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FUZZY GENERALIZED SEMI GENERALIZED CLOSED SETS

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Abstract: In this paper we introduce a new class of fuzzy set called fuzzy generalized semi generalized closed sets and its characterizations.Besides, we discuss fuzzy gsg-closure and fuzzy gsg-interior with its properties. As an application this set we also introduce fuzzy Tgsg-space. Further, we introduce Fgsg-continuity and Fgsg-irresolute mappings with some of its properties.

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1. Introduction

The usual notation of fuzzy set have been generalized with the introduction of fuzzy sets by Zadeh in his classical paper [22] of 1965. The concepts of fuzzy topological spaces have been introduced and developed by C.L.Chang[5]

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and since then several authors have extended various notions in classical topology to fuzzy topological spaces. The extensions of functions in fuzzy setting can very interestingly and effectively be carried out by the concept of quasi-coincidence and q-neighbourhoods introduced by Pu and Liu [13]. H.Maki et al [9] introduced the concept of generalized closed set in a fuzzy topological space . The concepts Fgα-closedsets, Fuzzy generalized semipre closed sets, Fuzzy gs-closed sets, Fuzzy sg-closed sets and Fuzzy semipre generalized closed sets have been investigated in [14-18]. The concept generalized sg-closed sets have been introduced and studied by Lellis et al [8] in classical topology. The purpose of this paper is to extend the notion of generalized sg-closed sets in fuzzy topological spaces. Section 3 is devoted to introduce the fuzzy version of generalized sg-closed sets, generalized sg-open sets and to study some of its properties. In Section 4, we study several interesting characterizations of fuzzy generalized sg-closed sets and fuzzy generalized sg-open sets. We also introduce fuzzy gsg-closure and fuzzy gsg-interior and obtain some of its properties in Section 5. As an application of fuzzy generalized sg-closed set, we introduce fuzzy Tgsg-space in Section 6. In Section 7, we introduce Fgsg-continuous and Fgsg-irresolute mappings by using fuzzy generalized sg-closed set and study some of their fundamental properties.

2. Preliminaries

By a fuzzy topological space we shall mean non empty set X together with fuzzy topology τ[in the sence of Chang] and denote it by (X,τ). Throughout this paper, (X, τ), (Y, σ) and (Z,η) (or simply X, Y and Z) always mean fuzzy topological spaces.For a fuzzy set A of (X,τ), Cl(A), Int(A) and $1 - A$ denote fuzzy closure, fuzzy interior and fuzzy complement of A respectively. The fuzzy semiclosure (resp. fuzzy α-closure , fuzzy semi-preclosure) of a fuzzy set A of (X,τ) is the intersection of all Fs-closed (resp. Fα-closed, Fsp-closed) sets that contain A and is denoted by sCl(A) (resp. α Cl(A) and spCl(A)).

Definition 2.1. *A fuzzy set A of (X,τ) is called:*

1. *Fuzzy semiopen (in short, Fs-open) if A $\leq$ Cl(Int(A)) and a fuzzy semiclosed (in short, Fs-closed) if Int(Cl(A)) $\leq$ A [1];*
2. *Fuzzy preopen (in short, Fp-open) if A $\leq$ Int(Cl(A)) and a fuzzy preclosed (in short,Fp-closed) if Cl(Int(A)) $\leq$ A [4] ;*
3. *Fuzzy α-open (in short, Fα-open) if A $\leq$ Int (Cl(Int(A))) and a fuzzy α-closed (in short, Fα-closed) if Cl (Int(Cl(A))) $\leq$ A [4] ;*

4. *Fuzzy semi-preopen (in short, Fsp-open) if $A \leq Cl\ (Int(Cl(A)))$ and a fuzzy semi- preclosed (in short, Fsp-closed) if $Int(Cl(Int(A))) \leq A$ [22] .*

Lemma 2.2. *[11] Let A be a fuzzy set in a fuzzy toplogical space (X, τ). Then*

1. *$spCl(A) \leq sCl(A) \leq \alpha Cl(A) \leq Cl(A) \leq rCl(A)$*
2. *$spCl(A) \leq pCl(A) \leq \alpha Cl(A)$*

