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ECCENTRICITY BASED TOPOLOGICAL INDICES OF THE JOIN OF k GRAPHS

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Abstract: The join of k vertex disjoint graphs $G_1, G_2, \cdots, G_k$ denoted by $G = G_1 \vee G_2 \vee G_3 \vee \cdots \vee G_k$, is obtained from their union by including all edges between the vertices in G_i and the vertices in G_j, where $i, j = 1, 2, \cdots, k$ and $i \neq j$. In this paper, we given an explict formula to calculate the eccentric connectivity index and eccentric distance sum index of the join of k graphs.

AMS Subject Classification: 05C07, 05C12
Key Words: join of graphs, eccentric distance sum index, eccentric connectivity index

1. Introduction

Let G be a simple connected graph with the vertex set $V(G)$. For a graph G, let $deg_G(v)$ be the degree of a vertex v in G. The distance between two vertices u and v, namely, the length of the shortest path between u and v, in a graph G is denoted by $d_G(u,v)$. The eccentricity of a vertex v in a connected graph G is defined as $ecc_G(v) = max\{d_G(u,v) : u \in V(G)\}$. Let $D_G(v)$ be the sum of distances of all vertices in G from v, equivalently, $D_G(v) = \sum_{u \in V(G)} d_G(u,v)$.

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Two eccentricity-based topological indices, the eccentric connectivity index, defined as

$$\xi^c(G) = \sum_{v \in V(G)} ecc_G(v) deg_G(v)$$

and the eccentric distance sum is defined as

$$\xi^d(G) = \sum_{v \in V(G)} ecc_G(v) D_G(v)$$

For more mathematical properties about these two indices, the reader is refered to the papers [1, 2, 3, 4, 5, 6]. L. Jing and W. Fuyi [7] calculated the Wiener indices, hyper-wiener indices and reverse Wiener indices of the join of k graphs. In this paper, we compute the eccentric connectivity index and eccentric distance sum of the join of k graphs.

2. Eccentric Connectivity Index of the Join of k Graphs

We define the join of k graphs as given in [7].

Definition 1. The join of k vertex disjoint graphs $G_1, G_2, \cdots, G_k$ denoted by $G = G_1 \vee G_2 \vee G_3 \vee \cdots \vee G_k$, is obtained from their union by including all edges between the vertices in G_i and the vertices in G_j, where $i, j = 1, 2, \cdots, k$ and $i \neq j$.

Let the order and size of the graph G_i, $1 \leq i \leq k$ be n_i and m_i respectively. From the definition of join of k graphs, it is clear that the eccentricity of any vertex in G is either one or two. Let us denote A_i as the set of vertices in the graph G_i with degree $n_i - 1$, $1 \leq i \leq k$. Let A denote the union of the sets $A_i^{\prime}s$. Let $n = n_1 + n_2 + \cdots + n_k$. We now calculate the eccentric connectivity index of join of k graphs.

Lemma 2. *Let $G = G_1 \vee G_2 \vee G_3 \vee \cdots \vee G_k$. For some $i, 1 \leq i \leq k$, $\sum_{v \in G_i} ecc_G(v) deg_G(v) = 4m_i - (n-1)|A_i| + 2n_i(n - n_i)$.*

Proof.

$$\sum_{v \in V(G_i)} ecc_G(v) deg_G(v) = \sum_{v \in A_i} ecc_G(v) deg_G(v) + \sum_{v \in V(G_i) - A_i} ecc_G(v) deg_G(v)$$

Every vertex in A_i has eccentricity one and the other vertices in G_i has eccentricity two. The degree of the vertices in G are

$$deg_G(v) = \begin{cases} n-1 & if \quad v \in A_i \\ deg_{G_i}(v) + n - n_i & if \quad v \in V(G_i) \setminus A_i \end{cases}$$

$$\begin{aligned} \sum_{v \in V(G_i)} ecc_G(v) deg_G(v) &= \sum_{v \in A_i} (n-1) + \sum_{v \in V(G_i) - A_i} 2[d_{G_i}(v) + n - n_i] \\ &= (n-1)\left|A_i\right| + 2[2m_i - \left|A_i\right|(n-1)] + 2n_i(n-n_i) \\ &= 4m_i - (n-1)\left|A_i\right| + 2n_i(n-n_i) \end{aligned}$$

