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# A NOTE ON THE RELATIVE ORDER OF ENTIRE FUNCTIONS OF TWO COMPLEX VARIABLES

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**Abstract:** In this paper we discuss some growth rates of entire functions of two complex variables on the basis of the definition of relative order of entire functions of two complex variables as introduced by Banerjee and Dutta [2].

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**Key Words:** entire function of two complex variables, order, relative order, composition, growth

## 1. Introduction, Definitions and Notations

Let $f$ be an entire function of two complex variables holomorphic in the closed polydisc

$$U = \{(z_1, z_2) : \ |z_i| \leq r_i, \ i = 1, 2 \text{ for all } \ r_1 \geq 0, r_2 \geq 0\}$$

and $M_f(r_1, r_2) = \max\{|f(z_1, z_2)| : |z_i| \leq r_i, \ i = 1, 2\}$. Then in view of maxi-

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mum principal and Hartogs theorem {[3], p. 2, p. 51}, $M_f(r_1, r_2)$ is an increasing functions of $r_1, r_2$. We do not explain the standard definitions and notations of the theory of entire functions as those are available in [3].

The following definition is well known:

**Definition 1.** {[3], p. 339, (see also [1])} The *order* ${}_{v_2}\rho_f$ and the *lower order* ${}_{v_2}\lambda_f$ of an entire function $f$ of two complex variables are defined as

$$ {}_{v_2}\rho_f = \limsup_{r_1, r_2 \to \infty} \frac{\log^{[2]} M_f(r_1, r_2)}{\log(r_1 r_2)} \text{ and } {}_{v_2}\lambda_f = \liminf_{r_1, r_2 \to \infty} \frac{\log^{[2]} M_f(r_1, r_2)}{\log(r_1 r_2)} . $$

An entire function of two complex variables for which *order* and *lower order* are the same is said to be of *regular growth.* The function $\exp(z_1 z_2)$ is an example of *regular growth* of entire function of two complex variables. Further the functions which are not of *regular growth* are said to be of *irregular growth.*

Banerjee and Dutta [2] introduced the notion of *relative order* between two entire functions of two complex variables denoted by ${}_{v_2}\rho_g(f)$ as follows:

$$ \begin{aligned} {}_{v_2}\rho_g(f) &= \inf\{\mu > 0 : M_f(r_1, r_2) < M_g(r_1^{\mu}, r_2^{\mu}); r_1 \geq R(\mu), r_2 \geq R(\mu)\} \\ &= \limsup_{r_1, r_2 \to \infty} \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} \end{aligned} $$

where $g$ is also an entire function holomorphic in the closed polydisc

$$ U = \{(z_1, z_2) : \ |z_i| \leq r_i, \ i = 1, 2 \text{ for all } \ r_1 \geq 0, r_2 \geq 0\} $$

and the definition coincides with the classical one [2] if $g(z) = \exp(z_1 z_2)$.

Similarly, one can define the *relative lower order* of $f(z_1, z_2)$ with respect to $g(z_1, z_2)$ denoted by ${}_{v_2}\lambda_g(f)$ as follows :

$$ {}_{v_2}\lambda_g(f) = \liminf_{r \to \infty} \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} . $$

In this paper we wish to prove some results related to the growth rates of composite entire functions of two complex variables on the basis of *relative order* and *relative lower order* of entire functions of two complex variables as introduced by Banerjee and Dutta [2].

## 2. Theorems

In this section we present the main results of the paper.

**Theorem 1.** *Let $f$, $g$ and $h$ be any three entire functions of two complex variables such that ${}_{v_2}\rho_h(f) < \infty$ and ${}_{v_2}\lambda_h(f \circ g) = \infty$. Then*

$$\lim_{r_1,r_2\to\infty} \frac{\log M_h^{-1} M_{f\circ g}(r_1,r_2)}{\log M_h^{-1} M_f(r_1,r_2)} = \infty .$$

*Proof.* Let us suppose that the conclusion of the theorem do not hold. Then we can find a constant $\beta > 0$ such that for a sequence of values of $r_1, r_2$ tending to infinity,

$$\log M_h^{-1} M_{f\circ g}(r_1,r_2) \le \beta \log M_h^{-1} M_f(r_1,r_2) . \tag{1}$$

