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MANUFACTURING LOGISTICS AND SUPPLY CHAIN: RELATIONSHIP AND ECONOMIC IMPORTANCE

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Abstract: The Supply Chain (SC) describes the succession of suppliers, manufacturers, storage facilities, distributors and customers that allow products to be ordered, produced and delivered. Supply-Chain Management (SCM) is another aspect of Advanced Planning and Scheduling. It administers the flow of supplies, logistics, services and information through the supply-chain, from suppliers, manufacturers, sub-contractors, stores and distributors to customers and end-users. It involves business strategy, information flow and systems compatibility. Manufacturing is the activity consisting in to make a good with tools and/or machines by effecting chemical, mechanical, or physical transformation of materials, substances, or components, or by simulating natural processes, usually repeatedly and on a large scale with a division of labor. Due to the economic and strategic importance of knowing in advance the client's needs to develop the master production planning, we propose in this paper a mathematical model to link the activities of the supply chain with the development of the master plan of production using a mixed integer linear programming model with a stochastic approach. In our proposal we include a set of constrains tha represents the real necessities of the customers in markets of high density.

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1. Importance of the Chain Supply and Manufacturing Systems

Optimal production planning and scheduling is essential for manufacturing companies. It ensures efficient production, reliable delivery, and lower costs. Companies often find themselves trying to deal with large fluctuations in demand on one hand and a strong desire for balanced production on the other. Rigid production plans often make flexible reactions impossible. Stochastic methods of optimizing sequencing are often non-existent. As a result, lead times are long, processes are inefficient, and in-process inventories are high.

To manage their supply chain successfully and help their companies transform to "ruthless competitor" status, some manufacturers should consider developing a corporate vision that optimizes operational excellence, provides insightful analytics, deploys and measures strategic initiatives, and identifies and manages risks. To do that, companies should focus on consistent business practices and develop well-defined and aligned business, technical and operational strategies that increase their flexibility and speed and allow them to adapt to market dynamics in a profitable way. These initiatives should include supply management strategies such as sourcing and commodity and supplier development, as well as total cost management and performance measurements. In addition, corporate executives, business unit managers, program managers and operations personnel must support the company's overall strategic vision in order for that vision to succeed [1].

Master Production Scheduling (MPS) is the process of scheduling over time items that are critical in their impact on lower level components or in their requirements for capacity [2]. Items that are master scheduled may be end items, intermediate components, or a pseudo item that represents items grouped for the purpose of planning. Making the production plan requires a variety of inputs from supply chain, operations through to capacity planning for the assembly line. Coupled with this complexity, there is often a significant disconnection between the sales order input team and the production line orders taken without reviewing appropriate lead times or dependencies leaving manufacturing with an uphill battle.

Ensuring the maximization of productivity whilst at the same time managing costs is not easy. It is very common in manufacturing that demand profiles can fluctuate and customers may only provide a short term horizon of orders which makes long term business planning more difficult, and in many cases it presents an ideal opportunity to create a smoothed production plan. A SC is a system of organizations, people, activities, information, and resources involved in moving a product or service from supplier to customer. SC activities involve the transformation of natural resources, raw materials, and components into a finished product that is delivered to the end customer. The SCM is the process of planning, executing and managing the actions of the supply chain [3]. A supply chain constitutes the movement and storage of the reserves, supplies and finished goods from the point-of-origin to the end point, i.e., the point-of-consumption.

The SCM synchronize and amalgamate these flows both within and among companies. The main job of a Supply Chain executive or Manager is to manage the supply chain so that the cycle time can be reduced. The supply chain should be planned and implemented in a manner that there is coordination in the supply system. Thus, supply chain management responsible plays a key role in capturing customer demands, creating forecasts, developing schedules, ordering and managing inventory, controlling production orders, and maximizing customer satisfaction.

Traditionally, the design of a MPS and managing the SCM are analyzed separately and each problem is solved without linking their results. This leads to problems of synchronization operations, loss and theft of products, delay in delivery, poor inventory management, high transportation costs and others. Therefore, it is important to coordinate the planning activities of the production plant with the plant capacity, storage capacity, shipment capacity and disaggregated demand at retail level, important rule to meet in the MPS-SCM system is the customer service level. This concept is a function of several different performance indices. The first one is the order fill rate, which is the fraction of customer demands that are met from stock. For this fraction of customer orders, there is no need to consider the supplier lead times and the manufacturing lead times. The order fill rate could be with respect to a central warehouse or a field warehouse or stock at any level in the system. Stock out rate is the complement of fill rate and represents the fraction of orders lost due to a stock out. The backorder level is the number of orders waiting to be filled. To maximize customer service level, one needs to maximize order fill rate, minimize stock out rate, and minimize backorder levels. Another measure is the probability of on-time delivery, which is the fraction of customer orders that are fulfilled on time, i.e., within the agreed-upon due date [19]. In our proposal we use the probability of on-time delivery approach, which is the fraction of customer orders that are fulfilled on-time, i.e., within the agreed-upon due date [4].

In this project, we are interested in building the MPS of goods-producing firm that owns several manufacturing facilities that provide a variable set of products to meet customer demand from a random perspective. Here, each source, i.e., each plant has different demands, production capabilities and fixed and variable costs of production. This proposal is an extension of the document developed in [4], and by [5]. The problem is defined as establishing short-term production levels (specifying what to produce and how much to produce) of all manufacturing which provides a continuous supply of goods to a network. It is interconnected with manufacturing plants, distribution centers and customers to meet standards care with a success probability (retailers).

We are motivated in to develop the program of operation for each period specifying the daily production level, subject to fundamental constraints that must be satisfied such as the covering of each hourly demand, satisfaction of inventory level, and others. It is concerned with setting production rates by product group or other broad categories for short-term (days). It is also interested in showing some alternative mathematical models associated to the logistics of the system as their more representative variations.