□

Theorem 3. *Let $G = G_1 \vee G_2 \vee G_3 \vee \cdots \vee G_k$. The $\xi^c(G) = 4\sum_{i=1}^k m_i - (n-1)\left|A\right| + 2n^2 - 2\sum_{i=1}^k n_i^2$.*

Proof. We have

$$\xi^c(G) = \sum_{i=1}^{k} \sum_{v \in V(G_i)} ecc_G(v) deg_G(v)$$

From Lemma: 2, we have $\sum_{v \in G_i} ecc_G(v) deg_G(v) = 4m_i - (n-1)\left|A_i\right| + 2n_i(n-n_i)$. Therefore,

$$\begin{aligned} \xi^c(G) &= \sum_{i=1}^{k} [4m_i - (n-1)\left|A_i\right| + 2n_i(n-n_i)] \\ &= 4\sum_{i=1}^{k} m_i - (n-1)\left|A\right| + 2n^2 - 2\sum_{i=1}^{k} n_i^2. \end{aligned}$$

□

Corollary 4. *If $\Delta(G_i) < n_i - 1$ for every i, then $\xi^c(G) = 4\sum_{i=1}^k m_i + 2n^2 - 2\sum_{i=1}^k n_i^2$.*

Corollary 5. *If G_i is complete graph for every i, then $\xi^c(G) = n^2 - n$.*

Proof. If each G_i is complete, we have $\sum_{i=1}^k m_i = \sum_{i=1}^k \frac{n_i(n_i-1)}{2}$ and $\left|A\right| = n$. □

3. Eccentric Distance Sum of the Join of k Graphs

We now compute the eccentric distance sum of join of k graphs. The sum of the distances of other vertices from v is

$$D_G(v) = \begin{cases} n-1 & if \quad v \in A_i \\ n + n_i - deg_{G_i}(v) - 2 & if \quad v \in V(G_i) \setminus A_i \end{cases}$$

Lemma 6. *Let $G = G_1 \vee G_2 \vee G_3 \vee \cdots \vee G_k$. For some $i, 1 \le i \le k$, $\sum_{v \in G_i} ecc_G(v) D_G(v) = 2nn_i - (n-1)|A_i| + 2n_i^2 - 4n_i - 4m_i$.*

Proof.

$$\sum_{v \in V(G_i)} ecc_G(v) D_G(v) = \sum_{v \in A_i} ecc_G(v) D_G(v) + \sum_{v \in V(G_i) - A_i} ecc_G(v) D_G(v)$$

Every vertex in A_i has eccentricity one and the other vertices in G_i has eccentricity two.

$$\begin{aligned} \sum_{v \in V(G_i)} ecc_G(v) D_G(v) &= \sum_{v \in A_i} (n-1) + \sum_{v \in V(G_i) - A_i} 2[n + n_i - d_{G_i}(v) - 2] \\ &= (n-1)|A_i| + 2(n + n_i - 2)(n_i - |A_i|) - 2[2m_i - |A_i|(n_i - 1)] \\ &= 2nn_i - (n-1)|A_i| + 2n_i^2 - 4n_i - 4m_i \end{aligned}$$

□

Theorem 7. *Let $G = G_1 \vee G_2 \vee G_3 \vee \cdots \vee G_k$. The $\xi^d(G) = 2n^2 - (n-1)|A| + 2\sum_{i=1}^k n_i^2 - 4\sum_{i=1}^k m_i - 4n$.*

Proof. We have

$$\xi^d(G) = \sum_{i=1}^k \sum_{v \in V(G_i)} ecc_G(v) D_G(v)$$

From Lemma: 6, we have $\sum_{v \in G_i} ecc_G(v) D_G(v) = 2nn_i - (n-1)|A_i| + 2n_i^2 - 4n_i - 4m_i$. Therefore,

$$\xi^d(G) = \sum_{i=1}^k [2nn_i - (n-1)|A_i| + 2n_i^2 - 4n_i - 4m_i]$$

$$= 2n^2 - (n-1)\,|A| + 2\sum_{i=1}^{k} n_i^2 - 4\sum_{i=1}^{k} m_i - 4n.$$

□

Corollary 8. *If $\Delta(G_i) < n_i - 1$ for every i, then $\xi^d(G) = 2n^2 + 2\sum_{i=1}^{k} n_i^2 - 4\sum_{i=1}^{k} m_i - 4n$.*

Corollary 9. *If G_i is complete graph for every i, then $\xi^d(G) = n^2 - n$.*

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