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SIMPLE MOTION PURSUIT DIFFERENTIAL GAME OF MANY PURSUERS AND ONE EVADER ON CONVEX COMPACT SET

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Abstract: We study a differential game of many pursuers and single evader in nonempty closed bounded convex subset of $\mathbb{R}^n$. In this game, all players cannot leave the given set. Control parameters of all players are subjected to geometric constraints. Maximum speeds of all players are equal to 1. Pursuit is said to be completed if geometric position of at least one pursuer coincides with that of the evader. Pursuers try to complete the pursuit. Problem is to find estimate for guaranteed pursuit time. To solve the problem, first, we study the same problem in an n-dimensional cube. Then, we reduce the main problem to the game in the cube. To this end, we use method of fictitious pursuers. In this paper, we improve the estimate for guaranteed pursuit time from $O(n^3)$ to $O(n^2)$.

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1. Introduction

Differential game is a field that has particular interest in constructing the most favorable strategies for all players. A large number of papers were devoted to differential game of many pursuers and one evader or several evaders (see, for example [1] –[21]). Simple motion differential game of many pursuers attract the attention of many researchers (see, for example, [1] –[19] and [21]). Among them, papers [3], [4], [8], [10], [15] and [18] are devoted to games with geometric constraints.

Pshenichnii[18] was one of the early researchers who studied simple motion differential game of many pursuers and one evader. In this game, maximum speeds of all players are equal to 1. It was proved that if the initial position of the evader y_0 belongs to the interior of convex hull of the initial positions of m pursuers, $x_{10}, ..., x_{m0}$, that is, $y_0 \in intconv\{x_{10}, ..., x_{m0}\}$, then pursuit can be completed, otherwise evasion is possible.

Chikrii and Prokopovich[8] found sufficient solvability conditions of pursuit and evasion for the simple motion differential game of a group of pursuers and one evader. The game is described by the following differential equations,

$$\dot{z}_i = u_i - v, \quad z_i \in \mathbb{R}^k, \quad u_i \in U_i, \quad v \in V, \quad z_i(0) = z_i^0, \quad i = 1, ..., n,$$

where $\mathbb{R}^k$, U_i and V are nonempty compact sets. A family of nonempty convex compacta $M_1, ..., M_n$ are given in $\mathbb{R}^k$, defining the terminal set M^*.

Chernous'ko[4] solved a simple motion evasion differential game of many pursuers and one evader. Here maximum speed of the evader is assumed to be equal to $\alpha > 1$, whereas maximum speeds of all pursuers are equal to 1. It was shown that evasion is possible from any finite number of pursuers. Result of this work further was extended to a wide class of problems by Chernous'ko and Melikyan[5], and Chernous'ko and Zak [6].

Blagodatskikh and Petrov [3] obtained necessary and sufficient conditions of completion of pursuit in a simple motion differential game of many pursuers and many evaders, provided that all the evaders use the same control, and the maximum speeds of all players are equal to 1. It should be noted that the papers [10] and [15] were studied under the state constraint.

Kuchkarov et. al[15] solved a problem of simple motion pursuit-evasion differential game that involved m pursuers and single evader on the surface of a given cylinder. The movements of all players are described by equations,

$$\begin{aligned} P_i &: \ \dot{x}_i = u_i, \ x_i(0) = x_{i0}, \ |u_i(t)| \leq 1, \ i = 1, 2, ..., m, \\ E &: \ \ \dot{y} = v, \quad y(0) = y_0, \quad |v(t)| \leq 1, \end{aligned}$$

where $x_i, x_{i0}, y, y_0, u_i, v \in \mathbb{R}^3$. Two cases were considered in this paper: (a) all pursuers move freely and unbounded with any phase constraint (b) the movement of all pursuers bounded on the surface of the cylinder. Meanwhile, the evader always moves on the surface of a given cylinder M, for both cases, where M is described by the following equation,

$$M = \{q = (q_{(1)}, q_{(2)}, q_{(3)}) | q_{(1)}^2 + q_{(2)}^2 = R^2, q_{(3)} \in \mathbb{R}\}.$$

Furthermore, the pursuers apply a counter strategy to guarantee pursuit while the evader applies a positional strategy to ensure evasion. In this paper, it was proved that if there exist some $i, j \in \{1, 2, ..., m\}$ such that $x_{i0(3)} < y_{0(3)} < x_{j0(3)}$, then pursuit can be completed for some time T in both cases (a) and (b). However, if $i, j \in \{1, 2, ..., m\}$ do not satisfy the condition $x_{i0(3)} < y_{0(3)} < x_{j0(3)}$, then evasion is possible for both cases.