In this proposal, the main purpose of MPS is to specify the optimal combination of production rate, and inventory on hand when the demand forecast D_t for each period t in the planning horizon T is given, and then, obtain the optimal delivery of products through the corresponding distribution network. The problem is addressed from a stochastic viewpoint because the demands per retailer are considered random variables. As in [4], we also consider two types or random variables representing the demand, the discrete and continuous case. In our proposal, we introduce new constrains associated to the problem, such as availability of inventory in the distribution centers, request that only p plants remain open during the production run or for certain time, and locate q plants and r warehouses simultaneously by using transshipment nodes.

Our document is organized as follows. In Section 2 we develop an analysis of the literature on the modeling and integration of (MPS) with (SCM). In Section 3, we describe the mathematical model by mentioning the new set of constraints associated with it. In Section 4 we do a thorough analysis of the statistics associated with the proposed models, and finally, in Section 5 we present the conclusions of our model.

2. Background

Next, we present a review of current literature on the subject. In [6], authors consider a stochastic crew-scheduling model and devise a solution methodology for integrating disruptions in the evaluation of crew schedules. Their goal was to use that information to find robust solutions that better withstand disruptions, and by identifying more robust schedules, cascading delay effects are minimized. They used the branching algorithm to solve their proposal. Meanwhile, in [7], authors scheduling problems of flexible chemical batch processes with a special emphasis on their real-time character. They used a large mixed-integer linear technique to approach this instance using a dual decomposition approach.

In [8], They consider the issue of call center scheduling in an environment where arrivals rates are highly variable, aggregate volumes are uncertain, and the call center is subject to a global service level constraint. They formulate the problem as a mixed-integer stochastic program and combine the server sizing and staff scheduling steps into a single optimization program. Also in [9], authors used a multi-stage stochastic programming approach in order to come up with the maximum expected profit given the demand scenarios. Controllable processing times enlarge the solution space so that the limited capacity of production resources is utilized. They showed that simple heuristic methods and multi-stage stochastic programming could be used effectively to solve the MPS problems.

More recently, [10] developed a novel integrated method for sequential batch processes under uncertainty. The integrated problem is formulated into a two-stage stochastic program. The first-stage decisions are modeled with binary variables for assignment and sequencing while the second-stage decisions are the remaining ones. They solve the resulting complicated integrated problem by using the framework of generalized Benders decomposition.

In relation to the problem of linking MPS with SCM, in [11] authors developed a two-stage 0-1 model to represent the supply chain management under uncertainty. Authors split the problem in two stages, in the first one they obtain the solution for the strategic decisions determining the production topology, plan sizing, product selection, product allocation among plants and vendor selection for raw materials. They related the second scenario with the tactical decisions for a better utilization of the supply chain along a time horizon with uncertainty in the product demand and price, and production and raw material costs. They proposed a two-stage version of a branch and fix coordination algorithmic approach for stochastic 0-1 program solving. In this sense, [12] developed a multi-objective two stage stochastic programming model to deal with a multi-period multi-product and multi-site production-distribution planning problem for a midterm-planning horizon. They involve majority of supply chain cost parameters such as transportation cost, inventory holding cost and shortage cost. Also, production cost, lead-time, outsourcing, employment, dismissal, workers-productivity and training are considered. They assumed that cost parameters and demand fluctuations are random variables departing from a pre-defined probability distribution and considering the traditional production-distribution-planning problem. This is one of the most important documents when authors includes (i) the minimization of the expected total cost of supply chain, (ii) the minimization of the variance of the total cost of supply chain and (iii) the maximization of the workers-productivity through training courses that could be held during the planning horizon. They solved the model applying a hybrid algorithm, that is, a combination of Monte Carlo sampling method.

A novel alternative to analyze the SPS problems is stochastic Petri nets. This tool is a convenient formalism for the representation and evaluation of Discrete Event Dynamic Systems (DEDS). The distribution of the stochastic process representing the time-evolution of a DEDS system at a certain given time is usually the basis for the quantitative evaluation of the behavior of the system. Often the transient analysis of these systems is mathematically very complex and simulation becomes the only viable technique. Carl Adam Petri introduced this approach in 1962. The stationary distribution of the stochastic process representing the time-evolution of a DEDS system is usually the basis for the quantitative evaluation of the behavior of the system expressed in terms of performance indices. Performance indices can be computed using a unifying approach in which proper index functions (also called reward functions) are defined over the states of the stochastic process and an expected reward is derived using the stationary distribution of the process.

In another order of ideas, SCM is an important competitive strategies used by modern enterprises. Effective design and management of supply chains assists in the production and delivery of a variety of products at low costs, high quality, and short lead times. Recently, Data Envelopment Analysis (DEA) has been extended to examine the efficiency of supply chain operations. Due to the existence of intermediate measures, the usual procedure of adjusting the inputs or outputs, as in the standard DEA approach, does not necessarily yield a frontier projection [13]. There are great quantities of papers in the literature that mention the application of technical analysis DEA in supply chain. In [14], authors developed a deep state of art about this topic; they show a list of industries where they found the most recent and successful applications of the DEA tool. For a detailed review of the DEA models applied in the domain of

SCM see [15, 16, 17, 18, 19].

3. The Mathematical Model

In our model we assume that the firm can produced any kind of product i, in any plant j, and all the production generated is sent immediately to any of the k Distribution Centers (DC), for $i \in I, j \in J$, and $k \in K$. Demand of product i in the distribution center k is consolidated from the sum of demands from l retailers, $l \in L$, and once manufactured, the finished products are sent immediately to the distribution center, i.e., in the first instance, inventories are not allowed at the production plants. Furthermore, each product has assigned a storage volume previously known. Finally, each product i should be delivered at each retailer l, to satisfy their demand at time t, (Figure 1).