Definition 2.3. *A fuzzy set A of (X,τ) is called:*

1. *Fuzzy generalized closed (in short, Fg-closed) [2] if $Cl(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X;*
2. *Fuzzy generalized semiclosed (in short, Fsg-closed) [17] if $sCl(A) \leq H$, whenever $A \leq H$ and H is Fs-open set in X. In[6], Hakeim called this set as Generalized fuzzy weakly semiclosed set;*
3. *Fuzzy generalized semiclosed (in short,Fgs-closed)[16] if $sCl(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X;*
4. *Fuzzy α-generalized closed (in short,Fαg-closed)[12] if $\alpha Cl(A) \leq H$, whenever $A \leq H$ and H is Fα-open set in X;*
5. *Fuzzy generalized α-closed (in short,Fgα-closed)[12] if $\alpha Cl(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X;*
6. *Fuzzy generalized semi-preclosed (in short, Fgsp-closed) [10] if $spCl(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X;*
7. *Fuzzy gegneralized pre closed set (in short, Fgp-closed) [7] if $pCl(A) \leq H$, whenever $A \leq H$ and H is fuzzy open set in X;*
8. *Fuzzy ω-closed(in short, Fω-closed) [19] if $Cl(A) \leq H$, whenever $A \leq H$ and H is Fs-open set in X.*

Definition 2.4. *A fuzzy topological space (X,τ) is called a*

1. *Fuzzy $T_{\frac{1}{2}}$ space [2] if every Fg-closed set in it is fuzzy closed.*
2. *Fuzzy T_ω space if every Fω-closed set in it is fuzzy closed.*
3. *Fuzzy T_b space if every Fgs-closed set in it is fuzzy closed.*

Definition 2.5. *A mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ is said to be:*

1. *Fg-continuous [2] if $f^{-1}(V)$ is fuzzy closed set in X, for every fuzzy closed set V in Y ;*

2. *Fsg-continuous [17] if $f^{-1}(V)$ is Fsg-closed in X, for each fuzzy closed set V in Y;*

3. *Fgsp-continuous [12] if $f^{-1}(V)$ is Fgsp-closed in X, for every fuzzy closed set V in Y ;*

Definition 2.6. *[10] A fuzzy point $x_\lambda \in A$ is said to be quasi-coincident (in short q-coincident) with the fuzzy set A is denoted by $x_\lambda qA$ if and only if $\lambda + A(x) > 1$. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by AqB if and only if there exists $x \in X$ such that $A(x) + B(x) > 1$. If the fuzzy sets A and B are not quasi-coincident then we write $A\bar{q}B$. A fuzzy set B is said to be a q-neibourhood (in short, q-nbd) of a fuzzy set A if there is a fuzzy open sets U with AqU $\leq$B.*

Lemma 2.7. *[21] Let A,B,C are fuzzy sets in (X, τ). Then $Aq(B \vee C)$ if and only if AqB or AqC.*

Definition 2.8. *[5] Let f be a mapping from X into Y. If A is a fuzzy set of X and B is a fuzzy set of Y , then*

1. *$f(A)$ is a fuzzy set of Y, where*

$$f(A) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} A(x), & \text{for } f^{-1}(y) \neq 0, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

2. *$f^{-1}(B)$ is fuzzy set of X, where $f^{-1}(B)(x) = B(f(x))$ for each $x \in X$.*

3. *$f^{-1}(1 - B) = 1 - f^{-1}(B)$.*

3. Fuzzy Generalized sg-Closed sets and Fuzzy Generalized sg-Open Sets

In this section we introduce and study the fuzzy generalized sg-closed sets, generalized sg-open sets and some of its properties.

Definition 3.1. *A fuzzy set A of (X,τ) is called a fuzzy generalized sg-closed set (in short, Fgsg-closed) if Cl(A) $\leq$ H whenever A $\leq$ H and H is Fsg-open in X.*

Proposition 3.2. *Every fuzzy closed set is Fgsg-closed.*

Proof. Let A be fuzzy closed set and H be any Fsg-open set such that A $\leq$ H. Since A is fuzzy closed, Cl(A) = A $\leq$ H. Hence A is Fgsg-closed. □

The reverse implication of the above proposition is not true as shown in the following example.