Again from the definition of ${}_{v_2}\rho_h(f)$, it follows for all sufficiently large values of $r_1, r_2$ that

$$\log M_h^{-1} M_f(r_1,r_2) \le ({}_{v_2}\rho_h(f) + \epsilon) \log(r_1 r_2) . \tag{2}$$

Thus from (1) and (2), we have for a sequence of values of $r_1, r_2$ tending to infinity that

$$\log M_h^{-1} M_{f\circ g}(r_1,r_2) \le \beta ({}_{v_2}\rho_h(f) + \epsilon) \log(r_1 r_2)$$

$$i.e., \; \frac{\log M_h^{-1} M_{f\circ g}(r_1,r_2)}{\log(r_1 r_2)} \le \frac{\beta ({}_{v_2}\rho_h(f) + \epsilon) \log(r_1 r_2)}{\log(r_1 r_2)}$$

$$i.e., \; \liminf_{r_1,r_2\to\infty} \frac{\log M_h^{-1} M_{f\circ g}(r_1,r_2)}{\log(r_1 r_2)} = {}_{v_2}\lambda_h(f\circ g) < \infty .$$

This is a contradiction.

Thus the theorem follows. □

**Remark 1.** Theorem 1 is also valid with "limit superior" instead of "limit" if ${}_{v_2}\lambda_h(f\circ g) = \infty$ is replaced by ${}_{v_2}\rho_h(f\circ g) = \infty$ and the other conditions remain the same.

**Corollary 1.** *Under the assumptions of Theorem 1 and Remark 1,*

$$\lim_{r_1,r_2\to\infty} \frac{M_h^{-1} M_{f\circ g}(r_1,r_2)}{M_h^{-1} M_f(r_1,r_2)} = \infty \text{ and } \limsup_{r_1,r_2\to\infty} \frac{M_h^{-1} M_{f\circ g}(r_1,r_2)}{M_h^{-1} M_f(r_1,r_2)} = \infty$$

*respectively hold.*

The proof is omitted.

Analogously one may also state the following theorem and corollaries without their proofs as those may be carried out in the line of Remark 1, Theorem 1 and Corollary 1 respectively.

**Theorem 2.** *Let $f$, $g$ and $h$ be any three entire functions of two complex variables with ${}_{v_2}\rho_h(g) < \infty$ and ${}_{v_2}\rho_h(f \circ g) = \infty$. Then*

$$\limsup_{r_1,r_2\to\infty} \frac{\log M_h^{-1} M_{f\circ g}(r_1,r_2)}{\log M_h^{-1} M_g(r_1,r_2)} = \infty$$

**Remark 2.** Theorem 2 is also valid with "limit" instead of "limit superior" if ${}_{v_2}\rho_{\mathrm{h}}(f \circ g) = \infty$ is replaced by ${}_{v_2}\lambda_{\mathrm{h}}(f \circ g) = \infty$ and the other conditions remain the same.

**Corollary 2.** *Under the assumptions of Theorem 2 and Remark 2,*

$$\limsup_{r_1,r_2\to\infty} \frac{M_h^{-1} M_{f\circ g}(r_1,r_2)}{M_h^{-1} M_g(r_1,r_2)} = \infty \text{ and } \lim_{r_1,r_2\to\infty} \frac{M_h^{-1} M_{f\circ g}(r_1,r_2)}{M_h^{-1} M_g(r_1,r_2)} = \infty$$

*respectively hold.*

**Theorem 3.** *Let $f$ and $g$ be any two entire functions of two complex variables. Then*

$$\frac{{}_{v_2}\lambda_f}{{}_{v_2}\rho_g} \leq {}_{v_2}\lambda_g(f) \leq \min\left\{\frac{{}_{v_2}\lambda_f}{{}_{v_2}\lambda_g}, \frac{{}_{v_2}\rho_f}{{}_{v_2}\rho_g}\right\} \leq \max\left\{\frac{{}_{v_2}\lambda_f}{{}_{v_2}\lambda_g}, \frac{{}_{v_2}\rho_f}{{}_{v_2}\rho_g}\right\} \leq {}_{v_2}\rho_g(f) \leq \frac{{}_{v_2}\rho_f}{{}_{v_2}\lambda_g}.$$