Ivanov[10] studied a pursuit-evasion differential game of many pursuers and one evader in a compact set. All players have the same dynamical possibilities and motions of all players are described by equations,

$$\dot{x}_i = u_i, \ x_i(0) = x_{i0}, \ |u_i| \leq 1, \ i = 0, 1, 2, ..., m,$$

where $x_i, \ u_i \in \mathbb{R}^n, \ u_i, \ i = 1, 2, ...m$, are control parameters of the pursuers and u_0 is control parameter of the evader. During the game, all players must not leave a given compact convex subset A of $\mathbb{R}^n$, $n \geq 2$. It is assumed that A has nonempty interior. It was proved that if the number pursuers less than the dimension of the space ($m < n$), then evasion is possible. If $m \geq n$, then it was proved that pursuit can be completed. Moreover, in this paper guaranteed pursuit time T is estimated by a third degree polynomial in n, more precisely $T \leq (n^3 - 2n^2 + n + 1)d$, where d is the diameter of A.

In the present paper, we also study a differential game of many pursuers and single evader in nonempty closed bounded convex subset A of $\mathbb{R}^n$, $n \geq 2$. Similar to the game studied by Ivanov[10], all players cannot leave the given set A and control parameters of all players are subjected to geometric constraints. We will improve the estimation of guaranteed pursuit time given by Ivanov[10].

2. Statement of the problem

We consider a pursuit differential game described by the following equations:

$$P_i \ : \ \dot{x}_i = u_i, \ x_i(0) = x_{i0}, \ |u_i| \leq 1, \ i = 1, 2, ..., m, \tag{1}$$
$$E \ : \ \dot{y} = v, \quad y(0) = y_0, \quad |v| \leq 1, \tag{2}$$

where $x_i, y, u_i, v, x_{i0}, y_0 \in \mathbb{R}^n, \ m \geq n$.

Definition 1. A measurable function $u_i(\cdot) = u_i(t), \ t \geq 0$, is called a control of the pursuer x_i if $|u_i(t)| \leq 1, \ t \geq 0$, and the solution $x_i(\cdot) = x_i(t)$, $t \geq 0$, of the initial value problem $\dot{x}_i = u_i(t), \ x_i(0) = x_{i0}, \ i \in \{1, 2, ..., m\}$, satisfies the inclusion

$$x_i(t) \in A.$$

Definition 2. A measurable function $v(\cdot) = v(t), t \geq 0$, is called a control of the evader y if $|v(t)| \leq 1, \ t \geq 0$, and the solution $y(\cdot) = y(t), \ t \geq 0$, of the initial value problem $\dot{y} = v(t), \ y(0) = y_0$, satisfies the inclusion

$$y(t) \in A.$$

Let $H(0, 1)$ be the ball of radius 1 and centered at the origin.

Definition 3. A function $U_i(x_i, y, v)$, $U_i : \mathbb{R}^n \times \mathbb{R}^n \times H(0, 1) \to H(0, 1)$, is called strategy of the pursuer x_i if for any control of the evader $v(t)$ the initial value problem

$$\dot{x}_i = U_i(x_i, y, v(t)), \ x_i(0) = x_{i0}, \tag{3}$$
$$\dot{y} = v(t), \qquad y(0) = y_0, \tag{4}$$

has a unique absolutely continuous solution $(x_i(t), y(t))$, $t \geq 0$, with $x_i(t), \ y(t) \in A, \ t \geq 0$.