Example 3.3. *Let X = {a,b,c} and the fuzzy sets A, B and C from X to [0,1] be defined as A(a) = 0.0, A(b) = 0.0, A(c) = 0.4; B(a) = 0.9, B(b) = 0.6, B(c) = 0.0; C(a) = 1.0, C(b) = 0.7, C(c) = 1.0. Let τ = {0, A, B, A $\vee$ B, 1}. Then the set C is Fgsg-closed but not fuzzy closed in (X,τ).*

Proposition 3.4. *Every Fgsg-closed set is Fg-closed.*

Proof. Let A be any Fgsg-closed set and H be any fuzzy open set such that A $\leq$ H. Since every fuzzy open set is Fsg-open and A is Fgsg-closed, we have Cl(A)$\leq$ H. Hence A is Fg-closed. □

Proposition 3.5. *Every Fgsg-closed set is Fω-closed.*

Proof. Let A be any Fgsg-closed set and H be any Fs-open set such that A $\leq$ H. Since every fuzzy semi open set is Fsg-open and A is Fgsg-closed, we have Cl(A)$\leq$ H. Hence A is Fω-closed. □

Proposition 3.6. *Every Fgsg-closed set is Fgα-closed.*

Proof. Let A be any Fgsg-closed set and H be any fuzzy open set such that A $\leq$ H. Since every fuzzy open set is Fsg-open and A is Fgsg-closed, we have αCl(A) $\leq$ Cl(A) $\leq$ H. Hence A is Fgα-closed. □

Proposition 3.7. *Every Fgsg-closed set is Fαg-closed.*

Proof. Let A be any Fgsg-closed set and H be any Fα-open set such that A $\leq$ H. Since every Fα-open set is fuzzy semi open set which is Fsg-open and A is Fgsg-closed, we have αCl(A) $\leq$ Cl(A) $\leq$ H. Hence A is Fαg-closed. □

Proposition 3.8. *Every Fgsg-closed set is Fsg-closed and Fsp-closed.*

Proof. Let A be any Fgsg-closed set and H be any Fs-open set such that A $\leq$ H. Since every fuzzy semi open set is Fsg-open and A is Fgsg-closed, we have sCl(A)$\leq$ Cl(A) $\leq$ H. Hence A is Fsg-closed. Since every Fsg-closed set is Fsp-closed, A is Fsp-closed. □

Proposition 3.9. *Every Fgsg-closed set is Fgs-closed,Fgsp-closed and Fgp-closed.*

Proof. Let A be any Fgsg-closed set and H be any fuzzy open set such that A $\leq$ H. Since every fuzzy open set is Fsg-open and A is Fgsg-closed, we have sCl(A) $\leq$ Cl(A)$\leq$ H. Hence A is Fgs-closed.
Similarly we have spCl(A)$\leq$ Cl(A)$\leq$ H and pCl(A)$\leq$ Cl(A) $\leq$ H. Hence A is Fgsp-closed and Fgp-closed . □

The following example serves the reverse implications of the above propositions are not true.

Example 3.10. *Let X = {a,b,c} and the fuzzy sets A, B, C,D and E from X to [0,1] be defined as A(a) = 0.7, A(b) = 0.3, A(c) = 1.0; B(a) = 0.7, B(b) = 0.0, B(c) = 0.0 ; C(a) = 0.9, C(b) = 0.2, C(c) = 0.1; D(a) = 0.2, D(b) = 0.7, D(c) = 0.0; E(a) = 0.2, E(b) = 0.7, E(c) = 0.2. Let* $\tau = \{0, A, B, 1\}$*. Then C is Fg-closed and hence Fg*α*-closed, Fgp-closed, Fgs-closed and Fgsp-closed, but not Fgsg-closed in (X,* τ*). The fuzzy set D is F*ω*-closed but not Fgsg-closed in (X,* τ*). And the set E is F*α*g-closed and hence Fsg-closed and Fsp-closed but not Fgsg-closed in (X,* τ*).*

Definition 3.11. *A fuzzy set A of a fuzzy toplogical space (X,*τ*) is called Fgsg-open set if and only if 1 − A is Fgsg-closed.*

Proposition 3.12. *Every fuzzy open set is Fgsg-open.*

Proof. Let A be any fuzzy open set. Let H be any Fsg-open set such that 1−A $\leq$ H. Since A is fuzzy open ,we have Cl(1 −A) = 1 − Int(A) = 1− A $\leq$ H. This gives 1−A is Fgsg-closed and hence A is Fgsg-open . □

Proposition 3.13. *Every Fgsg-open set is Fg-open and F*ω*-open.*

Proof. Let A be any Fgsg-open set. Then 1− A is Fgsg-closed. By Propositions 3.4 and 3.5, 1− A is Fg-closed and Fω-closed. Hence A is Fg-open and Fω-open. □

Proposition 3.14. *Every Fgsg-open set is Fgs-open, Fsg-open, Fsp-open, Fgsp-open, Fg*α*-open and F*α*g-open.*

Proof. Similar to above Proposition. □

4. Characterization of Fgsg-Closed Sets and Fgsg-Open Sets

In this section we study several interesting characterizations of fuzzy generalized sg-closed sets and fuzzy generalized sg-open sets.

