International Journal of Pure and Applied Mathematics
Volume 99 No. 1 2015, 23-36
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v99i1.3

ijpam.eu

# FUZZY $\omega$-AUTOMATA AND ITS RELATIONSHIPS

R. Cokilavany[1], R. Venkatesan[2], J. Kavikumar[3 §]

[1,2]Department of Mathematics
Anna University
Chennai, 600 025, INDIA

[3]Department of Mathematics and Statistics
Faculty of Science, Technology and Human Development
Universiti Tun Hussein Onn Malaysia
84600 Batu Pahat, Johor, MALAYSIA

**Abstract:** A notion of finite $\omega$ - automata with single initial state is proposed. The concept of fuzzy deterministic Buchi automaton and Muller automaton with full acceptance component which is recognize the same fuzzy language are studied. We also establish the relationship between fuzzy deterministic Rabin automaton and Muller automaton. Further, we define the transition fuzzy $\omega$ - automata and show that these automata recognize the same fuzzy language as in the fuzzy $\omega$ - automata. Finally, we give some closure properties of fuzzy deterministic $\omega$ - automata.

**AMS Subject Classification:** 68Q70, 68Q45, 18B20, 03E72
**Key Words:** fuzzy $\omega$-automata, fuzzy power set, fuzzy Buchi automaton, fuzzy Muller automaton, fuzzy Rabin automaton

## 1. Introduction

A fuzzy $\omega$ - automaton is a fuzzy automaton, which is a nonterminating computational machine. It has a different acceptance components unlike a set of accepting states as in usual fuzzy automaton [2] and [6]. A $\omega$ -language is a

Received: April 18, 2014



§Correspondence author

collection of infinite strings over a finite alphabet. Fuzzy languages have been originally introduced in [5] where the weight function $W_L : A^* \longrightarrow \{0,1\}$ of a language $L$ over an alphabet $A$ has been generalized to $W_L : A^* \longrightarrow [0,1]$. A notion of fuzzy $\omega$ - automaton was introduced by several authors [3], [4] and [9]. Although there are different definitions and notations used in the literature for classical $\omega$ - automaton, it was originally introduced by Buchi, Muller and Rabin [1], [7] and [8].

In this paper the notion of Rozenberg and Salomaa [10] has been taken to define fuzzy $\omega$ - automaton. In Section 2, we introduce the classes of fuzzy $\omega$ - automata. In Section 3, we study the relationships among fuzzy deterministic $\omega$ - automata. In Section 4, we define the transition fuzzy $\omega$ - automata and prove that the same fuzzy language can be recognized by both the transition fuzzy $\omega$ - automata and fuzzy $\omega$ automata. In Section 5, we prove that the fuzzy languages recognized by fuzzy deterministic $\omega$ - automata are closed under intersection. The paper concludes with Section 6.

## 2. Classes of Fuzzy $\omega$-Automata

In this section, we introduce the concept of fuzzy $\omega$ - automaton with different acceptance component. The classes of fuzzy $\omega$ - automaton differ only in terms of acceptance component.

**Definition 1.** A (nondeterministic) fuzzy $\omega$ - automaton is a 5 - tuple $\mathfrak{F} = (S, A, f, I, Acc)$, where

- $S$ is the finite set of states.
- $A$ is the finite set of input alphabets.
- $f : S \times A \times S \longrightarrow F$ is the fuzzy transition function. Where $F = [0,1]$, the fuzzy interval.
- $I : s_1 \longrightarrow F$ is an initial state, $s_1 \in S$.
- $Acc$ is the acceptance component.

A fuzzy $\omega$ - automaton $\mathfrak{F}$ is said to have deterministic transition, if for every state $s \in S$ and every symbol $a \in A$ , there is at most one state $t \in S$ and one constant $c > 0 \in F$ such that $(s, a, t, c)$ is a transition. It is deterministic if it has deterministic transition. Let $A^*$ denote the set of finite words on $A$ and $A^+$

denote the set of nonempty finite words on $A$ and $\Lambda$ denote an empty word. The extension of the transition function $g : S \times A^* \times S \longrightarrow F$ is defined as follows, for every $u \in A^*$ and $s \in S$, if $u = \Lambda$, then $s \bullet \Lambda = s$ and for $u = u_1u_2 \dots u_k \neq \Lambda$

$$g(s, u, s_k) = \wedge\{f(s, u_1, s_1), f(s_1, u_2, s_2), \cdots, f(s_{k-1}, u_k, s_k)| \\ s \bullet u_1 = s_1, s_1 \bullet u_2 = s_2, \cdots, s_{k-1} \bullet u_k = s_k\}$$