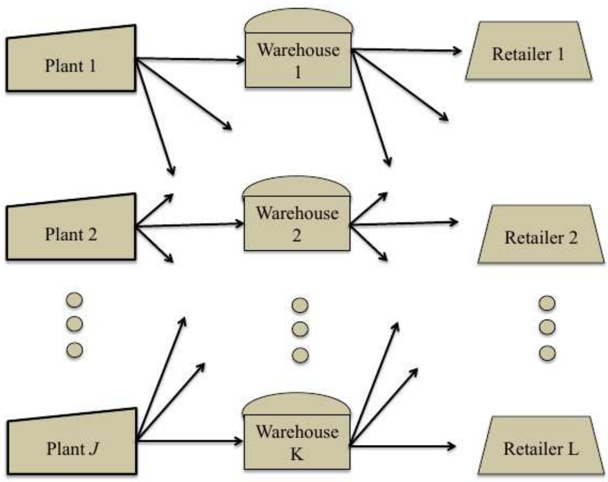

Figure 1: The system considered: Manufacturing plants, distribution centers and retailers.

Let the planning horizon be discretized into $t \in T$ uniform subintervals, we define the sets I, J, K, and L to identify the products, manufacturing plants, distribution centers, and retailers, respectively.

3.1. Notation and Problem Definition

Throughout this document we will use the following notation.

$x_{ijt} \equiv$ Amount of product i manufactured in plant j during time t, in pieces.

$I_{ikt} \equiv$ Inventory level of product i, in the distribution center k, at the end of time t, in pieces.

$c_{ijt} \equiv$ Variable cost of producing one unit of i product in plant j, at time t.

$h_{ikt} \equiv$ Inventory holding cost per day of product i, in the distribution center k, at time t.

$C_{ijt} \equiv$ Fixed cost associated with the production of product i in the plant j at time t.

$Q_r \equiv$ Fixed cost associated with the operation of the warehouse r at time t.

$y_{ijkt} \equiv$ Amount of product i shipped from plant j to distribution center k, at time t, in pieces.

$D_{ilt} =$ Demand of product i requested by the retailer l, at time t, in pieces/day.

$z_{iklt} =$ Amount of product i shipped from distribution center k to retailer l at time t, in pieces.

$\xi_{ijkt} =$ Unit cost of shipping product i from plant j to distribution center k at time t.

$\zeta_{iklt} =$ Unit cost of shipping product i from distribution center k to retailer l at time t.

$\vartheta_{ijt} =$ Production capacity of product i in plant j at time t, in pieces.

$\theta_{ikt} =$ Storage capacity of product i in distribution center k at time t, in pieces.

Thus, the problem can be defined as follows. For each $D_{ilt},\ i \in I,\ l \in L,\ t \in T$, we should obtain the optimal vector

$$\eta^* = (x^*, I^*, y^*, z^*, \theta^*, \beta^*)$$

such that $g(\eta^*) \leftarrow \min$, where each component of η^* is itself a vector whose entries are the variables already defined. Subsequently, we present a more detailed discussion about the size of the components $x^*, I^*, y^*, z^*, \theta^*, \beta^*$. The entire collection of η^* values are an optimized realization of the stochastic process

$$\Upsilon = \{\Upsilon_t := (x_{ijt}, I_{ikt}, y_{ijkt}, z_{kilt}, \theta_{ikt}, \beta_{ijt})\}_{t=1}^{T}$$

and due the randomness of demand, this means that even if the initial condition (or starting point) is known, there are many possibilities the process might go to, but some paths may be more probable and others less so. Our task is to develop an optimal schedule for each t in a horizon of length $\mid T \mid$.

Then, the mathematical model can be written as the minimization of all the costs generated by the operation of the system, subject to their technological constrains. Thus, we should minimize

$$\begin{aligned} g(\eta) = \; & \textstyle\sum_{t\in T}\sum_{j\in J}\sum_{i\in I}\left(c_{ijt}x_{ijt} + \beta_{ijt}C_{ijt}\right) + \\ & \textstyle\sum_{t\in T}\sum_{k\in K}\sum_{i\in I} h_{ikt}\, I_{ikt} + \\ & \textstyle\sum_{t\in T}\sum_{k\in K}\sum_{j\in J}\sum_{i\in I} \xi_{ijkt}\, y_{ijkt} + \\ & \textstyle\sum_{t\in T}\sum_{l\in L}\sum_{k\in K}\sum_{i\in I} \zeta_{iklt}\, z_{iklt}. \end{aligned} \tag{1}$$

Subject to

$$x_{ijt} \leq \beta_{ijt}\,\vartheta_{ijt},\; i\in I,\; j\in J,\; t\in T \tag{2}$$

$$\mathbf{P}\left[\sum_{l\in L} D_{ilt} \leq \sum_{j\in J} x_{ijt}\right] = 1-\alpha,\; i\in I,\; t\in T \tag{3}$$

$$\sum_{k\in K} y_{ijkt} = x_{ijt},\;\; i\in I,\;\; j\in J,\;\; t\in T \tag{4}$$

$$\sum_{j\in J} y_{ijkt} \leq \theta_{ikt},\;\; i\in I,\; k\in K,\; t\in T \tag{5}$$

$$\begin{aligned} \theta_{ikt} = \; & \theta_{i,k,(t-1)} + \textstyle\sum_{j\in J} y_{ijkt} - \sum_{l\in L} z_{iklt}, \\ & i\in I,\; k\in K,\; t\in T \end{aligned} \tag{6}$$

$$\begin{aligned} I_{ikt} = \; & I_{ik,(t-1)} + \textstyle\sum_{j\in J} y_{ijkt} - \sum_{l\in L} z_{iklt}, \\ & i\in I,\; k\in K,\; t\in T \end{aligned} \tag{7}$$

$$\frac{\varrho}{\mid L \mid}\sum_{l\in L} D_{ilt} \leq I_{ikt} \leq \theta_{ikt},\;\; i\in I,\; k\in K,\; t\in T \tag{8}$$

$$\sum_{j\in J} y_{ijkt} = \sum_{l\in L} z_{iklt},\;\; i\in I,\; k\in K,\; t\in T \tag{9}$$

$$\sum_{l\in L}\sum_{k\in K} z_{iklt} \geq \sum_{l\in L} D_{ilt},\;\; i\in I,\;\; t\in T \tag{10}$$

$$\sum_{k\in K} z_{iklt} = D_{ilt},\;\; i\in I,\;\; l\in L,\;\; t\in T \tag{11}$$

$$\beta_{ijt} = \begin{cases} 1, & \text{if product } i \text{ is manufactured at the} \\ & j\text{−th plant during } t, \\ 0, & \text{in other case,} \end{cases} \tag{12}$$

$$x_{ijt},\ I_{ikt},\ y_{ikt},\ z_{iklt} \geq 0 \text{ and integers, } \alpha \in (0,1) \tag{13}$$

Where $\mathbf{P}[x]$ represents the probability of x.