*Proof.* From definitions of ${}_{v_2}\rho_f$ and ${}_{v_2}\lambda_f$ we have for all sufficiently large values of $r_1, r_2$ that

$$M_f(r_1,r_2) \leq \exp\{({}_{v_2}\rho_f + \varepsilon)\log(r_1 r_2)\}, \tag{3}$$
$$M_f(r_1,r_2) \geq \exp\{({}_{v_2}\lambda_f - \varepsilon)\log(r_1 r_2)\} \tag{4}$$

and also for a sequence of values of $r_1, r_2$ tending to infinity we get that

$$M_f(r_1,r_2) \geq \exp\{({}_{v_2}\rho_f - \varepsilon)\log(r_1 r_2)\}, \tag{5}$$
$$M_f(r_1,r_2) \leq \exp\{({}_{v_2}\lambda_f + \varepsilon)\log(r_1 r_2)\}. \tag{6}$$

Similarly from the definitions of ${}_{v_2}\rho_g$ and ${}_{v_2}\lambda_g$, it follows for all sufficiently large values of $r_1, r_2$ that

$$M_g(r_1,r_2) \leq \exp\{({}_{v_2}\rho_g + \varepsilon)\log(r_1 r_2)\}$$

$$
\begin{aligned}
i.e.,\ \ (r_1 r_2) &\leq M_g^{-1}\left[\exp\left\{\left({}_{v_2}\rho_g+\varepsilon\right)\log\left(r_1 r_2\right)\right\}\right] \\
i.e.,\ M_g^{-1}\left(r_1,r_2\right) &\geq \exp\left[\frac{\log\left(r_1 r_2\right)}{\left({}_{v_2}\rho_g+\varepsilon\right)}\right]\ , \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
M_g\left(r_1,r_2\right) &\geq \exp\left\{\left({}_{v_2}\lambda_g-\varepsilon\right)\log\left(r_1 r_2\right)\right\} \\
i.e.,\ \ (r_1 r_2) &\geq M_g^{-1}\left[\exp\left\{\left({}_{v_2}\lambda_g-\varepsilon\right)\log\left(r_1 r_2\right)\right\}\right] \\
i.e.,\ M_g^{-1}\left(r_1,r_2\right) &\leq \exp\left[\frac{\log\left(r_1 r_2\right)}{\left({}_{v_2}\lambda_g-\varepsilon\right)}\right] \qquad (8)
\end{aligned}
$$

and for a sequence of values of $r_1, r_2$ tending to infinity we obtain that

$$
\begin{aligned}
M_g\left(r_1,r_2\right) &\geq \exp\left\{\left({}_{v_2}\rho_g-\varepsilon\right)\log\left(r_1 r_2\right)\right\} \\
i.e.,\ \ (r_1 r_2) &\geq M_g^{-1}\left[\exp\left\{\left({}_{v_2}\rho_g-\varepsilon\right)\log\left(r_1 r_2\right)\right\}\right] \\
i.e.,\ M_g^{-1}\left(r_1,r_2\right) &\leq \exp\left[\frac{\log\left(r_1 r_2\right)}{\left({}_{v_2}\rho_g-\varepsilon\right)}\right]\ , \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
M_g\left(r_1,r_2\right) &\leq \exp\left\{\left({}_{v_2}\lambda_g+\varepsilon\right)\log\left(r_1 r_2\right)\right\} \\
i.e.,\ \ (r_1 r_2) &\leq M_g^{-1}\left[\exp\left\{\left({}_{v_2}\lambda_g+\varepsilon\right)\log\left(r_1 r_2\right)\right\}\right] \\
i.e.,\ M_g^{-1}\left(r_1,r_2\right) &\geq \exp\left[\frac{\log\left(r_1 r_2\right)}{\left({}_{v_2}\lambda_g+\varepsilon\right)}\right]\ . \qquad (10)
\end{aligned}
$$

