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A MOSQUITOES POPULATION DYNAMICS MODEL WITH PESTICIDE CONTROL

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Abstract: A deterministic differential equation model for the population dynamics of the mosquito vector is derived and studied. The life cycle of mosquito involves four stages Egg, Larva, Pupa & Adult. The mosquito population is divided into 3 compartments; larvae (L), indoor population (I) and outdoor population (O). The effect of pesticide applied to control the vector, is incorporated in the model with both periodic application and constant application. Taking mean effectiveness β_0 of pesticide as constant rate of pesticide control, the dynamical behaviour is studied. The system is bounded. Conditions for the existence and stability of a non-zero steady-state vector population density are derived. Numerical simulation is performed to verify the analytical result.

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1. Introduction

For many centuries, vector-borne diseases, among all infectious diseases of humans, have constituted a major cause of human mortality and morbidity [1-8]. For example, about 1.5-3 million people, die of malaria every year (WHO, 2005) [9]. Mosquitoes are foremost in man's war against insects. The blood sucking creatures not only cause nuisance by their irritating bites but also create health menace, as they are about responsible for spreading serious diseases like Malaria, Dengue, yellow fever, Japanese Encephalitis, Chikun Gunya etc. Mosquitoes belong to order Diptera [3]. For a disease vector to feed on a human, it must actively seek the human host. In the case of the mosquito, the vector responsible for the transmission of several human diseases, including malaria, only the adult female needs blood meals. This mosquito can systematically target and identify human beings [6] and once attracted, it may seek and bite humans as many times as possible until it takes a blood meal. Some species of the mosquito prefer human blood (anthropophilic) while others prefer animal blood (zoophilic). Some prefer to bite indoors (endophagic), and others prefer to bite outdoors (ex- ophagic). Here we suppose that mosquito approaches to indoor human population for blood. Some species prefer to rest within the habitat of the human from which they take their blood meal (endophilic) while others prefer to rest outdoors (exophilic). Once a good blood meal has been taken, the vector searches for, and moves to, a convenient breeding site: Usually a swamp or humid area. Such breeding sites may host several species of mosquitoes [4].

The vector's preference for a particular swamp depends on the distance from a human settlement and perhaps also on safety from predators [6,9]. Once the mosquito takes blood meal form indoor population, we assume that mosquito searches for outdoor sight for breeding and safety point of view. In human populated area mosquitoes breed in stored exposed water collection like drums, jars, pots, buckets, discarded bottles, tins, tyres etc. & a lot more places where rainwater gets collected and stored. The eggs are laid on water and after about 2-3 days, they hatch into larva. In about 4-10 days, the larva changes into pupa. The pupa then changes into the adult mosquito in about 2-4 days. The duration of the whole cycle, from egg laying to an adult mosquito eclosion varies between 7 and 20 days, depending on the ambient temperature of the swamp and the mosquito species involved [6].

Due to increasing drug-resistance and mosquito insecticide resistance, more attention is required for control of mosquito. Comparative knowledge of the effectiveness and efficacy of different control strategies is necessary to design useful and cost-effective mosquito control programs [6,9]. Mathematical modelling of mosquito dynamics can play a unique role in comparing the effects of control strategies, used individually or in packages.

2. Modelling Assumptions

We propose a mathematical model for the mosquito population dynamics under the following assumptions:

1. In this model it is assumed that stagnant water is available though out the year for the growth of mosquitoes. Although it may be very strong assumption, yet it is applicable for the place where the duration of rainy season is large.

2. The life cycle of mosquitoes involves four stages: Egg, Larva, Pupa and Adult. First three stages are associated with water, we call the combined stage as larval stage. In the model broadly there are two stages, i.e., larva and adult.

3. Due to the presence of zooplankton and fish population larval population will decay.

4. The growth of larval directly depends on population of female mosquitoes. Most female mosquitoes have to feed on an animal and get a sufficient blood meal before she can develop eggs. If they do not get this blood meal, then they will die without laying viable eggs.

5. Growth of larval population depends on adult population.

6. There is rapid movement of mosquito population from indoor to outdoor and vice-versa.

7. Mosquitoes have average life span of about 30 days. So we assume the average natural death rate is reciprocal to the average life span.