Let $A^\omega$ denote the set of infinite words ($\omega$ - words) on $A$. The extension of the transition function $S \times A^\omega \times S \longrightarrow F$ is defined by the similar way. A run of the fuzzy $\omega$ - automaton $\mathfrak{F}$ on a $\omega$ - word $\omega = \omega_1\omega_2 \cdots$ is a $\omega$ - word $R = s_1s_2 \cdots \in S^\omega$ such that $I(s_1) > 0$ and $f(s_i, \omega_i, s_{i+1}) > 0$ for $i \geq 1$. The set of infinitely often states occurs in the run $R$ is denoted by $inf(R)$. That is

$$inf(R) = \{s \in S|\text{there exists infinitely many i such that } s_i = s\} \tag{1}$$

A $\omega$ - word is called accepted word by $\mathfrak{F}$, if the corresponding run $R$ is successful. The set of $\omega$ - words recognized by $\mathfrak{F}$ is a set, denoted by L($\mathfrak{F}$), of labels of successful runs in $\mathfrak{F}$. A set X of $\omega$ - words is recognizable if there exists a fuzzy Buchi automaton $\mathfrak{F}$ such that X=L($\mathfrak{F}$).

**Definition 2.** A fuzzy Buchi automaton is a 5 - tuple $\mathfrak{F} = (S, A, f, I, \mathfrak{B})$, where

- $(S, A, f, I)$ is a fuzzy $\omega$ - automaton.
- $\mathfrak{B} : S \longrightarrow F$ is the set of fuzzy final states.

A run in $\mathfrak{F}$ is successful if it visits $\mathfrak{B}$ infinitely often, that is $inf(R) \cap \mathfrak{B} \neq \phi$. The weight of the accepted word $\omega$ is calculated as follows

$$W(\omega) = \vee\{\wedge\{\{I(s_1)\} \bigcup \{f(s_i, \omega_j, s_k)|i, j, k \geq 1\} \\ \bigcup \{\mathfrak{B}(t)|t \in inf(R) \bigcap \mathfrak{B}\}\}\} \tag{2}$$

A fuzzy Buchi automaton is said to be deterministic fuzzy Buchi automaton if it has deterministic transition.

**Definition 3.** The fuzzy power set $F(S)$ of a set $S$ is the set of fuzzy subsets of $S$. That is

$$F(S) = \{P : S \longrightarrow F| \ P \text{ is a fuzzy subset of } S\}$$

**Definition 4.** A fuzzy Muller automaton is a 5 - tuple $\mathfrak{F} = (S, A, f, I, \mathfrak{M})$, where

- $(S, A, f, I)$ is a fuzzy $\omega$ - automaton.

- $\mathfrak{M}$ is a set of fuzzy subsets of S, that is $\mathfrak{M} \subset F(S)$.

A run in $\mathfrak{F}$ is successful if the set of infinitely often states occurs is a member of $\mathfrak{M}$, that is $inf(R) \in \mathfrak{M}$.
The weight of the accepted word $\omega$ is calculated as follows

$$W(\omega) = \vee\{\wedge\{\{I(s_1)\} \bigcup \{f(s_i, \omega_j, s_k) | i, j, k \geq 1\} \bigcup \{\mathfrak{M}(t) | t \in inf(R)\}\}\} \quad (3)$$

A fuzzy Muller automaton is said to be deterministic fuzzy Muller automaton if it has deterministic transition.

**Theorem 5.** *Let X be a subset of $A^\omega$. If X is recognized by a fuzzy Muller automaton, then X is the finite union of differences of languages recognized by fuzzy deterministic Buchi automaton.*

