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SOFT STABILITY OF S-FUNCTIONS

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Abstract: In the present paper we introduce soft stability of s-functions and soft sequences and their convergence which are defined over an initial universe with a fixed set of parameters with norm.

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1. Introduction and Preliminaries

The real world is too complex for our immediate and direct understanding. We create models of reality that are simplifications of aspects of the real word. Unfortunately these mathematical models are too complicated and we cannot find the exact solutions. The uncertainty of data while modeling the problems in engineering, physics, computer sciences, economics, social sciences, medical

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sciences and many other diverse fields makes it unsuccessful to use the traditional classical methods. These may be due to the uncertainties of natural environmental phenomena, of human knowledge about the real world or to the limitations of the means used to measure objects. For example, vagueness or uncertainty in the boundary between states or between urban and rural areas or the exact growth rate of population in a country's rural area or making decisions in a machine based environment using database information. Thus classical set theory, which is based on the crisp and exact case may not be fully suitable for handling such problems of uncertainty. There are several theories, for example, theory of fuzzy sets [1], theory of intuitionistic fuzzy sets [2], theory of vague sets, theory of interval mathematics [3, 4] and theory of rough sets [5]. These can be considered as tools for dealing with uncertainties but all these theories have their own difficulties. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory as it was mentioned by Molodtsov in [6]. He initiated the concept of soft set theory as a new mathematical tool which is free from the problems mentioned above. In his paper [6], he presented the fundamental results of the new theory and successfully applied it to several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability etc. A soft set is a collection of approximate descriptions of an object. He also showed how soft set theory is free from the parametrization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. Soft systems provide a very general framework with the involvement of parameters. Research works on soft set theory and its applications in various fields are progressing rapidly.

In 1940 S. Ulam [10] proposed the general Ulam stability problem: *Let G_1 be a group, G_2 a metric group with the metric d. Given $\varepsilon > 0$, does there exists $\delta > 0$ such that if a function $h : G_1 \to G_2$ satisfies the inequality*

$$d\Big(h(xy) - h(x)h(y)\Big) < \delta, \quad (x, y \in G_1),$$

then there is a homomorphism $H : G_1 \to G_2$ with

$$d(h(x), H(x)) < \varepsilon, \quad (x \in G_1)?$$

D. H. Hyers [8] gave a partial affirmative answer to the question of Ulam in the context of Banach spaces. In 1950, a generalized version of Hyers' theorem for approximate additive mappings was given by T. Aoki [7]. In 1978, T. M. Rassias [9] extended the theorem of Hyers by considering the unbounded cauchy difference inequality

$$\|f(x+y)-f(x)-f(y)\| \leqslant \varepsilon(\|x\|^p+\|y\|^p) \quad (\varepsilon \geq 0,\, p \in [0,1)) \tag{1.1}$$

2. Soft Sets

Definition 2.1. Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of $\varepsilon-$approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Definition 2.2. For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of(G, B) if

(1) $A \subseteq B$ and

(2) for all $E \in A$, $F(e)$ and $G(e)$ are identical approximations.

We write $(F, A)\widetilde{\subset}(G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A)\widetilde{\supset}(G, B)$.

Definition 2.3. Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4. Let $E = \{e_1, e_2, ..., e_n\}$ be a set of parameters. The NOT set of E denoted by $\rceil E$ is defined by $\rceil E = \{\rceil e_1, \rceil e_2, ..., \rceil e_n\}$ where, $\rceil e_i =$ not e_i for all i.

Definition 2.5. The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \rceil A)$ where, $F^c :\rceil A \to P(U)$ is a mapping given by $F^c(a) = U\backslash F(\rceil a)$, for all $a \in\rceil A$. Let us call F^c to be the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.

Definition 2.6. ([8]) A soft set (F, A) over U is said to be a NULL soft set denoted by $\varnothing$ if for all $\varepsilon \in A$, $F(\varepsilon) = \varnothing$(null set).

Definition 2.7. A soft set (F, A) over U is said to be an absolute soft set denoted by $\tilde{A}$ if for all $\varepsilon \in A$, $F(\varepsilon) = U$. Clearly $(\tilde{A})^c = \varnothing$ and$\varnothing^c = \tilde{A}$.

