Then the collection $U = \{\phi, Y, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6\}$ is a Fuzzy Topology on Y. Hence (Y, U) is a fuzzy topological system.
Let $f : X \to Y$ be the function given by,
$f(0) = 0, f(1) = a, f(2) = c, f(3) = b, f(4) = b$, Then
$f^{-1}(0) = 0, f^{-1}(a) = 1, f^{-1}(c) = 2, f^{-1}(b) = 3, f^{-1}(b) = 4$

$$f(\mu_1) = \sigma_5 \quad f(\mu_2) = \sigma_6 \quad f(\mu_3) = \sigma_3 \quad f(\mu_4) = \sigma_2$$

Hence the image of each T-open fuzzy set is U-open.
Therefore the function f is Fuzzy Open.

Definition 3.7. Let $(X, *)$,$(Y, *)$ be two TM-Algebras. Let (X, T),(Y, U) be two fuzzy topological systems. Let $(A, T_A), (B, U_B)$ be the fuzzy topological subsystems on X and Y. f is said to be a mapping of (A, T_A) into (B, U_B) if $f(A) \subset B$.

Definition 3.8. Let $(X,*)$,$(Y,*)$ be two TM-Algebras. Let (X,T),(Y,U) be two fuzzy topological systems . Let $(A,T_A),(B,U_B)$ be the fuzzy topological subsystems on X and Y. A mapping f from (A,T_A) into (B,U_B) is relatively fuzzy continuous if and only if $f^{-1}(\sigma_B)\cap A$ is in T_A where $\sigma_B\in U_B$.

Example 3.9. Consider the TM-algebra $(X,*)$ as in the example 3.4. Let the fuzzy subsets $\mu_i : X\to[0,1], i=1,2,3,4$ be given by

$$\mu_1(x)=\begin{cases} .9 & if\ x=0\\ .6 & if\ x=1,2\\ .8 & if\ x=3,4\end{cases}\qquad \mu_2(x)=\begin{cases} .6 & if\ x=0\\ .2 & if\ x=1,2\\ .3 & if\ x=3,4\end{cases}$$

$$\mu_3(x)=\begin{cases} .5 & if\ x=0\\ .2 & if\ x=2,4\\ 0 & if\ x=1,3\end{cases}\qquad \mu_4(x)=\begin{cases} .4 & if\ x=0\\ 0 & if\ x=1,2\\ .1 & if\ x=3,4\end{cases}$$

$$\mu_5(x)=\begin{cases} .8 & if\ x=0\\ .5 & if\ x=1,2\\ .7 & if\ x=3,4\end{cases}\qquad \mu_6(x)=\begin{cases} .7 & if\ x=0\\ .5 & if\ x=3,4\\ .4 & if\ x=1,2\end{cases}$$

Then the collection $T=\{\phi,X,\mu_1,\mu_2,\mu_3,\mu_4,\mu_5\}$ is a Fuzzy Topology on X. Hence (X,T) is a fuzzy topological system. Choose $A=\mu_6$

Then $T_A=\{\phi,\mu_2,\mu_3,\mu_4,\mu_6,\}$ and $A=(\mu_6,T_A)$ is fuzzy topological subsystem of(X,T).

Consider the TM-algebra $(Y,*)$ as in the example 3.4.

Let the fuzzy subsets $\sigma_i : Y\to[0,1], i=1,2,3,4$ be given by

$$\sigma_1(y)=\begin{cases} .4 & if\ y=0\\ .1 & if\ y=b\\ 0 & if\ y=a,c\end{cases}\qquad \sigma_2(y)=\begin{cases} .9 & if\ y=0\\ .8 & if\ y=b\\ .6 & if\ y=a,c\end{cases}$$

$$\sigma_3(y)=\begin{cases} .7 & if\ y=0\\ .5 & if\ y=b\\ .4 & if\ y=a,c\end{cases}\qquad \sigma_4(y)=\begin{cases} .6 & if\ y=0\\ .3 & if\ y=b\\ .2 & if\ y=a,c\end{cases}$$

Then the collection $U=\{\phi,Y,\sigma_1,\sigma_3,\sigma_4\}$ is a Fuzzy Topology on Y. Hence (Y,U) is a fuzzy topological system. Choose $B=\sigma_2$. Then $T_A=\{\phi,\sigma_1,\sigma_3,\sigma_4\}$ and $B=(\sigma_2,U_B)$ is fuzzy topological subsystem of(Y,U).