Definition 4.1. *A fuzzy set A in (X,τ) is called Fgsg-nhd of a fuzzy point x_λ if there exixts a Fgsg-open set B such that $x_\lambda \in B \leq A$. A Fgsg-nhd A is said to be Fgsg-open-nhd (resp.Fgsg-closed-nhd) if and only if A is Fgsg-open(resp.Fgsg-closed). A fuzzy set A in (X,τ) is called fuzzy gsg-q-nhd of a fuzzy point x_λ (resp.fuzzy set B), if there exists a Fgsg-open set U in (X,τ) such that $x_\lambda qU \leq A$ (resp.$BqU \leq A$).*

Theorem 4.2. *If A and B are Fgsg-closed sets in (X,τ) then A $\vee$B is Fgsg-closed.*

Proof. Let A and B be two fuzzy Fgsg-closed sets in (X,τ) and let H be any Fsg-open set such that A $\leq$ H and B $\leq$ H. Therefore we have Cl(A) $\leq$ H and Cl(B) $\leq$ H. Since A $\leq$ H and B $\leq$ H, wehave A $\vee$ B $\leq$ H. Now Cl(A $\vee$ B) = Cl(A) $\vee$ Cl(B) $\leq$ H. Hence A $\vee$ B is Fgsg-closed. □

Theorem 4.3. *If A and B are Fgsg-open sets in (X,τ) then A $\wedge$ B is Fgsg-open.*

Proof. Let A and B be two fuzzy Fgsg-open sets in (X,τ). Then 1−A and 1−B are Fgsg-closed. By above Theorem 4.2, (1−A)$\vee$ (1−B) is Fgsg-closed. Since (1−A) $\vee$ (1−B) = 1−(A $\wedge$B). Hence A $\wedge$ B is Fgsg-open. □

Theorem 4.4. *If a fuzzy set A is Fgsg-closed in (X,τ) and Cl(A)$\wedge$ (1 − Cl(A)) = 0 then Cl(A)− A does not contain any non-zero Fsg-closed set in (X,τ).*

Proof. Let A be Fgsg-closed in (X,τ) and Cl(A) $\wedge$ (1−Cl(A)) = 0. We prove the result by contradiction. Let B be any Fsg-closed set in (X,τ) such that B $\leq$ Cl(A) −A and B $\neq$ 0. This gives B $\leq$ Cl(A) and B $\leq$ 1−A. We have A $\leq$ 1− B, which is Fsg-open. Since A is Fgsg-closed, we have Cl(A) $\leq$ 1−B. This implies B $\leq$ 1−Cl(A). Therefore B $\leq$Cl(A) $\wedge$ 1−Cl(A) = 0. That is B = 0, which is a contradiction. Hence Cl(A)− A does not contain any non-zero Fsg-closed set in (X,τ). □

Theorem 4.5. *If a fuzzy set A is Fgsg-closed in (X,τ) and Cl(A)$\wedge$ (1$-$Cl(A)) = 0 then Cl(A)$-$A does not contain any non-zero fuzzy closed set in (X,τ).*

Proof. It follows from the above theorem and the fact that every fuzzy closed set is Fsg-closed. □

Theorem 4.6. *If A is Fsg-open and Fgsg-closed in (X,τ) then A is fuzzy closed in (X,τ).*

Proof. Since A $\leq$ A and A is Fsg-open and Fgsg-closed, we have Cl(A) $\leq$ A. Since A $\leq$ Cl(A), we have A = Cl(A). Hence A is fuzzy closed. □

Theorem 4.7. *A fuzzy set A of (X,τ) is Fgsg-closed if and only if $A\bar{q}U$ $\Rightarrow$ Cl(A)$\bar{q}U$, for every F_{sg}-closed set U of (X,τ).*

Proof. (Necessity) Let U be Fsg-closed set and $A\bar{q}U$. Then A $\leq$ 1$-$U . Since A is Fgsg-closed and 1$-$U is Fsg-open, we have Cl(A)$\leq$ 1$-$U. Hence Cl$(A)\bar{q}U$. (Sufficiency) Let H be Fsg-open set such that A $\leq$ H. By hypothesis $A\bar{q}(1-H)$ $\Rightarrow$ Cl$(A)\bar{q}(1-H)$, as 1$-$H is Fsg-closed. Then Cl(A) $\leq$ H. Hence A is Fgsg-closed. □