Now from (5) and in view of (7), we get for a sequence of values of $r_1, r_2$ tending to infinity that

$$
\log M_g^{-1}M_f\left(r_1,r_2\right) \geq \log M_g^{-1}\left[\exp\left\{\left({}_{v_2}\rho_f-\varepsilon\right)\log\left(r_1 r_2\right)\right\}\right]
$$

$$
i.e.,\ \ \log M_g^{-1}M_f\left(r_1,r_2\right) \geq \log\exp\left[\frac{\log\exp\left\{\left({}_{v_2}\rho_f-\varepsilon\right)\log\left(r_1 r_2\right)\right\}}{\left({}_{v_2}\rho_g+\varepsilon\right)}\right]
$$

$$
\begin{aligned}
i.e.,\ \ \log M_g^{-1}M_f\left(r_1,r_2\right) &\geq \frac{\left({}_{v_2}\rho_f-\varepsilon\right)}{\left({}_{v_2}\rho_g+\varepsilon\right)}\log\left(r_1 r_2\right) \\
i.e.,\ \ \frac{\log M_g^{-1}M_f\left(r_1,r_2\right)}{\log\left(r_1 r_2\right)} &\geq \frac{\left({}_{v_2}\rho_f-\varepsilon\right)}{\left({}_{v_2}\rho_g+\varepsilon\right)}\ .
\end{aligned}
$$

As $\varepsilon\,(>0)$ is arbitrary, it follows that

$$
\limsup_{r_1,r_2\to\infty}\frac{\log M_g^{-1}M_f\left(r_1,r_2\right)}{\log\left(r_1 r_2\right)} \geq \frac{{}_{v_2}\rho_f}{{}_{v_2}\rho_g}
$$

$$i.e.,\ {}_{v_2}\rho_g(f) \geq \frac{{}_{v_2}\rho_f}{{}_{v_2}\rho_g}\ . \tag{11}$$

Analogously, from (4) and in view of (10), it follows for a sequence of values of $r_1, r_2$ tending to infinity that

$$\log M_g^{-1} M_f(r_1, r_2) \geq \log M_g^{-1}\left[\exp\left\{\left({}_{v_2}\lambda_f - \varepsilon\right)\log(r_1 r_2)\right\}\right]$$

$$i.e.,\ \log M_g^{-1} M_f(r_1, r_2) \geq \log\exp\left[\frac{\log\exp\left\{\left({}_{v_2}\lambda_f - \varepsilon\right)\log(r_1 r_2)\right\}}{\left({}_{v_2}\lambda_g + \varepsilon\right)}\right]$$

$$i.e.,\ \log M_g^{-1} M_f(r_1, r_2) \geq \frac{\left({}_{v_2}\lambda_f - \varepsilon\right)}{\left({}_{v_2}\lambda_g + \varepsilon\right)}\log(r_1 r_2)$$

$$i.e.,\ \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} \geq \frac{\left({}_{v_2}\lambda_f - \varepsilon\right)}{\left({}_{v_2}\lambda_g + \varepsilon\right)}\ .$$

Since $\varepsilon\,(> 0)$ is arbitrary, we get from above that

$$\limsup_{r_1, r_2 \to \infty} \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} \geq \frac{{}_{v_2}\lambda_f}{{}_{v_2}\lambda_g}$$

$$i.e.,\ {}_{v_2}\rho_g(f) \geq \frac{{}_{v_2}\lambda_f}{{}_{v_2}\lambda_g}\ . \tag{12}$$

Again in view of (8) we have from (3) for all sufficiently large values of $r_1, r_2$ that

$$\log M_g^{-1} M_f(r_1, r_2) \leq \log M_g^{-1}\left[\exp\left\{\left({}_{v_2}\rho_f + \varepsilon\right)\log(r_1 r_2)\right\}\right]$$

$$i.e.,\ \log M_g^{-1} M_f(r_1, r_2) \leq \log\exp\left[\frac{\log\exp\left\{\left({}_{v_2}\rho_f + \varepsilon\right)\log(r_1 r_2)\right\}}{\left({}_{v_2}\lambda_g - \varepsilon\right)}\right]$$

$$i.e.,\ \log M_g^{-1} M_f(r_1, r_2) \leq \frac{\left({}_{v_2}\rho_f + \varepsilon\right)}{\left({}_{v_2}\lambda_g - \varepsilon\right)}\log(r_1 r_2)$$

$$i.e.,\ \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} \leq \frac{\left({}_{v_2}\rho_f + \varepsilon\right)}{\left({}_{v_2}\lambda_g - \varepsilon\right)}\ .$$

Since $\varepsilon\,(> 0)$ is arbitrary, we obtain that

$$\limsup_{r_1, r_2 \to \infty} \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} \leq \frac{{}_{v_2}\rho_f}{{}_{v_2}\lambda_g}$$

$$i.e.,\ {}_{v_2}\rho_g(f) \leq \frac{{}_{v_2}\rho_f}{{}_{v_2}\lambda_g}\ . \tag{13}$$