In the above formulation, Equation (2) defines the limits of production capacity for each product in each plant. Equation (3) represents the customer service level measure, understood here, as the probability of on-time delivery or the fraction of customer orders that are fulfilled on-time, i.e. within the agreed-upon due date. Equation (4) ensures that shipments to the distribution centers are equal to the manufactured products. In turn, shipments received in the distribution centers are limited by the storage capacity of these, Equation (5). Regarding the storage capacity, Equation (6) represents the balance equation of the storage capacity linking the products received and the demand required by the retailers. The inventory balance Equation (7), says that the amount of inventory in the next time period must equal to the current inventory, plus the received products from all the production plants, minus the demand required by the retailers. Equation (8) ensures that inventories are lower bounded by a level of security given by a percentage ϱ of the average demand for each product, and upper bounded by its storage capacity at the corresponding distribution center. The flow balance at the distribution centers is given by Equation (9), i.e., all the shipments to the retailers must be equal to the availability of inventory at the distribution centers [4].
Equations (10) and (11) ensure compliance of total demand D_{it} and per retailer respectively, where

$$D_{it} = \sum_{l \in L} D_{ilt},\ i \in I,\ \ t \in T. \tag{14}$$

Equation (12) is an indicator variable related with the fixed charges of the problem. Finally, Equation (13), expresses the non-negativity and integrity conditions, and the range of α. The integrity conditions can be easily removed in cases where the model required a continuous demand.
For treatment of Equation (3), assume that the probability density function (pdf) of the random variable D_{it} is known for all $t \in T$, and it is given by $f_{D_{\mathrm{it}}}(\xi)$. Then, Equation (3) is equivalent to

$$\mathbf{P}\left[D_{it} \leq \sum_{j\in J} x_{ijt}\right] = \int_0^{\rho} d\,F_{D_{\rm it}}(\xi) = 1-\alpha, \tag{15}$$

where $\rho = \sum_{j\in J} x_{ijt}$, $\alpha \in (0,1)$, and $F_{D_{\rm it}}$ is the cumulative density function (cdf) of the random variable D_{it}.

If the pdf of D_{it} is a "nice" theoretical distribution then, Equation (3) is well defined. Otherwise, when we only have sample realizations of it, the pdf of D_{it} can be estimated from historical data or using Monte Carlo Method, and Equation (3) can be get from simple rules of an (S,s) inventory system.

Other significant constrains regularly considered in the model are (Assuming the existence of fixed costs for each plan open):

1. Availability of inventory in the distribution centers. Suppose that the amount requested is g_l, then it is sufficient to add a constraint over the model given by

$$I_{ikt} \geq g_l, \;\; i \in I,\, j \in J,\, t \in T$$

2. Request that only p plants remain open during the production run or for certain time.

$$\sum_{t\in T}\sum_{j\in J}\sum_{i\in I} \beta_{ijt} = p$$

3. Locating q plants and r warehouses simultaneously. Here, we assume that exist another fix cost μ for each warehouse open. Then, the objective function must include the fixed costs for each open plan i.e.,

$$C_{ij}\,\beta_{ij} + C_{jk}\,\beta_{jk} + \ldots + C_{pq}\,\beta_{pq}$$

and the costs for each warehouse open are

$$Q_{ij}\,\mu_{ij} + Q_{jk}\,\mu_{jk} + \ldots + Q_{pr}\,\mu_{pr}$$

where $i, j, p,$ and q are transshipment nodes

$$\sum_{k\in K} y_{ijkt} = x_{ijkt}\,\mu_{jk}, \;\; i \in I,\, j \in J,\, t \in T.$$

3.2. About the Number of Constraints and Variables

The solution of the proposed instance is generally difficult. So, there have been several attempts to approach the problem from different approaches, for example Ant Colony Optimization (ACO). ACO is a population-based met heuristic for the solution of difficult combinatorial optimization problems. In ACO, each individual of the population is an artificial agent that builds incrementally and stochastically a solution to the considered problem. Agents build solutions by moving on a graph-based representation of the problem. At each step their moves define which solution components are added to the solution under construction. A probabilistic model is associated with the graph and is used to bias the agents' choices. The probabilistic model is updated on-line by the agents so as to increase the probability that future agents will build good solutions [20]. In our proposal we formulated a mathematical model and it has solved by using the simplex algorithm. In practice, this tool is quite efficient and can be guaranteed to find the global optimum if certain precautions against cycling are taken. The simplex algorithm has been proved to solve 'random" problems efficiently, i.e., in a cubic number of steps [5], which is similar to its behavior on practical problems.

Our model uses integer variables due to some situations involving MPS and SCM cannot be modeled by linear programming, but are easily handled by integer programming, however it can be solved using swarm intelligent methods. Although one can model a binary decision in linear programming with a variable that ranges between 0 and 1, there is nothing that keeps the solution from obtaining a fractional value such as 0.5, hardly acceptable to a decision maker. Integer programming requires such a variable to be either 0 or 1, but not in between. Unfortunately, integer-programming models of practical size are often very difficult or impossible to solve. Linear programming methods can solve problems orders of magnitude larger than integer programming methods. In practice, models must be both tractable, capable of being solved and valid, representative of the original situation. These dual goals are often contradictory and are not always attainable. It is generally true that the most powerful solution methods can be applied; the simplest or most abstract the model.