8. Periodic control measure is also used for larval density and outdoor population using insecticide.

3. Parameter Description

Keeping in view the mosquito dynamics and modelling assumptions, we have identified following major parameters:

Symbol	Descriptions
c	growth rate of larval(L) population due to migrant mosquitoes
μ	growth rate of larval population due to adult female
ω	natural washout rate of larvae due to fish and zooplankton
a	fraction of the Indoor contributing towards the growth of L
r	conversion rate of larval to adult mosquito
α_1	average rate of movement from outdoor(O) to indoor(I)
α_2	average rate of movement from indoor to outdoor
n	natural death rate of adult mosquitoes
β_0	mean effectiveness of pesticide used to control larvae
β_1	mean effectiveness of pesticide used to control Outdoor
$\beta_0\delta_0$	the periodic perturbation in L due to the insecticide
$\beta_1\delta_1$	the periodic perturbation in O due to the insecticide
ω_1	frequency of pesticide used to control L and O populations

4. Model with Periodic Control

Keeping in view of the assumption, the schematic flow shown in Figure 1. Considering the set of parameters and Figure 1, we propose a mathematical model of population dynamics of mosquitoes with periodic control using insecticide for outdoor and larva population is given by the following differential equations:

$$\frac{dL}{dt} = c + \mu(O + aL) - \omega L - rL - \beta_0(1 + \delta_0 sin(\omega_0 t))L,$$

$$\frac{dO}{dt} = rL - \alpha_1 O + \alpha_2 I - nO - \beta_1(1 + \delta_1 sin(\omega_1 t))O,$$

$$\frac{dI}{dt} = \alpha_1 O - \alpha_2 I - nI,$$

with initial populations $L(0) = L_0 > 0$, $O(0) = O_0 > 0$, $I(0) = I_0 > 0$. Here β_0 and β_1 are average rate of decay of larva and outdoor mosquito population due to continuous application of insecticides.

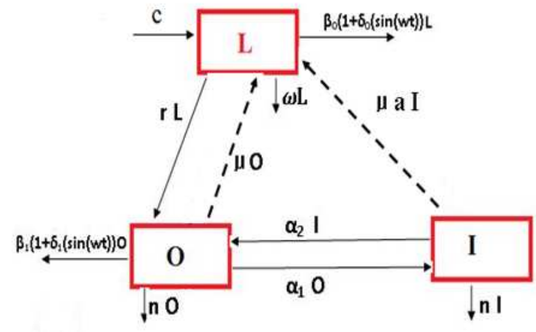

Figure 1: Schematic Diagram of model

5. Model with Continuous Control

Instead of periodic control of the vector, here we assumed a continuous control with average effort is same as in the periodic control shown in above model. Hence the modified system is given by the following differential equations:

$$\frac{dL}{dt} = c + \mu(O + aL) - \omega L - rL - \beta_0 L, \tag{1}$$

$$\frac{dO}{dt} = rL - \alpha_1 O + \alpha_2 I - nO - \beta_1 O, \tag{2}$$

$$\frac{dI}{dt} = \alpha_1 O - \alpha_2 I - nI, \tag{3}$$

with initial conditions $L(0) = L_0 > 0, O(0) = O_0 > 0, I(0) = I_0 > 0$. Here β_0 and β_1 are average rate of decay of larva and outdoor mosquito population due to continuous application of insecticides.

6. Analysis of the Model

Following subsection, we will analysis the above model step by step.