*Proof.* Let $\mathfrak{F} = (S, A, f, I, \mathfrak{M})$ be a fuzzy Muller automaton and let $Y_i$ and $Z_i$ are subsets of $A^\omega$ recognized by fuzzy deterministic Buchi automaton. Then we have to show that

$$X = \bigcup_{1 \leq i \leq n} (Y_i \setminus Z_i)$$

The relation between Muller's and Buchi's language is

$$L(\mathfrak{F}) = \bigcup_{Q \in \mathfrak{M}} L(S, A, f, I, Q)$$

Since $(S, A, f, I, Q)$ has a single fuzzy final states set, it is fuzzy deterministic Buchi automaton. For $Q \in \mathfrak{M}$

$$L(S, A, f, I, Q) = \bigcap_{q \in Q} L(S, A, f, I, q) \setminus \bigcup_{q \notin Q} L(S, A, f, I, q)$$

$$X = L(\mathfrak{F}) = \bigcup_{Q \in \mathfrak{M}} (\bigcap_{q \in Q} L(S, A, f, I, q) \setminus \bigcup_{q \notin Q} L(S, A, f, I, q))$$

□

A subset P of S is said to be admissible if there exists a run R such that $inf(R) = P$. A fuzzy Muller automaton $\mathfrak{F} = (S, A, f, I, \mathfrak{M})$ is said to have full acceptance component if for every admissible set $P \in \mathfrak{M}$ and for every admissible set Q such that $Q \supset P$, we have $Q \in \mathfrak{M}$.

**Definition 6.** A fuzzy Rabin automaton is a 5 - tuple $\mathfrak{F}$=$(S, A, f, I, \mathfrak{R})$, where

- $(S, A, f, I)$ is a fuzzy $\omega$ - automaton.
- $\mathfrak{R}$=$\{(E_1, F_1\ ),(E_2, F_2\ ),\cdots,(E_n, F_n\ )\ \}$ is a pairs of fuzzy set of states. That is
$$\mathfrak{R} = \{(E_i : Q \to F, F_i : Q \to F)|1 \leq i \leq n\}$$

A run in $\mathfrak{F}$ is successful if it visits $F_i$ infinitely often and visits $E_i$ finitely often, that is

$$\bigvee_{i=1}^{n}(inf(R) \cap E_i = \phi \wedge inf(R) \cap F_i \neq \phi)$$

The weight of the accepted word $\omega$ is calculated as follows

$$\begin{aligned} W(\omega) = \vee\{\wedge\{\{I(s_1)\}\bigcup\{f(s_i, \omega_j, s_k)|i, j, k \geq 1\} \\ \bigcup\max_i\{\min\{E_i(s), F_i(t)|s \in inf(R)\bigcap E_i, \\ t \in inf(R)\bigcap F_i\}\}\}\} \quad (4) \end{aligned}$$

A fuzzy Rabin automaton is said to be deterministic fuzzy Rabin automaton if it has deterministic transition.

## 3. Relationships in Fuzzy $\omega$-Automata

In this section, we prove the fuzzy language recognized by a fuzzy deterministic Muller automaton with full acceptance component is also recognized by a fuzzy deterministic Buchi automaton. We also prove that the fuzzy language recognized by a fuzzy deterministic Rabin automaton is recognized by a fuzzy deterministic Muller automaton. We provide examples to illustrate the results obtained .

**Theorem 7.** *A fuzzy language recognized by a fuzzy deterministic Muller automaton with full acceptance component is also recognized by a fuzzy deterministic Buchi automaton and conversely.*

*Proof.* Let $\mathfrak{F} = (S, A, f, I, \mathfrak{M})$ be a fuzzy deterministic Muller automaton with full acceptance component. Construct a fuzzy deterministic Buchi automaton

$$\mathfrak{F}' = (P(S) \times S, A, f', \Phi \times I, \mathfrak{B})$$

where the transition function $f' : (P(S) \times S) \times A \times (P(S) \times S) \to F$ is defined as follows, for $a \in A$, $(T, s)$ & $(T', s') \in P(S) \times S$

$$f'((T, s), a, (T', s')) = f(s, a, s')$$

such that

$$(T, s) \bullet a = \begin{cases} (\phi, s \bullet a), & \text{if } T \cup \{s \bullet a\} \text{ contains an element of } \mathfrak{M}; \\ (T \cup \{s \bullet a\}, s \bullet a), & \text{otherwise.} \end{cases}$$