Theorem 4.8. *If A is Fgsg-closed set in (X,τ) and A $\leq$ B $\leq$ Cl(A) then B is Fgsg-closed in (X,τ).*

Proof. Let H be Fsg-open set such that B $\leq$ H. Since A $\leq$ B, we have A $\leq$ H. Since A is Fgsg-closed set, Cl(A) $\leq$ H. But B $\leq$ Cl(A) implies Cl(B) $\leq$ Cl(Cl(A)) = Cl(A)$\leq$ H. Hence B is Fgsg-closed. □

Theorem 4.9. *If A is Fgsg-open set in (X,τ) and Int(A) $\leq$ B $\leq$ A, then B is Fgsg-open in (X,τ).*

Proof. Let A is Fgsg-open set in (X,τ) and Int(A)$\leq$ B $\leq$ A. Then 1$-$A is Fgsg-closed and $1-A \leq 1-B \leq$ Cl(1$-$A). Then by theorem 4.8, 1$-$B is Fgsg-closed. Hence B is Fgsg-open. □

Theorem 4.10. *A fuzzy set A is Fgsg-open if and only if F $\leq$ Int(A) where F is Fgsg-closed and F $\leq$ A.*

Proof. Let F $\leq$ Int(A) where F is Fgsg-closed and F $\leq$ A. Then $1-A \leq 1-F$ and $1-F$ is Fsg-open. Now Cl($1-A$) = 1−Int(A)$\leq 1-F$, by hypothesis. Then $1-A$ is Fgsg-closed. Hence A is Fgsg-open.
Conversely, let A is Fgsg-open and F is Fsg-closed and F $\leq$ A. Then $1 - A \leq 1-F$. Since $1-A$ is Fgsg-closed and $1-F$ is Fsg-open, we have Cl($1-A$) $\leq 1-F$. Then F $\leq$ Int(A). □

Theorem 4.11. *If A be a Fgsg-closed set in (X,τ) and x_λ be a fuzzy point of X such that $x_\lambda q$Cl(A)then Cl(x_λ)qA.*

Proof. Suppose Cl(x_λ)$\bar{q}$A then A $\leq$ 1−Cl(x_λ). Since 1−Cl(x_λ) is Fsg-open and A is Fgsg-closed, we have Cl(A) $\leq$ 1−Cl(x_λ) = $1-x_\lambda$. This gives $x_\lambda\bar{q}$Cl(A), a contradiction. Hence Cl(x_λ)qA.

□

5. Fgsg-Closure and Fgsg-Interior

We introduce fuzzy gsg-closure and fuzzy gsg-interior and obtain some of its properties in this section.

Fuzzy gsg-closure and fuzzy gsg-interior of fuzzy set A in fuzzy topological space (X,τ) is denoted by gsg-Cl(A) and gsg-Int(A) respectively and defined as follows:

Definition 5.1. *Let A be any fuzzy set in (X,τ) then we define Fgsg-Closure and Fgsg-Interior as*

gsg-Cl(A) $= \wedge\{B : B$ *is Fgsg-closed and* $B \geq A\}$,
gsg-Int(A) $= \vee\ \{B : B$ *is Fgsg-open and* $B \leq A\}$.

It is evident that

1. gsg-Cl(A) = A if and only if A is Fgsg-closed.
2. gsg-Int(A) = A if and only if A is Fgsg-open.
3. gsg-Cl(A) is the smallest fuzzy set containing A.
4. gsg-Int(A) is the largest fuzzy set contained in A.

Theorem 5.2. *Let x_λ and A be a fuzzy point and fuzzy set respectively in (X,τ). Then $x_\lambda \in$ gsg-Cl(A) if and only if every fuzzy gsg-q-nhd of x_λ is q-coincident with A.*

Proof. We prove by contradiction. Let $x_\lambda \leq$ gsg-Cl(A). Suppose there exists a gsg-q-nhd U of x_λ such that U$\bar{q}$A. Since U is gsg-q-nhd of x_λ, there exists Fgsg-open set V in (X,τ) such that x_λqV $\leq$ U whish gives that V$\bar{q}$A and hence A $\leq$ 1− V. Then gsg-Cl(A) $\leq$ 1− V, as 1− V is Fgsg-closed. Since $x_\lambda \notin$ 1−V , we have $x_\lambda \notin$ gsg-Cl(A), a contradiction. Hence every fuzzy gsg-q-nhd of x_λ is q-coincident with A.
Conversely suppose $x_\lambda \notin$ gsg-Cl(A). Then There exists a Fgsg-closed set B such that A $\leq$ B and $x_\lambda \notin$ B.Then we have x_λq(1−B) and A$\bar{q}$(1−B), a contradiction. Hence $x_\lambda \in$gsg-Cl(A). □

Properties 5.3. *Let A be any fuzzy set in (X,τ). Then*
gsg-Int(1− A) = 1 −(gsg-Cl(A)).
gsg-Cl(1 −A) = 1 − (gsg-Int(A)).