In [4] authors showed that the instance here proposed is NP complex. In connection with the number of variables involved in this, they also probe that; the required variables in the model is given by: $x_{ij} =| I || J || T |$, $I_{ikj} =| I || K || (T+1) |$, $\beta_{ijt} =| I || J || T |$, $y_{ijkt} =| I || J || T || K |$, $z_{iklt} =| I || L || T || K |$, $\theta_{ikt} =| I || K || (T+1) |$. Here, $| A |$ represents the cardinality of the set A.

Integer programs often have the advantage of being more realistic than linear programs, but the disadvantage of being much harder to solve. Thanks to the advances in computing of the past decade, linear programs with a few thousand variables and constraints are nowadays viewed as "small". Problems having tens or hundreds of thousands of continuous variables are regularly solved; tractable integer programs are necessarily smaller, but are still commonly in the hundreds or thousands of variables and constraints. In our proposal, the availability of inventory at the distribution centers given by $I_{ikt} \geq C_i$, $i \in I, j \in J, t \in T$, increases $\mid I \mid\mid J \mid\mid T \mid$ constrains to the model (Table 2). Also, request that only p plants remain open during the production run or for certain time $\sum_{t \in T} \sum_{j \in J} \sum_{i \in I} \beta_{ijt} = p$, increases $\mid t \mid\mid j \mid\mid i \mid$ constrains more. Finally locating q plants and r warehouses simultaneously given by equations $C_{ij}\beta_{ij} + C_{jk}\beta_{jk} + \ldots + C_{pq}\beta_{pq}$ and $q_{ij}\mu_{ij} + q_{jk}\mu_{jk} + \ldots + q_{pr}\mu_{pr}$, add $\mid pq \mid + \mid pr \mid$ more constrains.

In short, the whole group of equations involved have the following number of associated constrains. Eq (2): $\mid I \mid\mid J \mid\mid T \mid$; Eq (3): $\mid I \mid\mid T \mid$; Eq (4): $\mid I \mid\mid T \mid\mid J \mid$; Eq (5): $2 \mid I \mid\mid T \mid\mid K \mid$, Eq (6): $\mid I \mid\mid (T+1) \mid\mid K \mid$; Eq (7): $\mid I \mid\mid (T+1) \mid\mid K \mid$; Eq (8): $2 \mid I \mid\mid K \mid$; Eq (9): $\mid I \mid\mid T \mid\mid K \mid$, Eq (10): $\mid I \mid\mid T \mid$; Eq (11) $\mid I \mid\mid T \mid\mid L \mid$; Eq (12) $\mid I \mid\mid T \mid\mid J \mid$.

4. Statistical Properties of the Model

Next, we discuss some important properties found in this model. From Equation (14), we enable the moment generating function of the random variable D_{it}, denoted by $m_{D_{it}}$ and defined as

$$m_{D_{it}} = \int_{-\infty}^{\infty} e^{\varphi D_{it}} dF_{D_{it}}, \quad -h \leq \varphi \leq h, \;\; h > 0, \; i \in I, \; t \in T.$$

Formally

Theorem 1: If $D_{i1t}, \ldots, D_{i|L|t}$ are independent random variables and the moment generating function of each exist for all $-h < \varphi < h$ for some $h > 0$, let $D_{it} = \sum_{l \in L} D_{ilt}$ then, the moment generating function of D_{it} is given by

$$m_{D_{it}} = \Pi_{l \in L}\, m_{D_{ilt}}(\varphi), -h \leq \varphi \leq h, \, h > 0, \; i \in I, \; t \in T. \tag{16}$$

Proof: From Equations (11) and (14), the proof is trivial see [21], □.

Theorem 2: For each $t \in T$, let Z be the random variable defined by $Z = \sum_{l \in L} \sum_{k \in K} \sum_{i \in I} z_{iklt}$, then

$$m_Z = \Pi_{l \in L}\, \Pi_{i \in I}\, m_{D_{ilt}}(\varphi), -h \leq \varphi \leq h, \, h > 0, \; t \in T. \tag{17}$$

Proof: From Equation (11) and (14) and by the definition of Z we have

$$Z = \sum_{l\in L}\sum_{k\in K}\sum_{i\in I} z_{iklt} = \sum_{l\in L}\sum_{i\in I} D_{ilt} = \sum_{i\in I} D_{it}. \tag{18}$$

Then for each $t \in T$, assuming that the moment generating function of each D_{it} exists, and applying the results of Theorem (1) we obtain

$$m_Z = m_{\sum_{i\in I} D_{it}} = \Pi_{i\in I}\, m_{D_{it}} = \Pi_{l\in L}\,\Pi_{i\in I}\, m_{D_{ilt}}(\varphi),$$

and the theorem is proved $\square$.

Corrollary 1: For each $t \in T$, let D', X', Y' and Z' be the random variables defined as the total demand required by retailers, the total production manufactured in plants, the total volume of shipments from manufacturing plants to distribution centers, and the total volume of shipments from distribution centers to retailers respectively, then, under the assumptions of Theorems (1) and (2) the four variables are identically distributed.
Proof: The proof is easily constructed from previously obtained results. From the definitions of X', Y' and Z' we have that, for each $t \in T$:

$$X' = \sum_{j\in J}\sum_{i\in I} x_{ijt},\ \ Y' = \sum_{k\in K}\sum_{j\in J}\sum_{i\in I} y_{ijkt},\ \text{and } Z' = \sum_{l\in L}\sum_{k\in K}\sum_{i\in I} z_{iklt}.$$

From Equation (18) it is obvious that $Z' = D'$ as $m_{Z'}(\varphi) = m_{\sum_{i\in I} D_{it}}(\varphi)$. Similarly, from Equation (9) it is satisfied

$$\sum_{k\in K}\sum_{j\in J}\sum_{i\in I} y_{ijkt} = \sum_{l\in L}\sum_{k\in K}\sum_{i\in I} z_{iklt}.$$

i.e., $Y' = Z'$ as $m_{Y'}(\varphi) = m_{Z'}(\varphi)$. Applying the same arguments to Equation (4), and by transitivity the results are evident $\square$.