If $s \bullet a$ is undefined in $\mathfrak{F}$, then $(T, s) \bullet a$ is undefined and $\mathfrak{B} : \Phi \times S \to F$.
Let $\omega \in L(\mathfrak{F})$, then $\omega$ is the label of a run $R$ such that $inf(R) \in \mathfrak{M}$. Let $R'$ be a run of label of $\omega$ in $\mathfrak{F}'$. We have to prove that the run $R'$ is successful. We will prove by the method of contradiction. Suppose there exist a prefix u of $\omega$, such that if uv is a prefix of $\omega$ the state $\mathfrak{B}((\phi, s_1) \bullet uv) = 0$. Let $(\phi, s_1) \bullet uv = (Q, s \bullet v)$, where $Q \supset inf(R')$. Since the fuzzy deterministic Muller automaton has full acceptance component, $inf(R') \in \mathfrak{M}$ implies $Q \in \mathfrak{M}$, which is a contradiction. Therefore $R'$ is successful and the weight of the word $\omega$ is also remains the same. Hence $\omega \in L(\mathfrak{F}')$. Conversely, let $\omega \in L(\mathfrak{F}')$, then $\omega$ is the label of a run $R'$. Suppose all the states of $P \in \mathfrak{M}$ is infinitely often in $R'$. Then $P \subset inf(R')$ and since $\mathfrak{F}$ has full acceptance component. Therefore $inf(R') \in \mathfrak{M}$ and the weight of the word $\omega$ is also remains the same. Hence $\omega \in L(\mathfrak{F})$. □

**Example 8.** *Consider the fuzzy deterministic Muller automaton* $\mathfrak{F} = (S, A, f, I, \mathfrak{M})$,*where*
$S = \{s, t\}$,
$A = \{a, b\}$,
$I = \{(s, 0.9)\}$,
$\mathfrak{M} = \{\{(t, 1)\}\}$

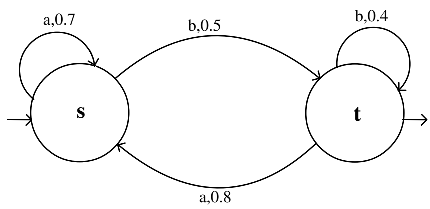

Figure 1. Fuzzy deterministic Muller automaton

*The fuzzy language accepted by* $\mathfrak{F}$ *is* $L(\mathfrak{F}) = \{\omega \in (a + b)^* b^\omega\}$ *and* $W(\omega) = 0.4$. *By the above construction procedure, we obtain a fuzzy deterministic Buchi* *automaton* $\mathfrak{F}'$

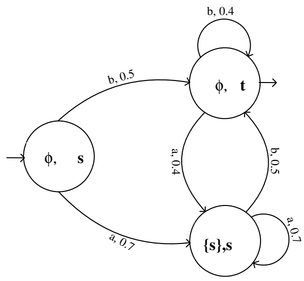


Figure 2. Fuzzy deterministic Buchi automaton

*Thus the fuzzy language* $L(\mathfrak{F})$ *is also recognized by* $\mathfrak{F}'$.
*With the above example, we have illustrated that a fuzzy language recognized by a fuzzy deterministic Muller automata with full acceptance component is also recognized by a fuzzy deterministic Buchi automata.*

**Theorem 9.** *A fuzzy language recognized by a fuzzy deterministic Rabin automaton is also recognized by a fuzzy deterministic Muller automaton and conversely.*

*Proof.* Let $L(\mathfrak{F})$ be recognized by a fuzzy deterministic Rabin automaton $\mathfrak{F} = (S, A, f, I, \mathfrak{R})$. Construct a fuzzy deterministic Muller automaton $\mathfrak{F}' = (S, A, f, I, \mathfrak{M})$ such that

$$\mathfrak{M} = \{P \in F(S) | P \cap E_i = \phi \text{ and } P \cap F_i \neq \phi \text{ for some } (E_i, F_i) \in \mathfrak{R}\}$$

By construction, $L(\mathfrak{F})$ is also recognized by the fuzzy deterministic Muller automaton $\mathfrak{F}'$. Conversely, let $X_n$ be the class of subsets of $A^\omega$ recognized by fuzzy deterministic Muller automaton. Then by lemma, $X_n$ is of the form $\bigcup_{1 \leq i \leq n}(Y_i \setminus Z_i)$ where $Y_i$ and $Z_i$ are recognized by fuzzy deterministic Buchi automaton. We have to show that $X_n$ is also recognized by fuzzy deterministic Rabin automaton. Let $\mathfrak{F} = (S, A, f, I, \mathfrak{R})$ be a fuzzy deterministic Rabin automaton with $\mathfrak{R} = (E_1, F_1), (E_2, F_2), \cdots, (E_n, F_n)$.