Proof. (i)By definition, gsg-Cl(A) = ∧{ B : B is Fgsg-closed and $B \geq A$}
1−gsg-Cl(A) = 1 −∧ { B : B is Fgsg-closed and B≥A}
= ∨{1− B : B is Fgsg-closed and B $\geq A$}
= ∨{U : U is Fgsg-open and U $\leq$ 1−A}
= gsg-Int(1− A)
(ii)The proof is similar to (i) □

Properties 5.4. *If A and B are Fuzzy sets in (X,τ). Then the following are true.*

1. $gsg - Cl(0) = 0, gsg - Cl(1) = 1.$
2. $gsg - Cl(A)$ *is* F_{gsg}*-closed in* (X, τ).
3. $gsg - Cl(A) \leq gsg - Cl(B)$ *when* $A \leq B$.
4. UqA *if and only if* $Uqgsg - Cl(A)$, *when* U *is* F_{gsg}*-open set in* (X, τ).
5. $gsg - Cl(A) = gsg - Cl(gsg - Cl(A))$.
6. $gsg - Cl(A \wedge B) \leq gsg - Cl(A) \wedge gsg - Cl(B)$.
7. $gsg - Cl(A \vee B) = gsg - Cl(A) \vee gsg - Cl(B)$.

Proof. 1. and

2. are obvious.

3. Let $x_\lambda \notin$ gsg-Cl(B).Then by above theorem 5.2, there exist fuzzy gsg-q-nhd V of x_λ such that V$\bar{q}$B. Since V is fuzzy gsg-q-nhd V of x_λ there exists fuzzy open set U such that x_λqU$\leq$ V. This gives U$\bar{q}$B. Since A $\leq$ B, then U$\bar{q}$A. Then by above theorem 5.2, $x_\lambda \notin$ gsg-Cl(A).Hence gsg-Cl(A)$\leq$ gsg-Cl(B).

4. Let U be any Fgsg-open set in (X,τ).Suppsoe U$\bar{q}$A, then A $\leq$ 1$-$U.Since 1$-$U is Fgsg-closed and by (iii), gsg-Cl(A) $\leq$ gsg-Cl(1$-U$) = 1$-$U. Thus U$\bar{q}$ gsg-Cl(A). Conversely, Let U$\bar{q}$ gsg-Cl(A). Then gsg-Cl(A) $\leq$ 1$-$ U. Since A $\leq$ gsg-Cl(A), we have A $\leq$ 1$-$ U. Thus U$\bar{q}$A. Hence UqA if and only if Uq gsg-Cl(A).

5. Since gsg-Cl(A) $\leq$ gsg-Cl(gsg-Cl(A)), it is enough to prove gsg-Cl(gsg-Cl(A))$\leq$ gsg-Cl(A). Let $x_\lambda \notin$ gsg-Cl(A). Then by theorem 5.2, there exist fuzzy gsg-q-nhd V of x_λ such that V$\bar{q}$A and so there is a Fgsg-open set U in (X,τ) such that x_λqU $\leq$ V and U$\bar{q}$A. By (iv), U$\bar{q}$ gsg-Cl(A).Then by theorem 5.2 $x_\lambda \notin$ gsg-Cl(gsg-Cl(A)). Hence gsg-Cl(A) = gsg-Cl(gsg-Cl(A)).

6. Since A$\wedge$B $\leq$ A and A$\wedge$B $\leq$ B, gsg-Cl(A$\wedge$B) $\leq$ gsg-Cl(A) and gsg-Cl(A$\wedge$B) $\leq$gsg-Cl(B). Hence gsg-Cl(A $\wedge$B) $\leq$ gsg-Cl(A)$\wedge$ gsg-Cl(B).