Other obvious and immediate result arising from the proposed model is also formalized.

Corrollary 2: In the developed model, the total inventory of product i at the distribution centers is constant for $t \in T$ and it is equal to the initial inventory proposed.
Proof: By corollary (1) and from Equation (7), we have that for $t \geq 1$

$$\begin{aligned}\textstyle\sum_{k\in K}\sum_{i\in I} I_{ikt} = \ & \textstyle\sum_{k\in K}\sum_{i\in I} I_{ik,(t-1)} + \sum_{k\in K}\sum_{j\in J}\sum_{i\in I} y_{ijkt} - \\ & \textstyle\sum_{l\in L}\sum_{k\in K}\sum_{i\in I} z_{iklt} \ = \\ & \textstyle\sum_{k\in K}\sum_{i\in I} I_{ik,(t-1)} + Y' - Z'\end{aligned}$$

i.e., $\sum_{k\in K}\sum_{i\in I} I_{ikt} = \sum_{k\in K}\sum_{i\in I} I_{ik,(t-1)}$, thus, solving this equality from $t = 1$ we obtain the proposed result $\square$.

In order to approximate the distribution of the costs function, we assume that the random variables z_{iklt} are independent and identically distribute

Corrollary 3: Assume that the variables z_{iklt} are independent and identically distributed. The pdf of the objective function (1) can be characterized through its moment generating function as follows.

$$m_{g(\eta)} = \hat{\upsilon}\,(m_\phi\, m_\psi\, m_\omega)\,(\varphi),\ -h \le \varphi \le h,\, h > 0. \tag{19}$$

where the estimator $\hat{\upsilon}$ can be approximated by

$$\hat{\upsilon} = \exp\left(\sum_{t\in T}\sum_{j\in J}\sum_{i\in I} \beta_{ijt}C_{ijt} + \sum_{t\in T}\sum_{k\in K}\sum_{i\in I} h_{ikt}I_{ikt}\right)(\varphi) \tag{20}$$

Proof: Let ω and ω^1 be the random variables defined as

$$\omega = \sum_{t\in T}\sum_{l\in L}\sum_{k\in K}\sum_{i\in I}\zeta_{iklt}z_{iklt},\quad \omega^1 = \sum_{i\in I}\zeta_{iklt}z_{iklt}.$$

Then, for $t \in T$, $l \in L$, $k \in K$, and $-h \le \varphi \le h,\ h > 0$ we have

$$m_{\omega^1} = \mathbf{E}\left[\exp\sum_{i\in I}\zeta_{iklt}\, z_{iklt}\right] = \Pi_{i\in I}\, m_{\zeta_{iklt}\, z_{iklt}}(\varphi),$$

Similarly, for $\omega^2 = \sum_{k\in K}\omega^1_k = \sum_{k\in K}\sum_{i\in I}\zeta_{iklt}\, z_{iklt}$

$$m_{\omega^2} = \mathbf{E}\left[\exp\sum_{k\in K}\omega^1_k\right] = \Pi_{k\in K}\, m_{\omega^1}(\varphi) = \Pi_{k\in K}\,\Pi_{i\in I}\, m_{\zeta_{iklt}\, z_{iklt}}(\varphi)$$

Then for ω, its moment generating function is

$$m_\omega = \Pi_{t\in T}\Pi_{l\in L}\Pi_{k\in K}\Pi_{i\in I}\, m_{\zeta_{iklt}\, z_{iklt}}(\varphi),\ -h \le \varphi \le h,\, h > 0. \tag{21}$$

Proceeding in a similar way with the variables

$$\psi = \sum_{t\in T}\sum_{k\in K}\sum_{j\in J}\sum_{i\in I}\xi_{ijkt}y_{jikt},\ \phi = \sum_{t\in T}\sum_{j\in J}\sum_{i\in I}c_{ijt}x_{ijt},$$

and for $-h \le \varphi \le h$, $h > 0$ we have

$$m_\psi = \Pi_{t\in T}\Pi_{k\in K}\Pi_{j\in J}\Pi_{i\in I}\ m_{\xi_{ijkt}\, y_{jikt}}(\varphi), \tag{22}$$

$$m_\phi = \Pi_{t\in T}\Pi_{j\in J}\Pi_{i\in I}\ m_{c_{ijt}\, x_{ijt}}(\varphi), \tag{23}$$

Therefore, Equation (1) can be rewritten as

$$\begin{aligned} g(\eta) = \quad & \phi + \psi + \omega + \textstyle\sum_{t\in T}\sum_{j\in J}\sum_{i\in I}\beta_{ijt}C_{ijt} + \\ & \textstyle\sum_{t\in T}\sum_{k\in K}\sum_{i\in I} h_{ikt}\, I_{ikt} = \phi + \psi + \omega + \upsilon \end{aligned} \tag{24}$$

where

$$\upsilon = \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} \beta_{ijt} C_{ijt} + \sum_{t\in T}\sum_{k\in K}\sum_{i\in I} h_{ikt}\, I_{ikt},$$

and by the arguments used above

$$m_{g(\eta)} = \hat{\upsilon}\,(m_\phi\, m_\psi\, m_\omega)\,(\varphi),\ -h \le \varphi \le h,\, h > 0,\ \Box \tag{25}$$