$$L(\mathfrak{F}) = \bigcup_{1 \leq i \leq n} L(\mathfrak{F}_i)$$

where $\mathfrak{F}_i = (S, A, f, I, \mathfrak{R}_i)$ is the fuzzy deterministic Rabin automaton with $\mathfrak{R}_i = (E_i, F_i)$. Now

$$L(\mathfrak{F}_i) = L(S, A, f, I, F_i) \setminus L(S, A, f, I, E_i)$$

where $(S, A, f, I, F_i)$ and $(S, A, f, I, E_i\ )$ are fuzzy deterministic Buchi automaton.

$$L(\mathfrak{F}) = \bigcup_{1\leq i\leq n} [L(S, A, f, I, F_i) \setminus L(S, A, f, I, E_i)]$$

Hence $L(\mathfrak{F}) \in X_n$.
Let $\Pi_1$ and $\Pi_2$ be two fuzzy languages recognized by fuzzy deterministic Buchi automaton $\mathfrak{F}_1$=$(S_1, A, f_1, I_1, \mathfrak{B}_1)$ and $\mathfrak{F}_2$=$(S_2, A, f_2, I_2, \mathfrak{B}_2)$ respectively. Construct a fuzzy deterministic one pair Rabin automaton $\mathfrak{F}$=$(S, A, f, I, \mathfrak{R})$, where $S = S_1 \times S_2$, $f = f_1 \times f_2$ defined as follows for $(s_1, s_2)$ & $(t_1, t_2) \in S_1 \times S_2$ and $a \in A$

$$f((s_1, s_2), a,(t_1, t_2)) = f_1(s_1, a, t_1) \wedge f_2(s_2, a, t_2)$$

$I = I_1 \times I_2$, $\mathfrak{R} = ((S_1 \times \mathfrak{B}_2),(\mathfrak{B}_1 \times S_2))$. By construction, $\Pi_1 \setminus \Pi_2$ is recognized by $\mathfrak{F}$. Hence $X_1$ is recognized by fuzzy deterministic one pair Rabin automaton. Now let $\Pi \in X_n$, then $\Pi$ is the union of n sets $\Pi_i$ and every $\Pi_i$ is recognized by fuzzy deterministic one pair Rabin automaton $\mathfrak{F}_i = (S_i, A, f_i, I_i, \mathfrak{R}_i)$ with $\mathfrak{R}_\mathrm{i} = (E_i, F_i)$. The fuzzy deterministic n pair Rabin automaton is $\mathfrak{F} = (S, A, f, I, \mathfrak{R})$, where $S = S_1 \times S_2 \times \cdots \times S_n$, $f = f_1 \times f_2 \times \cdots \times f_n$ defined as follows, for $(s_1, s_2, \cdots s_n)$ & $(t_1, t_2, \cdots t_n) \in S_1 \times S_2 \times \cdots \times S_n$ and $a \in A$

$$\begin{aligned} f((s_1, s_2, \cdots, s_n), a,(t_1, t_2, \cdots, t_n)) = f_1(s_1, a, t_1) \wedge f_2(s_2, a, t_2)\wedge \\ \cdots \wedge f_n(s_n, a, t_n) \end{aligned}$$

$$I = I_1 \times I_2 \times \cdots \times I_n,$$

$$\begin{aligned} \mathfrak{R} = (S_1 \times S_2 \times \cdots \times S_{i-1} \times E_i \times S_{i+1} \times \cdots \times S_n, \\ S_1 \times S_2 \times \cdots \times S_{i-1} \times F_i \times S_{i+1} \times \cdots \times S_n) \end{aligned}$$

By construction, $\Pi$ is recognized by $\mathfrak{F}$. Hence $X_n$ is recognized by fuzzy deterministic Rabin automaton. □

**Example 10.** *Consider the following two fuzzy deterministic Buchi automata* $\mathfrak{F}_1 = (S_1, A, f_1, I_1, \mathfrak{B}_1)$, *where* $S_1 = \{s_1, s_2\}$, $A = \{a, b\}$, $I_1 = \{(s_1, 1)\}$, $\mathfrak{B}_1 = \{(s_1, 0.8),(s_2, 0.9)\}$ *and* $\mathfrak{F}_2 = (S_2, A, f_2, I_2, \mathfrak{B}_2)$, *where* $S_2 = \{s_3, s_4\}$,

$A = \{a, b\}$, $I_2 = \{(s_3, 1)\}$, $\mathfrak{B}_2 = \{(s_4, 0.7)\}$.