Again from (4) and in view of (7) we get for all sufficiently large values of $r_1, r_2$ that

$$\log M_g^{-1} M_f(r_1, r_2) \geq \log M_g^{-1}\left[\exp\left\{\left({}_{v_2}\lambda_f - \varepsilon\right)\log(r_1 r_2)\right\}\right]$$

$$i.e., \ \log M_g^{-1} M_f(r_1, r_2) \geq \log\exp\left[\frac{\log\exp\left\{\left({}_{v_2}\lambda_f - \varepsilon\right)\log(r_1 r_2)\right\}}{\left({}_{v_2}\rho_g + \varepsilon\right)}\right]$$

$$\begin{aligned} i.e., \ \log M_g^{-1} M_f(r_1, r_2) &\geq \frac{\left({}_{v_2}\lambda_f - \varepsilon\right)}{\left({}_{v_2}\rho_g + \varepsilon\right)}\log(r_1 r_2) \\ i.e., \ \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} &\geq \frac{\left({}_{v_2}\lambda_f - \varepsilon\right)}{\left({}_{v_2}\rho_g + \varepsilon\right)}. \end{aligned}$$

As $\varepsilon\,(> 0)$ is arbitrary, it follows from above that

$$\begin{aligned} \liminf_{r_1, r_2 \to \infty} \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} &\geq \frac{{}_{v_2}\lambda_f}{{}_{v_2}\rho_g} \\ i.e., \ {}_{v_2}\lambda_g(f) &\geq \frac{{}_{v_2}\lambda_f}{{}_{v_2}\rho_g}. \end{aligned} \tag{14}$$

Also in view of (9), we get from (3) for a sequence of values of $r_1, r_2$ tending to infinity that

$$\log M_g^{-1} M_f(r_1, r_2) \leq \log M_g^{-1}\left[\exp\left\{\left({}_{v_2}\rho_f + \varepsilon\right)\log(r_1 r_2)\right\}\right]$$

$$i.e., \ \log M_g^{-1} M_f(r_1, r_2) \leq \log\exp\left[\frac{\log\exp\left\{\left({}_{v_2}\rho_f + \varepsilon\right)\log(r_1 r_2)\right\}}{\left({}_{v_2}\rho_g - \varepsilon\right)}\right]$$

$$\begin{aligned} i.e., \ \log M_g^{-1} M_f(r_1, r_2) &\leq \frac{\left({}_{v_2}\rho_f + \varepsilon\right)}{\left({}_{v_2}\rho_g - \varepsilon\right)}\log(r_1 r_2) \\ i.e., \ \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} &\leq \frac{\left({}_{v_2}\rho_f + \varepsilon\right)}{\left({}_{v_2}\rho_g - \varepsilon\right)}. \end{aligned}$$

Since $\varepsilon\,(> 0)$ is arbitrary, we get from above that

$$\begin{aligned} \liminf_{r_1, r_2 \to \infty} \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log^{[q]} r} &\leq \frac{{}_{v_2}\rho_f}{{}_{v_2}\rho_g} \\ i.e., \ {}_{v_2}\lambda_g(f) &\leq \frac{{}_{v_2}\rho_f}{{}_{v_2}\rho_g}. \end{aligned} \tag{15}$$

Similarly from (6) and in view of (8), it follows for a sequence of values of $r_1, r_2$ tending to infinity that

$$\log M_g^{-1} M_f(r_1, r_2) \leq \log M_g^{-1}\left[\exp\left\{\left({}_{v_2}\lambda_f + \varepsilon\right)\log(r_1 r_2)\right\}\right]$$

$$i.e.,\ \ \log M_g^{-1} M_f(r_1, r_2) \leq \log\exp\left[\frac{\log\exp\left\{\left({}_{v_2}\lambda_f + \varepsilon\right)\log(r_1 r_2)\right\}}{\left({}_{v_2}\lambda_g - \varepsilon\right)}\right]$$

$$\begin{aligned} i.e.,\ \ \log M_g^{-1} M_f(r_1, r_2) &\leq \frac{\left({}_{v_2}\lambda_f + \varepsilon\right)}{\left({}_{v_2}\lambda_g - \varepsilon\right)}\log(r_1 r_2) \\ i.e.,\ \ \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} &\leq \frac{\left({}_{v_2}\lambda_f + \varepsilon\right)}{\left({}_{v_2}\lambda_g - \varepsilon\right)}\ . \end{aligned}$$