6.1. Existence of Equilibrium Point

There is a unique equilibrium point of the system (1)-(3) given by $E^{\star}(L^{\star}, O^{\star}, I^{\star})$, where $L^{\star} = AI^{\star}$, $O^{\star} = (\frac{\alpha_2+n}{\alpha_1})I^{\star}$ and $I^{\star} = \frac{c}{(\omega+r+\beta_0)A-\mu(\frac{\alpha_2+n}{\alpha_1}+a)}$, where $A =$

$\frac{1}{r\alpha_1}[\alpha_1 n + (\beta_1 + n)(\alpha_2 + n)]$. Clearly, equilibrium point exists if and only if

$$[\alpha_1 n + (\beta_1 + n)(\alpha_2 + n)](\omega + r + \beta_0) > r\mu(\alpha_2 + n + a\alpha_1). \tag{4}$$

6.2. Boundedness of the System

Let $N = L + O + I$ be the total population, then

$$\frac{dN}{dt} = c - (\mu + \beta_0)L - (\beta_1 + n - \mu_0)O - (n - a\mu_0)I$$

Let $\theta = min\{\mu + \beta_0, \beta_1 + n - \mu_0, n - a\mu_0\}$. Hence

$$\frac{dN}{dt} = c - \theta N \quad \Rightarrow N(t) = \frac{c}{\theta} + C_1 e^{-\theta t},$$

where $C_1 = N_0 - \frac{c}{\theta}$, as $t \to \infty$, $N(t) \to \frac{c}{\theta}$. Let $\Omega = \{(L, O, I) : 0 \leq L, O, I, L + O + I \leq \frac{c}{\theta}\}$. Hence the system (1-3) is bounded in the region Ω.

6.3. Local Stability Analysis of E^*

The variational matrix of the system (1)-(3) about $E^{\star}$ is given by

$$J = \begin{bmatrix} -(\omega + r + \beta_0) & \mu & \mu a \\ r & -(\alpha_1 + \beta_1 + n) & \alpha_2 \\ 0 & \alpha_1 & -(\alpha_2 + n) \end{bmatrix}$$

Characteristic equation of J is

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0, \tag{5}$$

where $A_1 = \omega + r + \beta_0 + \beta_1 + \alpha_1 + \alpha_2 + 2n$,
$A_2 = (\beta_1+n)(\alpha_2+n)+n\alpha_1+(\alpha_2+n)(\omega+r+\beta_0)+(\omega+r+\beta_0)(\alpha_1+\beta_1+n)-r\mu$,
$A_3 = (\omega + r + \beta_0)(\alpha_2(\beta_1 + n) + n(\alpha_1 + \beta_1 + n)) - r\mu(\alpha_2 + n + a\alpha_1)$.
Clearly,

$$A_1 > 0, A_2 > 0 \text{if} \alpha_1 + \alpha_2 + \beta_1 + 2n > \mu \tag{6}$$

and $A_3 > 0$ from (4). Also, $A_1A_2 - A_3 = (\alpha_1 + \alpha_2 + \beta_1 + 2n)((\beta_1 + n)(\alpha_2 + n) + n\alpha_1) + (\omega + r + \beta_0 + \alpha_1 + \alpha_2 + \beta_1 + 2n)\{(\beta_0 + \omega)(\alpha_1 + \alpha_2 + \beta_1 + 2n) + r(\alpha_1 + \alpha_2 + \beta_1 + 2n - \mu)\} + r\mu(\alpha_2 + n + a\alpha_1)$

Hence $A_1A_2 - A_3 > 0$ hold under the condition (6). Hence by Routh Hurwitz criterion, the equilibrium point $E^{\star}$, if exists, is locally asymptotically stable if the condition $\alpha_1 + \alpha_2 + \beta_1 + 2n > \mu$ holds true.

7. Numerical Analysis

In this section, the models described in Section 5 and 6 are solved numerically with different sets of parametric values biologically relevant to mosquito's growth in different stages (L-O-I), shown in table 1.

Table 1: Values of the parameters

Parameter	Figure 2&3	Figure 4	Figure 5	Figure 6
c	1.5	3.5	3.5	3.5
μ	0.4	0.5	0.5	1.6
ω	0.13	0.25	0.25	0.25
a	0.3	0.3	0.3	0.3
r	0.15	0.2	0.2	0.2
α_1	0.5	0.5	0.5	0.3
α_2	0.5	0.5	0.5	0.3
n	0.15	0.15	0.15	0.15
β_0	0.5	0.5	0.5	0.5
β_1	0.5	0.5	0.5	0.5
δ_0	0.25	0.25	0.25	0.25
δ_1	0.5	0.5	0.5	0.5
ω_1	$\pi/3$	$\pi/3$	-	$\pi/3$