For modeling continuous demand, the normal distribution is the most used [2], [22]. And for discrete demands, some authors use the Poisson distribution [23]. In the first case, when demand is modeled by a normal distribution, i.e., when $D_{ilt} \sim N(\mu_{ilt}, \sigma_{ilt})$, for each $i \in I$, $l \in L$, $t \in T$ and when μ_{ilt} and σ^2_{ilt} are known, the results are as follows. As we showed, X', Y', and Z' are identically distributed for each $t \in T$, when D is defined as in Corollary (1), thus, for the entire sample on the horizon, and using Equations (18) and (21), it is reasonable to assume that $\omega \sim N(\mu_\omega, \sigma^2_\omega)$, where

$$\mu_\omega = \mu \sum_{t\in T}\sum_{l\in L}\sum_{k\in K}\sum_{i\in I} \zeta_{iklt} \tag{26}$$

$$\sigma^2_\omega = \sigma^2 \sum_{t\in T}\sum_{l\in L}\sum_{k\in K}\sum_{i\in I} \zeta^2_{iklt} \tag{27}$$

and

$$\mu \approx \mu_{z_{iklt}} = \frac{\sum_{t\in T}\sum_{l\in L}\sum_{i\in I} \mu_{ilt}}{\mid T \mid\mid L \mid\mid I \mid} \tag{28}$$

$$\sigma^2 \approx \sigma^2_{z_{ikl}} = \frac{\sum_{t\in T}\sum_{i\in I} \sigma^2_{ilt}}{\mid T \mid\mid L \mid\mid I \mid} \tag{29}$$

Similarly, $\phi \sim N(\mu_\phi, \sigma^2_\phi)$ and $\psi \sim N(\mu_\psi, \sigma^2_\psi)$ where

$$\mu_\psi = \mu \sum_{t\in T}\sum_{k\in K}\sum_{j\in J}\sum_{i\in I} \xi_{ijkt} \tag{30}$$

$$\sigma^2_\psi = \sigma^2 \sum_{t\in T}\sum_{k\in K}\sum_{j\in J}\sum_{i\in I} \xi^2_{ijkt} \tag{31}$$

$$\mu_\phi = \mu \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} c_{ijt} \tag{32}$$

$$\sigma^2_\phi = \sigma^2 \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} c^2_{ijt}. \tag{33}$$

Finally, by equation (25)

$$\begin{aligned} g(\eta) &\sim N\left(\mu_\omega + \mu_\psi + \mu_\phi + \upsilon, \sigma^2_\omega + \sigma^2_\psi + \sigma^2_\phi\right) \\ &= N\left(\mu_{tot}, \sigma_{tot}\right) \end{aligned} \tag{34}$$

The case when demand is discrete is also considered. Suppose now, z_{iklt} are independent Poisson random variables for $i \in I$, $k \in K$, $l \in L$, $t \in T$, i.e., each $z_{iklt} \sim P(\lambda)$. Thus, by Theorem (2) we have

$$\begin{aligned} m_{\tilde{Z}} &= \Pi_{t\in T}\,\Pi_{l\in L}\,\Pi_{i\in I}\, m_{D_{ilt}}(\varphi) \\ &= \textstyle\sum_{t\in T}\sum_{l\in L}\sum_{k\in K}\sum_{i\in I} \lambda_{iklt}\, \exp\left(e^{\varphi}-1\right), \\ &-h \leq \varphi \leq h,\ h>0,\ \lambda>0, \end{aligned} \tag{35}$$

where $\tilde{Z} = \sum_{t\in T}\sum_{l\in L}\sum_{k\in K}\sum_{i\in I} z_{iklt}$. Therefore, under the hypothesis of Corollary (3) we have that an estimator for λ is

$$\hat{\lambda} = \frac{\sum_{t\in T}\sum_{l\in L}\sum_{k\in K}\sum_{i\in I} \lambda_{iklt}}{\mid T \mid\mid L \mid\mid K \mid\mid I \mid}, \tag{36}$$

and by Equation (21)

$$m_{\omega} = \sum_{t\in T}\sum_{l\in L}\sum_{k\in K}\sum_{i\in I} \zeta_{iklt}\,\hat{\lambda}\,\left(e^{\varphi}-1\right),$$

$$-h \leq \varphi \leq h,\ h>0.$$

i.e.,

$$\lambda_{\omega} = \mathbf{Var}\,(\omega) = \hat{\lambda}\sum_{t\in T}\sum_{l\in L}\sum_{k\in K}\sum_{i\in I} \zeta_{iklt} \tag{37}$$

analogously

$$\lambda_{\psi} = \mathbf{Var}\,(\psi) = \hat{\lambda}\sum_{t\in T}\sum_{k\in K}\sum_{j\in J}\sum_{i\in I} \xi_{ijkt} \tag{38}$$

$$\lambda_{\phi} = \mathbf{Var}\,(\phi) = \hat{\lambda}\sum_{t\in T}\sum_{j\in J}\sum_{i\in I} c_{ijt} \tag{39}$$

finally, and by Equation (34)

$$g(\eta) \sim \mathrm{Exp}\left(\lambda_{\omega}+\lambda_{\psi}+\lambda_{\phi}+\upsilon\right). \tag{40}$$

These results are interesting and show that, for the model developed, we can estimate the probability distribution associated with the function of costs in terms of the parameters of the demand from retailers.

5. Conclusions

Supply chain management SCM has been a major component of competitive strategy to enhance organizational productivity and profitability. The role of the SCM measures and metrics in the success of an organization cannot be overstated because they affect strategic, tactical and operational planning and control. Performance measurement and metrics have an important role to play in setting objectives, evaluating performance, and determining future courses of actions. Performance measurement and metrics pertaining to SCM have not received adequate attention from researchers or practitioners. We developed a framework to promote a better understanding of the importance of SCM performance measurement and metrics. To do this, we have addressed the document presented in [4]. We extend the constrains from the original model, including the number of plants to be inserted, and the decisions of which plants open and what service centers must meet. We show that the new set of restrictions does not affect the statistical properties of the original model and the probability distribution of the cost function can be obtained from the probability distribution on the right side of the technological constraints of the linear programming model. Our self-results show that the magnitude of the problem (in terms of number of variables and constraints) increases markedly and can reach back to the model impractical. However, its structure is valuable for situations such as described herein and methods of solution are an interesting challenge to deal with emerging alternatives such as heuristic and meta-heuristic methods. The assumptions made about the probability distribution of retailers demand led to interesting results regarding the probability distribution of the shipments made from manufactures plants to distribution centers, and from them to retailers, as well as the costs function of the model. Future investigations in this area should consider the computational complexity of the model to include finer details in it, such as suppliers and/or subcontractors, cash flows or approaches to determine the probability distributions that are generated when the samples of the demand are rare (scattered or small). The estimation of bounds on the cost function is another promising line to be addressed. It should also be integrated also into future models, new constrains as times of movement of products on the network, quantities to be delivered, quality of the shipments and other obligations required in the Supply-chain operations reference-model (SCOR). The SCOR model was designed to enable companies to communicate, compare and learn from competitors and companies both within and outside of their industry. It not only measures supply chain performance but also effectiveness of supply chain reengineering. It is a process reference model. Although always difficult to do, SCOR sets the standard for delineating new constrains to consider in the design of mathematical models. The new optimization technology based on swarm intelligence is one alternative.