As $\varepsilon\,(> 0)$ is arbitrary, we obtain from above that

$$\begin{aligned} \liminf_{r_1, r_2 \to \infty} \frac{\log M_g^{-1} M_f(r_1, r_2)}{\log(r_1 r_2)} &\leq \frac{{}_{v_2}\lambda_f}{{}_{v_2}\lambda_g} \\ i.e.,\ \ {}_{v_2}\lambda_g(f) &\leq \frac{{}_{v_2}\lambda_f}{{}_{v_2}\lambda_g}\ . \end{aligned} \tag{16}$$

Thus the theorem follows from (11), (12), (13), (14), (15) and (16). □

**Corollary 3.** *Let $f$ and $g$ be any two entire functions of two complex variables such that $g$ is of regular growth. Then*

$${}_{v_2}\lambda_g(f) = \frac{{}_{v_2}\lambda_f}{{}_{v_2}\rho_g} \quad \text{and} \quad {}_{v_2}\rho_g(f) = \frac{{}_{v_2}\rho_f}{{}_{v_2}\rho_g}\ .$$

*In addition, if ${}_{v_2}\rho_f = {}_{v_2}\rho_g$, then*

$${}_{v_2}\rho_g(f) = {}_{v_2}\lambda_f(g) = 1\ .$$

**Corollary 4.** *Let $f$ and $g$ be any two entire functions of two complex variables with regular growth respectively . Then*

$${}_{v_2}\lambda_g(f) = {}_{v_2}\rho_g(f) = \frac{{}_{v_2}\rho_f}{{}_{v_2}\rho_g}\ .$$

**Corollary 5.** *Let $f$ and $g$ be any two entire functions of two complex variables with regular growth respectively. Also suppose that ${}_{v_2}\rho_f = {}_{v_2}\rho_g$. Then*

$${}_{v_2}\lambda_g(f) = {}_{v_2}\rho_g(f) = {}_{v_2}\lambda_f(g) = {}_{v_2}\rho_f(g) = 1\ .$$

**Corollary 6.** *Let $f$ and $g$ be any two entire functions of two complex variables with regular growth respectively. Then*

$$_{v_2}\rho_g(f) \cdot {}_{v_2}\rho_f(g) = {}_{v_2}\lambda_g(f) \cdot {}_{v_2}\lambda_f(g) = 1 \ .$$

**Corollary 7.** *Let $f$ and $g$ be any two entire functions of two complex variables such that either $f$ is not of regular growth or $g$ is not of regular growth. Then*

$$_{v_2}\lambda_g(f) \cdot {}_{v_2}\lambda_f(g) < 1 < {}_{v_2}\rho_g(f) \cdot {}_{v_2}\rho_f(g) \,.$$

**Corollary 8.** *Let $f$ and $g$ be any two entire functions of two complex variables. Then*

$$\begin{aligned}(i)\ {}_{v_2}\lambda_g(f) &= \infty \text{ when } {}_{v_2}\rho_g = 0 \ ,\\ (ii)\ {}_{v_2}\rho_g(f) &= \infty \text{ when } {}_{v_2}\lambda_g = 0 \ ,\\ (iii)\ {}_{v_2}\lambda_g(f) &= 0 \text{ when } {}_{v_2}\rho_g = \infty\end{aligned}$$

*and*

$$(iv)\ {}_{v_2}\rho_g(f) = 0 \text{ when } {}_{v_2}\lambda_g = \infty \ .$$

**Corollary 9.** *Let $f$ and $g$ be any two entire functions of two complex variables. Then*

$$\begin{aligned}(i)\ {}_{v_2}\rho_g(f) &= 0 \text{ when } {}_{v_2}\rho_f = 0 \ ,\\ (ii)\ {}_{v_2}\lambda_g(f) &= 0 \text{ when } {}_{v_2}\lambda_f = 0 \ ,\\ (iii)\ {}_{v_2}\rho_g(f) &= \infty \text{ when } {}_{v_2}\rho_f = \infty\end{aligned}$$

*and*

$$(iv)\ {}_{v_2}\lambda_g(f) = \infty \text{ when } {}_{v_2}\lambda_f = \infty \ .$$

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