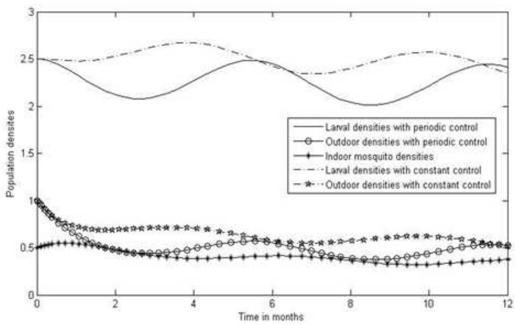

Figure 2: Population densities distributions for 12 months

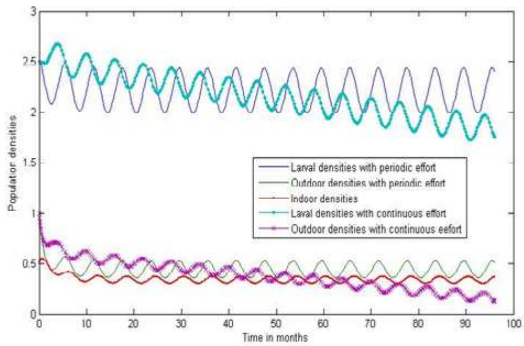

Figure 3: Population densities distributions for 100 months

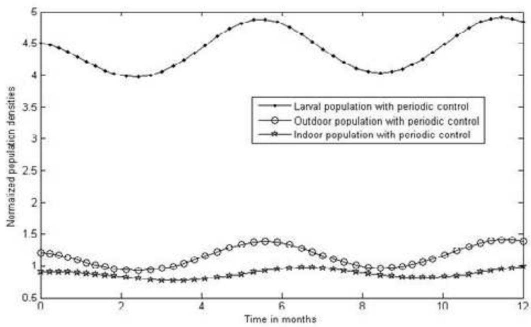

Figure 4: Population densities distributions for 12 months with periodic insecticide treatment

8. Conclusions

In this paper, two conceptual mathematical models were proposed to estimate the mosquitos population in different stages with or without insecticide control strategy. The dynamics were proposed keeping in view major parameters affecting the system. From Section 2 and 3, it is observed that coexistence of the system always occurs and the population size is bounded. Further, it is observed after comparison of figures 2 and 3 that in short time period, the periodic control is more effective than the continuous control of mosquito vector, but in long run continuous control is more beneficial compare to periodic with same amount of insecticide application. In figure 2-5 data sets are satisfy the

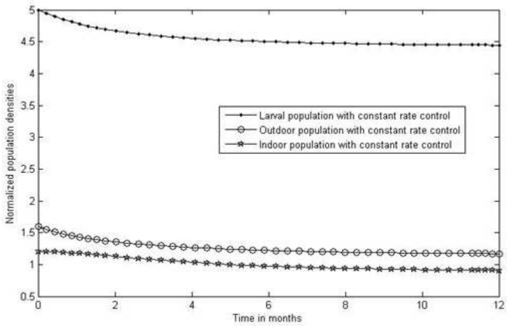

Figure 5: Population densities distributions for 12 months with constant rate insecticide treatment

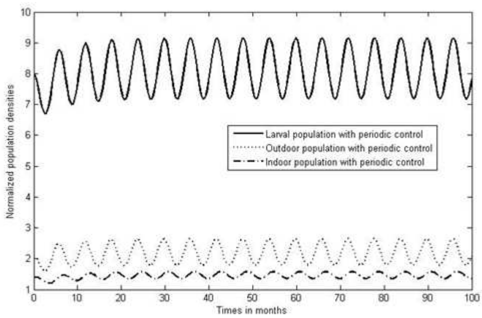

Figure 6: Population densities distributions for 100 months with periodic insecticide treatment

stability criterion $\alpha_1 + \alpha_2 + \beta_1 + 2n > \mu$, but in figure 6 dataset contradicts the stability criterion, i.e., $\alpha_1 + \alpha_2 + \beta_1 + 2n < \mu$. In future, these models can be validated using the actual experimental data collection by the field workers and scientists.

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