Definition 4. If $x_i(\tau) = y(\tau)$ at some $i \in \{1, 2, ..., m\}$ and $\tau \geq 0$, then we say that pursuit is completed at the time τ (by the pursuer x_i).

Definition 5. A number T is called guaranteed pursuit time if there exist strategies of pursuers $U_1, U_2, ..., U_m$ such that for any control of the evader $v(\cdot)$ an equality $x_i(\tau) = y(\tau)$ holds at some $i \in \{1, 2, ..., m\}$ and $0 \leq \tau \leq T$ where $(x_i(t), y(t))$, is the solution of (3), (4) at $v = v(t), \ t \geq 0$.

During the pursuit, all players may not leave the given set A. The aim of pursuers is to minimize the guaranteed pursuit time, and that of the evader is to maximize it.

Problem. Find a guaranteed pursuit time estimated from above by a second degree polynomial in n and construct strategies of the pursuers.

3. The main results

In this section, we obtain a guaranteed pursuit time. First, we study the problem in the case where the set A is a cube.

3.1. Differential game on cube

We first consider an auxiliary differential game when $m = n$ and

$$A = N = \{(q_1, q_2, ..., q_n) \,|\, 0 \leq q_i \leq a, i = 1, 2, ..., n\},$$

where a is a positive number. Clearly, N is an n-dimensional cube with edges equal to a. Thus, all the players move in N.

Lemma 6. *In the differential game on N, the number*

$$T = \frac{a}{2}(n^2 - n + \sqrt{n} + 1)$$

is a guaranteed pursuit time.

Proof. We construct the strategies of pursuers as follows. Let

$$u_i(t) = \frac{2(\xi_0 - x_{i0})}{a\sqrt{n}}, \quad 0 \leq t \leq t_1, \quad i = 1, 2, ..., n, \quad t_1 = \frac{a\sqrt{n}}{2}, \tag{5}$$

where $\xi_0 = \left(\frac{a}{2}, \frac{a}{2}, ..., \frac{a}{2}\right)$ is the center of the cube N. According to (5), each pursuer moves towards the point ξ_0 on the time interval $[0, t_1]$. Clearly,

$$x_i(t_1) = \xi_0,$$

that is, all the pursuers reach the point ξ_0 at the time t_1.

Next, we define strategies of the pursuers on the time interval $\left(t_1, t_1 + \frac{a}{2}\right]$ as follows:

$$\begin{aligned} &u_1(t) = (0, u_{1,2}(t), 0, ..., 0),\\ &u_2(t) = (0, 0, u_{2,3}(t), ..., 0),\\ &......................................,\\ &u_{n-1}(t) = (0, ..., 0, 0, u_{n-1,n}(t)),\\ &u_n(t) = (u_{n,1}(t), 0, 0, ..., 0), \end{aligned} \tag{6}$$

where

$$u_{i,i+1}(t) = \begin{cases} 1, & y_{i+1}(t_1) > \frac{a}{2}, \\ & \\ -1, & y_{i+1}(t_1) < \frac{a}{2}, \end{cases} \quad \text{if } t < \tau_{i+1}^1, \tag{7}$$

$$u_{i,i+1}(t) = \quad v_{i+1}(t), \quad t \geq \tau_{i+1}^1, \qquad i = 1, ..., n, \tag{8}$$

where τ_{i+1}^1 is a time for which $x_{i,i+1}(\tau_{i+1}^1) = y(\tau_{i+1}^1)$. In the sequel, we use notations $y_{n+i} = y_i$, $x_{i,n+j} = x_{i,j}$, $v_{n+i} = v_i$.

Let us illustrate the part of strategy defined in (6)-(8) for $i = 1$, that is, for the pursuer x_1. If $y_2(t_1) > \frac{a}{2}$ (see(7)), then $u_{1,2}(t) = 1$, and so the pursuer x_1 moves with the speed equal to 1 parallel to q_2-axis in positive direction of q_2-axis. Since the evader may not leave the set N, therefore $y_2(\tau_2^1) = x_{1,2}(\tau_2^1)$ at some $\tau_2^1 \in \left(t_1, t_1 + \frac{a}{2}\right]$. Then according to (8), $u_{1,2}(t) = v_2(t)$, $t \geq \tau_2^1$, which guarantees $x_{1,2}(t) = y_2(t)$, for all $t \geq \tau_2^1$.