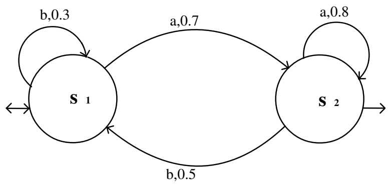


Figure 3. Fuzzy deterministic Buchi automaton

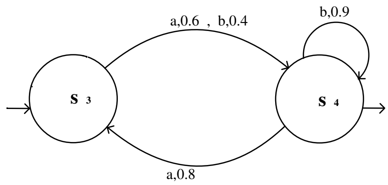


Figure 4. Fuzzy deterministic Buchi automaton

*The fuzzy language accepted by $\mathfrak{F}_1$ is $L(\mathfrak{F}_1) = \{\omega \in (a + b)^\omega\}$ and the fuzzy language accepted by $\mathfrak{F}_2$ is $L(\mathfrak{F}_2) = \{\omega \in (a+b)^* b^\omega\}$. By the above construction procedure, we obtain a fuzzy deterministic Rabin automaton $\mathfrak{F}$=$(S, A, f, I, \mathfrak{R})$, where $S = S_1 \times S_2$, $f = f_1 \times f_2$, $I = \{((s_1, s_3), 1)\}$ and $\mathfrak{R} = \{((s_1, s_4), (s_2, s_4)),$ $(s_2, s_3)\}$*

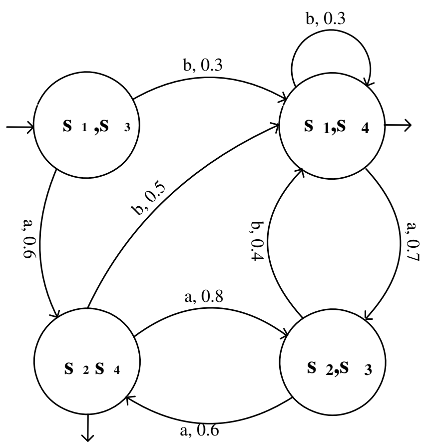


Figure 5. Fuzzy deterministic Rabin automaton

*The fuzzy language accepted by $\mathfrak{F}$ is $L(\mathfrak{F}) = \{\omega \in (a + b)^* a^\omega\}$.*
*The above example illustrates that the fuzzy languages $L(\mathfrak{F}_1)$ and $L(\mathfrak{F}_2)$ recognized by a fuzzy deterministic Buchi automata $\mathfrak{F}_1$ and $\mathfrak{F}_2$ respectively. Then the fuzzy language $L(\mathfrak{F}) = L(\mathfrak{F}_1) \setminus L(\mathfrak{F}_2)$ is recognized by a fuzzy deterministic Rabin automata.*

## 4. Fuzzy Transition Automata

In this section, we introduce the transition fuzzy $\omega$ - automata and prove that the transition fuzzy $\omega$ - automata and fuzzy $\omega$ - automata recognize the same fuzzy language.

**Definition 11.** A fuzzy transition Buchi automaton is a 5 - tuple $\mathfrak{F} = (S, A, f, I, \mathfrak{B})$, where

- $(S, A, f, I)$ is a fuzzy $\omega$ - automaton.
- $\mathfrak{B} \subseteq f$ is the set of fuzzy final transitions.

**Definition 12.** A fuzzy transition Muller automaton is a 5 - tuple $\mathfrak{F} = (S, A, f, I, \mathfrak{M})$, where

- $(S, A, f, I)$ is a fuzzy $\omega$ - automaton.
- $\mathfrak{M}$ is a set of subsets of $f$.

**Theorem 13.** *A fuzzy language recognized by a fuzzy deterministic Muller(Buchi) automaton is also recognized by a transition fuzzy deterministic Muller(Buchi) automaton.*

*Proof.* Let $\mathfrak{F} = (S, A, f, I, Acc)$ be a deterministic fuzzy Muller (Buchi) automaton. Construct a transition deterministic fuzzy Muller(Buchi) automaton $\mathfrak{F}' = (S', A, f', J, Acc')$, where
$J : p \to F$ is a new state.
$S' = f \cup \{J\}$

$$f' = \{(p, a, (s, a, t, c), c) | (s, a, t, c) \in f\} \\ \cup \{((t, a, t', c), b, (t', b, t'', d), d) | (t, a, t', c) \& (t', b, t'', d) \in f\}$$