3. Soft Sequences

Definition 3.1. Let (M, d) be a metric space, where d is a metric. We shall call space (M, d) a model space. Let U be a set and for every $m \in M$, we have a soft set $(F(m), E)$ over U. Such a pair (F, E) is said to be an s-function (soft function) and we shall use the notation $(F, E) : M \to U$.

Definition 3.2. Let (M, d) be a metric space and $(F, E) : M \to U$ be a s-function. Let $\{m_i\}_{i=1}^{\infty}$ be a sequence. We define $\{(F(m_i), E)\}_{i=1}^{\infty}$ is a soft sequence.

Definition 3.3. A soft sequence $\{(F(m_i), E)\}_{i=1}^{\infty}$ is said to converge to $\{(F(m), E)\}$ if the sequence $\{m_i\}_{i=1}^{\infty}$ converges to m. If the sequence $\{(F(m_i), E)\}_{i=1}^{\infty}$ converges to $\{(F(m), E)\}$ we write

$$\{(F(m_i), E)\} \to \{(F(m), E)\}$$

.

Example 3.4. Let $\left(M = \left\{\frac{1}{m} : m \in \mathbb{N}\right\}, d\right)$ where d is euclidean metric. Let $U_m = \{x : d(x, \frac{1}{m}) < \frac{1}{m}\}$ for every $m \in M$ and $(F, E) : M \to \bigcup_m U_m$ be s-function. We have $\frac{1}{m} \to 0$ then the soft sequence $\{(F(m_i), E)\} \to (F(0), E)$.

Definition 3.5. Let (M, d) be a metric space and $(F, E) : M \to U$ be a s-function. Let $\{m_i\}_{i=1}^{\infty}$ be a Cauchy sequence. We define $\{(F(m_i), E)\}_{i=1}^{\infty}$ as a soft Cauchy sequences.

4. Soft Topology

Let X be an initial universe set and E be the non-empty set of parameters.

Definition 4.1. The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E) \backslash (G, E)$, is defined as $H(e) = F(e) \backslash G(e)$ for all $e \in E$.

Definition 4.2. Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$. Note that for any $x \in X$, $x \notin (F, E)$ if $x \notin F(e)$ for some $e \in E$.

Definition 4.3. Let $x \in X$, then (x, E) denotes the soft set over X for which $x(\alpha) = \{x\}$, for all $\alpha \in E$.

Definition 4.4. The relative complement of a soft set (F, A) is denoted by $(F, A)'$ and is defined by $(F, A)' = (F', A)$ where $F' : A \in P(U)$ is a mapping given by $F'(\alpha) = U \backslash F(\alpha)$ for all $\alpha \in A$.

Definition 4.5. Assume that we have a binary operation, denoted by $*$, for subsets of the set U. Let (F, A) and (H, B) be two soft sets over U, then the operation $*$ of (F, A) and (G, B) is defined as $(F, A) * (G, B) = (H, A \times B)$ where $H : A \times B \to P(U \times U)$ and $H(a, b) = F(a) * G(b)$, where $\times$ is the Cartesian product of the sets A ,B and $(a, b) \in A \times B$.

Definition 4.6. Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if:

(1) $\varnothing$ and $\tilde{X}$ belong to τ

(2) the intersection of any number of soft closed sets is a soft closed set over X.

(3) the union of any two soft closed sets is a soft closed set over X.

The triplet (X, τ, E) is called a soft topology space over X.

5. Soft Stability

A mapping $f : (X, \tau, E) \to B$ is said to be *additive soft* if and only if it satisfies $f(x * y) = f(x) + f(y)$ for all $x, y \in (X, \tau, E)$ and B is Banach space.