Let $f : X\to Y$ be given by, $f(0)=0,f(1)=a,f(2)=c,f(3)=b,f(4)=b.$

Then $f^{-1}(0)=0,f^{-1}(a)=1,f^{-1}(c)=2,f^{-1}(b)=3,f^{-1}(b)=4$

$$f(A)=\begin{cases} .7 & if\ y=0\\ .5 & if\ y=b\\ .4 & if\ y=a,c\end{cases}\quad\subset B$$

$$(\sigma_1)_{f^{-1}}(x) = \begin{cases} .4 & if\ x = 0 \\ 0 & if\ x = 1,2 \\ .1 & if\ x = 3,4 \end{cases} \qquad (\sigma_1)_{f^{-1}}(x) \cap A = \mu_4(x), x \in X$$

$$(\sigma_3)_{f^{-1}}(x) = \begin{cases} .7 & if\ x = 0 \\ .4 & if\ x = 1,2 \\ .5 & if\ x = 3,4 \end{cases} \qquad (\sigma_3)_{f^{-1}}(x) \cap A = \mu_6(x), x \in X$$

$$(\sigma_4)_{f^{-1}}(x) = \begin{cases} .6 & if\ x = 0 \\ .2 & if\ x = 1,2 \\ .3 & if\ x = 3,4 \end{cases} \qquad (\sigma_4)_{f^{-1}}(x) \cap A = \mu_2(x), x \in X$$

Hence the inverse image of each U_B-open fuzzy set is T_A-open.
Therefore the function f is relatively fuzzy continuous.

Definition 3.10. Let $(X, *)$,$(Y, *)$ be two TM-Algebras. Let (X, T),(Y, U) be two fuzzy topological systems .Let (A, T_A), (B, U_B) be the fuzzy topological subsystems on X and Y. A mapping f from (A, T_A) into (B, U_B) is relatively fuzzy open if and only iff $f(\mu_A) \in U_B$ where $\mu_A \in T_A$.

Example 3.11. Consider the TM-algebra $(X, *)$ as in the example 3.4, Let the fuzzy subsets $\mu_i : X \to [0, 1], i = 1, 2, 3, 4$ be given by

$$\mu_1(x) = \begin{cases} .5 & if\ x = 0 \\ .1 & if\ x = 1,2 \\ .4 & if\ x = 3,4 \end{cases} \qquad \mu_2(x) = \begin{cases} .3 & if\ x = 0 \\ 0 & if\ x = 1,2 \\ .2 & if\ x = 3,4 \end{cases}$$

$$\mu_3(x) = \begin{cases} .6 & if\ x = 0 \\ .2 & if\ x = 1,2 \\ .4 & if\ x = 3,4 \end{cases} \qquad \mu_4(x) = \begin{cases} .7 & if\ x = 0 \\ .4 & if\ x = 1,2 \\ .5 & if\ x = 3,4 \end{cases}$$

Then the collection $T = \{\phi, X, \mu_1, \mu_2, \mu_3\}$ is a Fuzzy Topology on X.

Hence (X, T) is a fuzzy topological system. Choose $A = \mu_4$. Then $T_A = \{\phi, \mu_1, \mu_2, \mu_3, \}$ and $A = (\mu_4, T_A)$ is fuzzy topological subsystem of(X, T).
Consider the TM-algebra $(Y, *)$ as in the example 3.4. Let the fuzzy subsets $\sigma_i : Y \to [0, 1], i = 1, 2, 3, 4, 5, 6$ be given by

$$\sigma_1(y) = \begin{cases} .3 & if\ y = 0 \\ .2 & if\ y = b \\ 0 & if\ y = a, c \end{cases} \qquad \sigma_2(y) = \begin{cases} .5 & if\ y = 0 \\ .4 & if\ y = b \\ .1 & if\ y = a, c \end{cases}$$

$$\sigma_3(y) = \begin{cases} .6 & if\ y = 0 \\ .4 & if\ y = b \\ .2 & if\ y = a, c \end{cases} \qquad \sigma_4(y) = \begin{cases} .7 & if\ y = 0 \\ .4 & if\ y = b \\ .3 & if\ y = a, c \end{cases}$$

$$\sigma_5(y) = \begin{cases} .7 & if\ y = 0 \\ .5 & if\ y = b \\ .4 & if\ y = a, c \end{cases} \qquad \sigma_6(y) = \begin{cases} .8 & if\ y = 0 \\ .7 & if\ y = b \\ .5 & if\ y = a, c \end{cases}$$