7. Since A $\leq$ A$\vee$B and B $\leq$ A$\vee$B, gsg-Cl(A) $\leq$ gsg-Cl(A$\vee$B) and gsg-Cl(B) $\leq$ gsg-Cl(A$\vee$B). Then gsg-Cl(A)$\vee$gsg-Cl(B) $\leq$ gsg-Cl(A$\vee$ B).
   Conversely,$x_\lambda \in$ gsg-Cl(A$\vee$ B). Then by theorem 5.2, there exist fuzzy gsg-q-nhd U of x_λ such that Uq(A$\vee$B). By Lemma 2.7, either UqA or UqB. Then by theorem 5.2,$x_\lambda \in$ gsg-Cl(A) or $x_\lambda \in$ gsg-Cl(B).That is $x_\lambda \in$ gsg-Cl(A)$\vee$ gsg-Cl(B).Then gsg-Cl(A$\vee$B)$\leq$ gsg-Cl(A)$\vee$gsg-Cl(B). Hence gsg-Cl(A$\vee$ B) = gsg-Cl(A)$\vee$gsg-Cl(B). □

Properties 5.5. *If A and B are Fuzzy sets in (X,τ). Then the following are true.*

1. *gsg-Int(0) = 0, gsg-Int(1) = 1.*
2. *gsg-Int(A) is Fgsg-open in (X,τ).*
3. *gsg-Int(A)$\leq$ gsg-Int(B) when A $\leq$ B.*
4. *gsg-Int(A) = gsg-Int(gsg-Int(A)).*
5. *gsg-Int(A$\vee$B) $\geq$gsg-Int(A)$\vee$gsg-Int(B).*

6. gsg-Int(A∧B) = gsg-Int(A)∧ gsg-Int(B).

Proof. Obvious. □

6. Fuzzy Tgsg Space

In this section we introduce fuzzy Tgsg-space as an application of fuzzy generalized sg-closed set.

Definition 6.1. *A fuzzy topological space (X,τ) is called a Fuzzy T_{gsg} Space if every Fgsg-closet set in it is fuzzy closed.*

Proposition 6.2. *Every fuzzy $T_{\frac{1}{2}}$ space is fuzzy Tgsg space.*

Proof. Let (X,τ) be a fuzzy $T_{\frac{1}{2}}$ space and let A be Fgsg-closed set in (X,τ).Then A is Fg-closed,by proposition 3.4. Since (X,τ) is $T_{\frac{1}{2}}$ space, A is fuzzy closed in (X,τ). Hence (X,τ) is fuzzy T_{gsg} space. □

Proposition 6.3. *Every fuzzy T_ω space is a fuzzy Tgsg space.*

Proof. Let (X,τ) be a fuzzy T_ω space and let A be Fgsg-closed set in (X,τ).Then A is Fω-closed,by proposition 3.5. Since (X,τ) is T_ω space, A is fuzzy closed in (X,τ). Hence (X,τ) is fuzzy T_{gsg} space. □

Proposition 6.4. *Every fuzzy T_b space is a fuzzy Tgsg space.*

Proof. Let (X,τ) be a fuzzy T_b space and let A be Fgsg-closed set in (X,τ).Then A is Fgs-closed,by proposition 3.9. Since (X,τ) is T_b space, A is fuzzy closed in (X,τ). Hence (X,τ) is fuzzy T_{gsg} space. □

The following example shows that the converse of the above theorems is not true.

Example 6.5. *Let X = {a,b,c} and the fuzzy sets A and B from X to [0,1] be defined as A(a) = 0.7, A(b) = 0.3, A(c) = 1.0; B(a) = 0.7, B(b) = 0.0, B(c) = 0.0 ; Let τ= {0, A, B, 1}. Then (X, τ) is T_{gsg} but not $T_{\frac{1}{2}}$ space, T_ω space and T_b space.*

7. Fgsg-Continuous and Fgsg-Irresolute Mappings

We introduce Fgsg-continuous and Fgsg-irresolute mappings and study some of their fundamental properties in this section.

Definition 7.1. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is called fuzzy gsg-continuous (in short, Fgsg-continuous) if $f^{-1}(V)$ is Fgsg-closed in (X, τ) for every fuzzy closed set V of (Y, σ).*

Definition 7.2. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is called fuzzy gsg-irresolute(in short, Fgsg-irresolute) if $f^{-1}(V)$ is Fgsg-closed in (X, τ) for every Fgsg-closed set V of (Y, σ).*

Theorem 7.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be Fgsg-continuous. Then f is Fsg-continuous.*

Proof. Let V be a fuzzy closed set in (Y, σ). Since f is Fgsg-continuous, $f^{-1}(V)$ is Fgsg-closed in (X, τ). By proposition 3.8, $f^{-1}(V)$ is Fsg-closed in (X, τ). Hence f is Fsg-continuous. □