$Acc' = Acc$. Let $\omega \in L(\mathfrak{F})$, then $\omega = a_1, a_2, a_3, \cdots$ is the label of a run $R = s_1 s_i s_j s_k \cdots$ in $\mathfrak{F}$, there is a corresponding run $R' = p(s_1, a_1, s_i, c_i)(s_i, a_2, s_j, c_j)$ $(s_j, a_3, s_k, c_k) \cdots$ of $\omega$ in $\mathfrak{F}'$. From the construction the weight of the accepted word $\omega$ in $\mathfrak{F}'$ is same as in $\mathfrak{F}$. Hence $\omega \in L(\mathfrak{F}')$. Conversely, let $\omega \in L(\mathfrak{F}')$, then $\omega$ is the label of a run $R'$ in $\mathfrak{F}'$. $\omega$ visits the transition $(s_i, a, s_j, c_j)$ in $\mathfrak{F}$ if and only if $R'$ visits the state $(s_i, a, s_j, c_j)$ and from the construction the weight of the accepted word $\omega$ in $\mathfrak{F}$ is same as in $\mathfrak{F}'$. Hence $\omega \in L(\mathfrak{F})$. □

**Example 14.** *Consider the fuzzy deterministic Muller automaton of Figure 1. Then by the above construction procedure, we obtain a transition fuzzy deterministic Muller automaton*

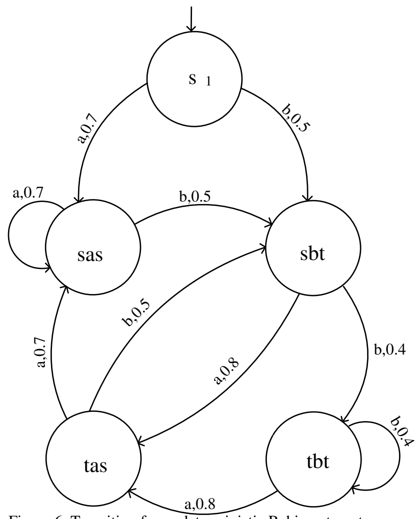


Figure 6. Transition fuzzy deterministic Rabin automaton

*The above example illustrates that a fuzzy language recognized by a fuzzy deterministic Muller automata is also recognized by a transition fuzzy deterministic Muller automata.*

## 5. Closure Poperties of Fuzzy Languages Recognized by Fuzzy $\omega$-Automata

In this section, we discuss the closure properties of fuzzy languages recognized by fuzzy deterministic Buchi, Muller and Rabin automata.

**Theorem 15.** *Let $L(\mathfrak{F}_1)$ and $L(\mathfrak{F}_2)$ be the fuzzy languages recognized by fuzzy deterministic Buchi automata $\mathfrak{F}_1$ and $\mathfrak{F}_2$ respectively. Then there exists a fuzzy deterministic Buchi automaton $\mathfrak{F}$ such that $L(\mathfrak{F}) = L(\mathfrak{F}_1) \cap L(\mathfrak{F}_2)$.*

**Proof:** Let $\mathfrak{F}_1$=$(S_1, A, f_1, I_1, \mathfrak{B}_1)$ and $\mathfrak{F}_2$=$(S_2, A, f_2, I_2, \mathfrak{B}_2)$ be fuzzy deterministic Buchi automata. Construct a fuzzy deterministic Buchi automaton $\mathfrak{F}$=$(S, A, f, I, \mathfrak{B})$, where
$S = S_1 \times S_2$,
$f((s_1, t_1), a, (s_2, t_2)) = f_1(s_1, a, s_2) \wedge f_2(t_1, a, t_2)$,
$I(s_1, t_1) = I_1(s_1) \wedge I_2(t_1)$,