In the case $y_2(t_1) < \frac{a}{2}$ also using a similar reasoning, we can obtain $x_{1,2}(\tau_2^1) = y_2(\tau_2^1)$ at some $\tau_2^1 \in \left(t_1, t_1 + \frac{a}{2}\right]$, and then the control $u_{1,2}(t) = v_2(t)$ guarantees that $x_{1,2}(t) = y_2(t)$ for all $t \geq \tau_2^1$. Thus, using the strategies (6)-(8), we obtain

$$x_{i,i+1}(t) = y_{i+1}(t), \quad t \geq \tau_{i+1}^1,$$

where $\tau_{i+1}^1 \in \left(t_1, t_1 + \frac{a}{2}\right]$ is some time.

Further, the strategy $u_i(t) = (u_{i1}(t), ..., u_{in}(t))$ of the pursuer x_i, $i = 1, 2, ..., n$, is constructed as follows. Starting from the time τ_{i+1}^1 we let

$$\begin{aligned} &u_{i,i+1}(t) = v_{i+1}(t), \;\; u_{i,i+2}(t) =^{+}_{-} \sqrt{1 - v_{i+1}^2(t)}, \\ &u_{i,i+j}(t) = 0, \qquad\qquad j = 3, 4, ..., n, \end{aligned} \tag{9}$$

until $x_{i,i+2}(t) = y_{i+2}(t)$. In (9), we take " + " if $x_{i,i+2}(\tau_{i+1}^1) < y_{i+2}(\tau_{i+1}^1)$, and we take " − " if $x_{i,i+2}(\tau_{i+1}^1) > y_{i+2}(\tau_{i+1}^1)$. Part of the strategy defined by (9) ensures that $x_{i,i+1}(t) = y_{i+1}(t)$ since $u_{i,i+1}(t) = v_{i+1}(t)$.

Afterwards, we construct the strategy of x_i inductively as follows. Let,

$$x_{i,i+j}(t) = y_{i+j}(t), \quad j = 1, 2, ..., (k-1),$$

and pursuer x_i be moving according to the strategy

$$u_i(t) = (u_{i,1}(t), u_{i,2}(t), ..., u_{i,n}(t)), \quad t \geq \tau_{i+k-1}^{k-1},$$

until $x_{i,i+k}(t) = y_{i+k}(t)$ with

$$\begin{aligned} &u_{i,i+1}(t) = v_{i+1}(t), \quad u_{i,i+2} = v_{i+2}(t), \quad ..., \quad u_{i,i+k-1}(t) = v_{i+k-1}(t), \\ &u_{i,i+k}(t) =^{+}_{-} \sqrt{1 - v_{i+1}^2(t) - v_{i+2}^2(t) - ... - v_{i+k-1}^2(t)}, \\ &u_{i,i+j}(t) = 0, \qquad\qquad j = (k+1), ..., n, \end{aligned} \tag{10}$$

where $t = \tau_{i+k-1}^{k-1}$, is the first time for which $x_{i,i+k}(t) = y_{i+k}(t)$.

In (10), " + " is taken if $x_{i,i+k}(\tau_{i+k-1}^{k-1}) < y_{i+k}(\tau_{i+k-1}^{k-1})$, and " − " is taken if $x_{i,i+k}(\tau_{i+k-1}^{k-1}) > y_{i+k}(\tau_{i+k-1}^{k-1})$. If $x_{i,i+k}(\tau_{i+k}^{k}) = y_{i+k}(\tau_{i+k}^{k})$ for the first time at some τ_{i+k}^{k}, then we set

$$\begin{aligned} &u_{i,i+1}(t) = v_{i+1}(t), \quad u_{i,i+2}(t) = v_{i+2}(t), \quad ..., \quad u_{i,i+k}(t) = v_{i+k}(t), \\ &u_{i,i+k+1}(t) =^{+}_{-} \sqrt{1 - v_{i+1}^2(t) - v_{i+2}^2(t) - ... - v_{i+k}^2(t)}, \\ &u_{i,i+j}(t) = 0, \qquad\qquad j = (k+2), ..., n, \quad \tau_{i+k}^{k} \leq t < \tau_{i+k+1}^{k+1}, \end{aligned}$$