References

[1] Gainingaltitude. (2012, February). Retrieved October 15, 2015, from Supply chain and manufacturing: Focus on three Vs to improve supply chain manage-

ment: https://www.pwc.com/us/en/industrial-products/assets/gaining-altitude-issue-1-feb-2012-.pdf

[2] D. Sipper and Robert L. Bulfin, J. . *Production: Planning, Control and Integration*, NY: McGraw-Hill Companies, Inc. (1997).

[3] Ballou, R. H. *Business Logistics: Supply Chain Management* (Fifth Edition ed.), New Jersey: Pearson Prentice Hall, (2003).

[4] Pérez-Lechuga, G., C.F. Flores-Rivera, A.A. Karelyn, A. Tarasenko. A multi-period stochastic model to link the supply chain with the production plan in a manufacturing organization, *Advances and Applications in Statistical Sciences*, Mili Publications, Volume 6, Issue 4, July 2011, Pages 255-283, (2011).

[5] Römish, N. G.-K. Stochastic Unit Commitment Power Production Planning, in *Applications of Stochastic Programming.* (S. W. Ziemba, Ed.) Philadelphia, PA., EEUU,: MPS-SIAM Series on Optimization, (2005).

[6] Yen, J. W., & Birge, J. R. A stochastic programming approach to the airline crew scheduling problem, *Transportation Science*, 3-14, (2006).

[7] Sand, G., & Engell, S. Modeling and solving real-time scheduling problems by stochastic integer programming. *Computers & Chemical Engineering*, 28 (6-7), 1087-1103, (2004).

[8] Robbins, T. R., & Terry P. Harrison. A stochastic programming model for scheduling call centers with global Service Level Agreements, *European Journal of Operational Research*, 207 (3), 1608-1619, (2010).

[9] Körpeoŏlu, E., Yaman, H., & Aktrk, M. S. A multi-stage stochastic programming approach in master production scheduling, *European Journal of Operational Research*, Stochastics and Statistics, Vol (213), 166-179, (2011).

[10] Chu, Y., & You, F. Integration of scheduling and dynamic optimization of batch processes under uncertainty: Two-stage stochastic programming approach and enhanced generalized Benders decomposition algorithm, *Industrial and Engineering Chemical Research*, 52 (47), 16851-16869, (2013).

[11] Alonso-Ayuso, P., Escudero, L. F., Ortuo, A. G., & Pérez, G. An approach for strategic chain planning under uncertainty based on stochastic 0-1 programming, *J. Global Optimization*, 26, 97-124, (2003).

[12] Mirzapour, S., Al-e-Hashem, Baboli, A., Sadjadi, S. J., & Aryanezhad, M. B. A multiple objective stochastic production-distribution planning problem in a uncertain environment considering risk and workers productivity. *Math. Prob. Engi.*, 1-14, (2011).

[13] Alireza A., L. K. A Data Envelopment Analysis approach to supply chain efficiency, *Advances in Decision Sciences*, 1-9, (2011).

[14] Gürdal, E., A.Nazli, & Gamze, T. DEA-Based benchmarking models in supply chain management: An application-oriented literature review, *International Logistics and Supply Chain Congress' 2012*. Istanbul, Turky, (2012).

[15] Ahn, H. J., & Lee, H. An agent-based dynamic information network for supply chain management. *BT Technology Journal*, 22 (2), 18-27, (2004).

[16] Akdeniz, A. H., & Turgutlu, T. Supplier selection on retail: Analysis with two multi-criteria evaluation methodologies. *Review of Social, Economic & Business Studies*, 9-10, 11-28, (2007).

[17] Azadeh, A., & Alem, S. M. A flexible deterministic, stochastic and fuzzy Data Envelopment Analysis approach for supply chain risk and vendor selection problem: Simulation analysis, *Expert Systems with Applications*, 37 (12), 7438-7448, (2010).

[18] Emrouznejad, A., Parker, B., & Tavares, G. Evaluation of research in efficiency and productivity: A survey and analysis of the first 30 years of scholarly literature in DEA, *Journal of Socio-Economic Planning Sciences*, 42 (3), 151-157, (2008).

[19] Ha, S., Kwon, E., Jin, J., & Park, H. Single and multiple sourcing in the Auto-Manufacturing industry, *International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing Engineering*, Vol (3), No (8), (2009).

[20] M. Dorigo, Optimization, Learning and Natural Algorithms, Ph.D. thesis, *Politecnico di Milano*, Italie, (1992).

[21] Mood, A. M., Graybill F. A., Boes, C. D., *Introduction to the Theory of Statistics*, Mc. Graw-Hill International Editions, (1974).

[22] Pérez-Lechuga, G., Karelyn, A., Orán, M. M., & Aragonés, L. P. Stochastic analysis of operations decoupling in flexible manufacturing systems, *WSEAS Trans. Syst.*, 4 (11), 2138-2143, (2005).

[23] V. Jeet, E Kutanoglu, and A. Partany, Logistics network design with inventory stocking for low-demand parts: Modeling and optimization, *IEE Transactions,* (41), 389-407, (2008).