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POINTWISE BINOMIAL APPROXIMATION TO THE GENERALIZED HYPERGEOMETRIC DISTRIBUTION

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Abstract: In this paper, we use the result in [2] and the w-function associated with the generalized hypergeometric random variable to give a pointwise bound for the point metric between the generalized hypergeometric distribution with parameters α, β and N and the binomial distribution with parameters $n = N-1$ and $p = 1 - q = \frac{\beta+1}{\alpha+\beta+2}$. With this bound, it is observed that the desired result gives a good binomial approximation when α is sufficiently large.

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1. Introduction

Let a non-negative integer-valued random variable X have the generalized hypergeometric distribution with parameters α, β and N. Its probability mass function is given by

$$\text{GH}_{\alpha,\beta,N}(x) = \frac{\binom{N-1}{x}\Gamma(N+\alpha-x)\Gamma(\beta+1+x)\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\alpha+\beta+N+1)},\ x = 0, ..., N-1,$$

where $N \in \mathbb{N} \setminus \{1\}, \alpha \geq 0$ and $\beta > -1$ and the mean and variance of X are

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$\mu = \frac{(N-1)(\beta+1)}{\alpha+\beta+2}$ and $\sigma^2 = \frac{(N-1)(\beta+1)(\alpha+1)(\alpha+\beta+N+1)}{(\alpha+\beta+2)^2(\alpha+\beta+3)}$, respectively [1]. We know that this distribution can be approximated by some appropriate discrete distributions if some conditions of their parameters are satisfied. In this case, Crosu [1] used Stein's method and the w-function associated with the generalized hypergeometric random variable to obtain a bound for the total variation distance between the generalized hypergeometric distribution and a Poisson distribution with mean $\mu = \frac{(N-1)(\beta+1)}{\alpha+\beta+2}$, where $\beta + 2 \geq N$. After that, Teerapabolarn [3] used the same tools to give a bound for the total variation distance between the generalized hypergeometric distribution and a binomial distribution with parameters $n = N-1$ and $p = \frac{\beta+1}{\alpha+\beta+2}$. In this paper, we extend the result in [3] to approximate the generalized hypergeometric probability function by determining an appropriate pointwise bound for the point metric $|\mathrm{GH}_{\alpha,\beta,N}(x) - \mathrm{B}_{n,p}(x)|$ when $x \in \{0, ..., n\}$, where $\mathrm{B}_{n,p}(x)$ is the binomial probability function with parameters n and p, which is in Section 2. The conclusion of this study is presented in the last section.

2. Result

The following lemma presents the w-function associated with the generalized hypergeometric random variable, which obtained from [1].

Lemma 2.1. *Let $w(X)$ be the w-function associated with the generalized hypergeometric random variable X. Then, we have the following:*

$$w(x) = \frac{(\beta + x + 1)(N - x - 1)}{(\alpha + \beta + 2)\sigma^2},\ x = 0, ..., N - 1. \tag{2.1}$$

The desired result of this study is a pointwise bound for the point metric between $\mathrm{GH}_{\alpha,\beta,N(x)}$ and $\mathrm{B}_{n,p}(x)$, which presents in the following theorem.

Theorem 2.1. *Let $n = N-1$ and $p = 1-q = \frac{\beta+1}{\alpha+\beta+2}$, for $x \in \{0, ..., N-1\}$, then we have*

$$|\mathrm{GH}_{\alpha,\beta,N}(x) - \mathrm{B}_{n,p}(x)| \leq \begin{cases} \frac{(1-q^{N-1})(N-2)(\alpha+1)}{(\alpha+\beta+2)(\alpha+\beta+3)} & \text{if } x = 0, \\ \min\left\{\frac{1-p^{N-1}}{x}, \frac{1-p^N-q^N}{Np}\right\} \frac{(N-1)(N-2)(\beta+1)}{(\alpha+\beta+2)(\alpha+\beta+3)} & \text{if } x > 0. \end{cases} \tag{2.2}$$

Proof. Because $(n-x)p - \sigma^2 w(x) = \frac{(\beta+1)(N-x-1)}{\alpha+\beta+2} - \frac{(\beta+x+1)(N-x-1)}{(\alpha+\beta+2)} \leq 0$ for every $0 \le x \le N-1$ and by following Corollary 3.1 in [2], we have that

$$|\mathrm{GH}_{\alpha,\beta,N}(x) - \mathrm{B}_{n,p}(x)| \le \begin{cases} \frac{1-q^n}{np}\left|\mu q - \sigma^2\right| & \text{if } x = 0, \\ \min\left\{\frac{1-p^n}{x_0 q}, \frac{1-p^{n+1}-q^{n+1}}{(n+1)pq}\right\}\left|\mu q - \sigma^2\right| & \text{if } x > 0. \end{cases}$$

Hence, by substituting these parameters, the inequality (2.2) is obtained. □

Corollary 2.1. *If $\beta = 0$, then, for $x \in \{0, ..., N-1\}$, we have*

$$|\mathrm{GH}_{\alpha,N}(x) - \mathrm{B}_{n,p}(x)| \le \begin{cases} \frac{(1-q^{N-1})(N-2)(\alpha+1)}{(\alpha+2)(\alpha+3)} & \textit{if } x = 0, \\ \min\left\{\frac{1-p^{N-1}}{x}, \frac{1-p^N-q^N}{Np}\right\}\frac{(N-1)(N-2)}{(\alpha+2)(\alpha+3)} & \textit{if } x > 0. \end{cases} \tag{2.3}$$

Remark. It can be seen that the result gives a good binomial approximation when p is small, or β is small and α is large, rather than p is large. Especially, in the case of $\beta = 0$, the bound is a good measurement of the accuracy of this approximation when $\frac{N}{\alpha}$ is small, or α is large.

3. Conclusion

In this study, a pointwise bound for the point metric between the generalized hypergeometric with parameters α, β and N and an appropriate binomial distribution with parameters $n = N-1$ and $p = \frac{\beta+1}{\alpha+\beta+2}$ was obtained. With this bound, it is observed that the binomial probability function can be used as an estimate of the generalized hypergeometric probability function when $\frac{N}{\alpha}$ is small, or α is sufficiently large.

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