where $t = \tau_{i+k+1}^{k+1}$ is the time for which $x_{i,i+k+1}(t) = y_{i+k+1}(t)$. Note that to obtain equality $x_{i,i+k+1}(t) = y_{i+k+1}(t)$, the pursuer x_i on the interval $[\tau_{i+k}^{k}, \tau_{i+k+1}^{k+1}]$ along q_{i+k+1}–axis has to travel the distance equal at most to $\frac{a}{2}$.

Therefore, starting from the time $t_1 + \frac{a}{2} = \frac{\sqrt{n}+1}{2}a$, each pursuer has to travel the distance equal at most to $\frac{a(n-1)}{2}$ exactly to complete the pursuit. All pursuers altogether have to travel the distance equal to $n\frac{a(n-1)}{2}$. This distance decreases with the rate equal to

$$\alpha(t) = \sum_{i\in I_1}\sqrt{1 - v_{i+1}^2(t)} + \sum_{i\in I_2}\sqrt{1 - v_{i+1}^2(t) - v_{i+2}^2(t)} + ...$$

$$+ \sum_{i\in I_n}\sqrt{1 - v_{i+1}^2(t) - v_{i+2}^2(t) - ... - v_{i+n-1}^2(t)}$$

where $I_1, I_2, ..., I_n$ are disjunctive subsets of the set $I = \{1, 2, ..., n\}$, and $I_1 \cup I_2 \cup ... \cup I_n = I$. We estimate $\alpha(t)$ as follows:

$$\alpha(t) \geq \sum_{i\in I_1}\sqrt{1 - v_{i+1}^2(t) - v_{i+2}^2(t) - ... - v_{i+n-1}^2(t)} + ...$$

$$+ \sum_{i\in I_n}\sqrt{1 - v_{i+1}^2(t) - v_{i+2}^2(t) - ... - v_{i+n-1}^2(t)}$$

$$= \sum_{i=1}^{n}\sqrt{1 - v_{i+1}^2(t) - v_{i+2}^2(t) - ... - v_{i+n-1}^2(t)}.$$

Clearly, if $0 \leq x \leq 1$, then $\sqrt{x} \geq x$, and therefore

$$\alpha(t) \geq (1 - v_2^2(t) - v_3^2(t) - ... - v_n^2(t)) + (1 - v_3^2(t) - v_4^2(t) - ... - v_1^2(t))$$

$$+ ... + (1 - v_1^2(t) - v_2^2(t) - ... - v_{n-1}^2(t))$$

$$= n - (n-1)(v_1^2(t) + v_2^2(t) + ... + v_n^2(t)$$

$$\geq n - (n-1)$$

$$= 1.$$

Thus, $\alpha(t) \geq 1$, meaning that the distance $\dfrac{n(n-1)a}{2}$ is decreasing with the rate $\alpha(t) \geq 1$ and, hence, it will have been traveled by all the pursuers for the time $\dfrac{n(n-1)}{2}a$. As the consequence, the guaranteed pursuit time in the differential game on N is

$$T \leq \frac{\sqrt{n}+1}{2}a + \frac{n(n-1)}{2}a = \frac{a}{2}(n^2 - n + \sqrt{n} + 1).$$

The proof of Lemma 6 is completed. □

3.2. General case

In this section, we consider the differential game on A. It is not difficult to construct a cube containing the set A. Without any loss of generality, we assume that the cube N defined by

$$N = \{(q_1, q_2, ..., q_n)|\ 0 \leq q_i \leq a, i = 1, 2, ..., n\}$$

contains the set A. Let $T_1 = \dfrac{a}{2}(n^2 - n + \sqrt{n} + 1)$.