Theorem 5.1. *Suppose that a mapping $f : (X, \tau, E) \to B$ satisfies the inequality*

$$||f(x * y) - f(x) - f(y)|| \leq \epsilon \tag{5.1}$$

for all $x, y \in (X, \tau, E)$. Then the limit

$$T(x) = \lim_{n\to\infty} \frac{1}{2^n} f(x^{2^n}) \tag{5.2}$$

exists for all $x \in (X, \tau, E)$ and the mapping $T : (X, \tau, E) \to B$ is a unique additive soft mapping satisfying

$$||T(x) - f(x)|| \leq \epsilon \tag{5.3}$$

Proof. Letting $x = y$ in (5.1), we obtain

$$||f(x^2) - 2f(x)|| \leq \epsilon \tag{5.4}$$

or

$$||\frac{1}{2}f(x^2) - f(x)|| \leq \frac{\epsilon}{2} \tag{5.5}$$

for all $x \in (X, \tau, E)$. Replacing x by x^2 in (5.5), we get the following inequalities

$$||\frac{1}{4}f(x^4) - f(x)|| \leq \frac{\epsilon}{2} + \frac{\epsilon}{4} \tag{5.6}$$

or all $x \in (X, \tau, E)$. Therefore

$$\begin{aligned} ||\frac{1}{2^n}f(x^{2^n}) - \frac{1}{2^m}f(x^{2^m})|| &\leq \sum_{k=m}^{n-1} \Big|| \frac{1}{2^{k+1}}f(x^{2^{k+1}}) - \frac{1}{2^k}f(x^{2^k})\Big|| \\ &\leq \sum_{k=m}^{n-1} \frac{2}{2^k}\epsilon \\ &< \epsilon \end{aligned} \tag{5.7}$$

for all $x \in (X, \tau, E)$ and all integers $n > m \geqslant 0$. Therefore the sequence $\frac{1}{2^n}f(x^{2^n})$ is a Cauchy sequence in B for all $x \in (X, \tau, E)$. Since B is complete, the sequence $\frac{1}{2^n}f(x^{2^n})$ converges in B for all $x \in (X, \tau, E)$. So one can define the mapping $T : (X, \tau, E) \to B$ by

$$T(x) := \lim_{n\to\infty} \frac{1}{2^n}f(x^{2^n}) \tag{5.8}$$

for all $x \in (X, \tau, E)$. Letting $m = 0$ and passing the limit $n \to \infty$ in (5.7) we get (5.3). On the other hand, from (5.8) we get

$$||T(x * y) - T(x) - T(y)|| = \lim_{n\to\infty} \frac{1}{2^n}||f(x^{2^n} * y^{2^n}) - f(x^{2^n}) - f(y^{2^n})|| = 0 \tag{5.9}$$

so T is mapping *additive soft*.

To prove the uniqueness of T, let $T' : X \to Y$ be another additive mapping satisfying (5.3) that

$$\begin{aligned} ||T(x) - T'(x)|| &= ||T(x) - f(x) + f(x) - T'(x)|| \\ &\leq ||T(x) - f(x)|| + ||T'(x) - f(x)|| < \epsilon \end{aligned} \tag{5.10}$$

for all $x \in (X, \tau, E)$. So $T = T'$. □

Theorem 5.2. *Let $f : (X, \tau, E) \to B$ be a mapping from a soft topology space over X into a Banach space B subject to the inequality and satisfies.*

Proof. Let $\varphi : (X, \tau, E) \times (X, \tau, E) \longrightarrow [0, \infty)$ be a function such

$$\lim_{n \longrightarrow \infty} \frac{1}{2^n} \varphi(x^{2^n}, y^{2^n}) = 0 \tag{5.11}$$

for all $x, y \in (X, \tau, E)$ and

$$\sum_{n=1}^{\infty} \frac{1}{2^n} \varphi(x^{2^n}, x^{2^n}) < \infty \tag{5.12}$$

for all $x \in (X, \tau, E)$. Moreover

$$\|f(x * y) - f(x) - f(y)\| \leq \varphi(x, y) \tag{5.13}$$

Then the limit

$$Q(x) := \lim_{n \longrightarrow \infty} \frac{1}{2^n} f(x^{2^n}) \tag{5.14}$$

all $x \in (X, \tau, E)$ exists for all $x \in (X, \tau, E)$ and $Q(x) : (X, \tau, E) \to B$ is a unique function *additive soft* mapping satisfying

$$||Q(x) - f(x)|| \leq \sum_{n=1}^{\infty} \varphi(x^{2^n}, x^{2^n}) \tag{5.15}$$

for all $x \in (X, \tau, E)$.

□

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