Then the collection $U = \{\phi, Y, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\}$ is a Fuzzy Topology on Y. Hence (Y, U) is a fuzzy topological system. Choose $B = \sigma_6$
Then $T_A = \{\phi, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\}$ and $B = (\sigma_6, U_B)$ is fuzzy topological subsystem of(Y, U).
Let $f : X \to Y$ be given by,$f(0) = 0, f(1) = a, f(2) = c, f(3) = b, f(4) = b$. Then $f^{-1}(0) = 0, f^{-1}(a) = 1, f^{-1}(c) = 2, f^{-1}(b) = 3, f^{-1}(b) = 4$.

$$f(A) \subset B \quad f(\mu_1) = \sigma_2 \quad f(\mu_2) = \sigma_1 \quad f(\mu_3) = \sigma_3$$

Hence the image of each T_A-open fuzzy set is U_B-open and hence the function f is relatively Fuzzy Open.

Theorem 3.12. Let $(X, *)$,$(Y, *)$ be two TM-Algebras. Let (X, T),(Y, U) be two fuzzy topological systems .Let $(A, T_A), (B, U_B)$ be the fuzzy topological subsystems on X and Y. Let f be a fuzzy continuous mapping of (X, T) in to (Y, U) such that $f(A) \subset B$. Then f is a relatively fuzzy continuous mapping of (A, T_A) in to (B, U_B).

Proof: Let $\sigma' \in U_B$
Then there exists $\sigma \in U$ such that $\sigma' = \sigma \cap B$
Hence

$$f^{-1}(\sigma') \cap A = f^{-1}(\sigma \cap B) \cap A = (f^{-1}(\sigma) \cap f^{-1}(B)) \cap A = f^{-1}(\sigma) \cap A \because f(A) \subset B$$

Since $\sigma \in U, f$ is fuzzy continuous and $f^{-1}(\sigma) \in T$
Therefore $f^{-1}(\sigma) \cap A$ is open in T_A, proving that $f^{-1}(\sigma') \cap A$ is open in T_A.
Therefore f is relatively fuzzy continuous mapping of (A, T_A) into (B, U_B).

Theorem 3.13. Let f be a fuzzy continuous mapping of a fuzzy topological system on a TM-algebra (X, T) in to (Y, U). Let g be a mapping of (Y, U) in to a fuzzy topological system on a TM-algebra (Z, V). Then the composition mapping $g \circ f$ is a fuzzy continuous mapping of (X, T) in to (Z, V).

Proof : Since the functions f,g are fuzzy continuous, by definition 3.3 , the inverse image of each U - open fuzzy set σ of U is T - open.

The inverse image of each open fuzzy set χ of V is an U-open fuzzy set.

$$(g \circ f)^{-1}(\chi) = (f^{-1} \circ g^{-1})(\chi) = f^{-1}(g^{-1}(\chi)) = f^{-1}(\sigma)$$

Since $f^{-1}(\sigma)$ is T-open fuzzy set, $(g \circ f)^{-1}(\chi)$ is T-open fuzzy set.

Hence $(g \circ f)$ is fuzzy continuous.

Theorem 3.14. Let $(A, T_A), (B, U_B), (C, V_C)$ be fuzzy topological subsystems on TM-algebras X, Y *and* Z respectively.Let f,g be the relatively fuzzy continuous mappings of (A, T_A) in to (B, U_B) and (B, U_B) in to (C, V_C). Then the composition $g \circ f$ is a relatively fuzzy continuous mapping of (A, T_A) in to (C, V_C)

Proof:

Let $\chi' \in V_C$. Then there exists $\chi \in V$ such that $\chi' = \chi \cap C$

Since g is relatively fuzzy continuous from (B, U_B) into (C, V_C), by definition 3.8 , $g^{-1}(\chi') \cap B$ is open in U_B.

Since f is relatively fuzzy continuous from (A, T_A) in to (B, U_B), by definition 3.8 , $f^{-1}((g^{-1}(\chi') \cap B)) \cap A$ is open in T_A.

But $(g \circ f)^{-1}(\chi') \cap A = f^{-1}(g^{-1}(\chi') \cap B) \cap A$ implying that $(g \circ f)^{-1}(\chi') \cap A$ is open in T_A.

Since $f(A) \subset B$, $g \circ f$ is relatively fuzzy continuous.

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