The converse of the above theorem is not true in general. For,

Example 7.4. *Let $X = \{a, b\}, Y = \{x, y\}$. Fuzzy sets A is defined as $A(a) = 0.4, A(b) = 0.6$. Let $\tau = \{0, A, 1\}$ and $\sigma = \{0, 1\}$. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = x, f(b) = y$ is Fgsg-continuous but not Fgsg-irresolute.*

Theorem 7.5. *Let$f : (X, \tau) \to (Y, \sigma)$ be Fgsg- continuous. Then f is Fgsp-continuous*

Proof. Let V be a fuzzy closed set in (Y, σ).Since f is Fgsg-continuous, $f^{-1}(V)$ is Fgsg-closed in (X, τ). By proposition 3.9, $f^{-1}(V)$ is Fgsp-closed in (X, τ). Hence f is Fgsp-continuous. □

The converse of the above theorem is not true in general. For,

Example 7.6. *Let $X = \{a, b\}, Y = \{x, y\}$. Fuzzy sets A and B are defined as $A(a) = 0.3, A(b) = 0.7; B(x) = 0.3, B(y) = 0.4$. Let $\tau = \{0, A, 1\}$ and $\sigma = \{0, B, 1\}$. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = x, f(b) = y$ is Fgsp-continuous but not Fgsg-continuous.*

Theorem 7.7. *Let $f : (X, \tau) \to (Y, \sigma)$ be Fgsg-continuous if and only if inverse image of each fuzzy open set of (Y, σ) is Fgsg-open in (X, τ).*

Proof. Let f be Fgsg-continuous. If V is any fuzzy open set in (Y,σ) then $f^{-1}(1-V) = 1 - f^{-1}(V)$ is Fgsg-closed. Hence $f^{-1}(V)$ is Fgsg-open in (X,τ). Conversely, Let V be a fuzzy closed set in (Y,σ). By hypothesis, $f^{-1}(1-V)$ is Fgsg-open in (X,τ).This gives $f^{-1}(V)$ is Fgsg-closed. Hence f is Fgsg-continuous. □

Theorem 7.8. *If $f : (X,\tau) \to (Y,\sigma)$ is F_{gsg}-continuous then for each fuzzy point x_λ of X and $A \in \sigma$ such that $f(x_\lambda) \in A$, there exists a F_{gsg}-open set B of X such that $x_\lambda \in B$ and $f(B) \leq A$.*

Proof. Let x_λ be a fuzzy point of X and A $\in \sigma$ such that $f(x_\lambda) \in A$. Take B $= f^{-1}(V)$. Since 1$-$ A is fuzzy closed in (Y,σ) and f is Fgsg-continuous, we have $f^{-1}(1-A) = 1 - f^{-1}(A)$ is Fgsg-closed in (X,τ). This gives B $= f^{-1}(A)$ is Fgsg-open in (X,τ) and $x_\lambda \in B$ and $f(B) = f(f^{-1}(A)) \leq A$. □

Theorem 7.9. *If $f : (X,\tau) \to (Y,\sigma)$ is Fgsg-continuous then for each fuzzy point x_λ of X and $A \in \sigma$ such that $f(x_\lambda)qA$, there exists a Fgsg-open set B of X such that $x_\lambda qB$ and $f(B) \leq A$.*

Proof. Let x_λ be a fuzzy point of X and $A \in \sigma$ such that $f(x_\lambda)qA$. Take $B = f^{-1}(A)$. By above thereon 7.10, B is Fgsg-open in (X,τ) and $x_\lambda qB$ and $f(B) = f(f^{-1}(A)) \leq$A. □

Theorem 7.10. *If $f : (X,\tau) \to (Y,\sigma)$ is F_{gsg}- continuous and $g : (Y,\sigma) \to (Z,\eta)$ is F_g-continuous and (Y,σ) is a fuzzy $T_{\frac{1}{2}}$ space. Then $gof : (X,\tau) \to (Z,\eta)$ is F_{gsg}-continuous.*

Proof. Let V be a fuzzy closed set in (Z,η). Since g is Fg-continuous and (Y,σ) is a fuzzy $T_{\frac{1}{2}}$ space, $g^{-1}(V)$ is fuzzy closed in (Y,σ). Since f is Fgsg- continuous,$f^{-1}(g^{-1}(V))$ is Fgsg-closed in (X,τ). Hence gof is Fgsg-continuous. □

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