Theorem 7. *Pursuit can be completed in the differential game (1)-(2) for the time T_1.*

Proof. Fictitious pursuers. Introduce fictitious pursuers. The fictitious pursuers can move either inside or outside the given closed bounded convex set A. However, they move only inside the cube N. Motions of the fictitious pursuers are described by the following equations:

$$\bar{P}_i : \dot{\bar{x}}_i = w_i, \quad \bar{x}_i(0) = x_{i0}, \quad |w_i| \leq 1, \quad i = 1, 2, ..., n,$$

where $\bar{x}_i$, w_i, $\in \mathbb{R}^n$.

According to this equation, the initial position of the fictitious pursuer $\bar{x}_i$ is the same as that of the pursuer x_i. The strategies of the fictitious pursuers on $[0, T_1]$ are defined as the strategies of real pursuers in the cube N (as described in the proof of Lemma above) and these strategies ensure the equality $\bar{x}_i(\tau) = y(\tau)$ at some $\tau \in [0, T_1]$ and $i \in \{1, 2, ..., n\}$. Real pursuers can move only inside the given set A. Thus, we introduced n fictitious pursuers, whose initial positions coincide with the initial positions of the real pursuers $x_{10}, x_{20}, ..., x_{n0}$, and move in N. By Lemma, they can complete the pursuit on

$[0, T_1]$.

Strategies of the real pursuers. Following Ivanov[10], construct a strategy for the pursuer x_i by means of the strategy of the fictitious pursuer $\bar{x}_i$. Define the control $u_i(t)$ of the pursuer x_i from the requirement of the equation $x_i(t) = F(\bar{x}_i(t))$, where $F(\bar{x}_i(t))$ is the projection of the point $\bar{x}_i(t)$ on the given set A, which is defined as follows:

$$|x - F(x)| = \min_{a \in A} |x - a|.$$

It is not difficult to show that the projection function $F(x), x \in \mathbb{R}^n$, for the closed convex set A has the following properties:

1. $F(x) = x$, if $x \in A$.
2. $|F(x) - F(y)| \leq |x - y|$ for all $x, y \in \mathbb{R}^n$.
3. If $\bar{x}(t)$, $t \geq 0$, is absolutely continuous, then $x(t) = F(\bar{x}(t))$, $t \geq 0$, is also absolutely continuous.

Since $\bar{x}_i(t)$ is absolutely continuous, then according to property 3 of projection so is $x_i(t)$. Admissibility of the pursuer's strategy can be shown as in the paper of Ivanov[10].

Proof that pursuit can be completed. Since $\bar{x}_i(\tau) = y(\tau)$ at some time $\tau \in [0, T_1]$ and $i \in \{1, 2, ..., n\}$ and $y(\tau) \in A$, then $\bar{x}_i(\tau) \in A$. This implies that $\bar{x}_i(\tau) = F(\bar{x}_i(\tau)) = x_i(\tau)$ (see Property 1). Thus, $y(\tau) = x_i(\tau)$. Hence, pursuit is completed as the position of a fictitious pursuer $\bar{x}_i$ coincides with the position of the evader y.

The proof of Theorem 7 is complete. □

4. Conclusion

We have studied a differential game of m pursuers and one evader on closed convex set A. The case $m < n$, where n is dimension of the space $\mathbb{R}^n$, was studied by Ivanov[10] and in this case evasion is possible. We have proved the theorem for n pursuers. Clearly, this implies validity of the theorem for $m \geq n$.

We have proved that guaranteed pursuit time is estimated by a second degree polynomial in n. Now, we can compare values of polynomials for guaranteed pursuit times. From the Figure 1 below, the first column of the table corresponds to Ivanov's guaranteed pursuit time, and the second column corresponds to ours one, where a is edge of the cube that contains the set A. We let $a = 1$ and obtained the result as follow.

n	$T = a(n^3 - 2n^2 + n + 1)$	$T = \frac{a}{2}(n^2 - n + \sqrt{n} + 1)$
1	1.00	1.00
2	3.00	2.21
3	13.00	4.37
4	37.00	7.50
5	81.00	11.62
6	151.00	16.73

Figure 1: The values of polynomials for guaranteed pursuit times.

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