$\mathfrak{B}(s,t) = \mathfrak{B}_1(s) \wedge \mathfrak{B}_2(t)$.
Let $\omega \in L(\mathfrak{F})$, then $\omega$ is the label of a run $R = p_1p_2p_3...$ such that $inf(R) \cap \mathfrak{B} \neq \phi$, where each $p_i$ is of the form $(s_i, t_i)$. If $f((s_i,t_i), \omega_j, (s_k, t_k,)) \neq 0$, then $f_1(s_i, \omega_j, t_k) \neq 0$ and $f_2(s_i, \omega_j, t_k) \neq 0$. The run $R_1 = s_1s_2s_3...$ such that $inf(R_1) \cap \mathfrak{B}_1 \neq \phi$ and the run $R_2 = s_1s_2s_3...$ such that $inf(R_2) \cap \mathfrak{B}_2 \neq \phi$. Therefore $\omega \in L(\mathfrak{F}_1)$ and $\omega \in L(\mathfrak{F}_2)$. Hence $\omega \in L(\mathfrak{F}) = L(\mathfrak{F}_1) \cap L(\mathfrak{F}_2)$. Conversely, let $\omega \in L(\mathfrak{F}_1) \cap L(\mathfrak{F}_2)$, then trere is a label of run $R_1 = s_1s2s_3...$ of $\omega$ in $\mathfrak{F}_1$ and trere is a label of run $R_2 = t_1t_2t_3...$ of $\omega$ in $\mathfrak{F}_2$. Consider the $\omega$-word $R = (s_1,t_1)(s_2,t_2)(s_3,t_3)....$ By construction the run $R$ is in $\mathfrak{F}$ such that $inf(R) \cap \mathfrak{B} \neq \phi$. Hence $\omega \in L(\mathfrak{F})$.

**Theorem 16.** *The class of fuzzy languages recognized by fuzzy deterministic Muller automata is closed under intersection.*

**Proof:** Let $L(\mathfrak{F}_1)$ and $L(\mathfrak{F}_2)$ be the fuzzy languages recognized by fuzzy deterministic Muller automata $\mathfrak{F}_1$=$(S_1, A, f_1, I_1, \mathfrak{M}_1)$ and $\mathfrak{F}_2$=$(S_2, A, f_2, I_2, \mathfrak{M}_2)$ respectively. We have to prove that there exists a fuzzy deterministic Muller automaton $\mathfrak{F}$ such that $L(\mathfrak{F}) = L(\mathfrak{F}_1) \cap L(\mathfrak{F}_2)$. Construct a fuzzy deterministic Muller automaton $\mathfrak{F}$=$(S, A, f, I, \mathfrak{M})$, where
$S = S_1 \times S_2$,
$f((s_1,t_1), a, (s_2,t_2)) = f_1(s_1, a, s_2) \vee f_2(t_1, a, t_2)$,
$I(s_1,t_1) = I_1(s_1) \wedge I_2(t_1)$,
$\mathfrak{M} = \{\{(s_1,t_1,c_1), ..., (s_n,t_n,c_n)\} | \mathfrak{M}_1(s_1) \wedge \mathfrak{M}_2(t_1) = c_1, ..., \mathfrak{M}_1(s_n) \wedge \mathfrak{M}_2(t_n) = c_n\}$.
By the same argument as in the above theorem 15, it is easy to show that $L(\mathfrak{F}) = L(\mathfrak{F}_1) \cap L(\mathfrak{F}_2)$.

**Theorem 17.** *The class of fuzzy languages recognized by fuzzy deterministic Rabin automata is closed under intersection.*

**Proof:** By theorem 9, the fuzzy language recognized by a fuzzy deterministic Muller automaton is recognized by a fuzzy deterministic Rabin automaton and conversely. Since the class of languages recognized by fuzzy deterministic Muller automata is closed under intersection. Hence the same is true for the fuzzy language recognized by fuzzy deterministic Rabin automata.

## 6. Conclusion

In this study, we have proved that a fuzzy language recognized by a fuzzy deterministic Muller automaton with full acceptance component is also recognized by a fuzzy deterministic Buchi automaton. We have obtained that a fuzzy language recognized by a fuzzy deterministic Rabin automaton is also recognized by a fuzzy deterministic Muller automaton. Further, we defined the transition fuzzy $\omega$ - automata and proved that the same fuzzy language can be recognized by both the transition fuzzy $\omega$ - automata and fuzzy $\omega$ automata. Also, we studied the closure properties of fuzzy languages recognized by fuzzy deterministic $\omega$ - automata. From this discussion, we conclude that we can easily construct one class of fuzzy deterministic $\omega$ - automaton from the other.

## References

[1] J.R. Buchi, On a decision method in restricted second - order arithmetic, *Proceedings of 1960 Int. Congr. for Logic, Methodology and philosophy of Science*, Stanford Univ. Press, Stanford, (1962) 1 - 11.

[2] J. Kavikumar, Azme Khamis and Mohd Saifullah Rusiman, $\mathcal{N}$ - Structures Applied to Finite State Machines, *IAENG International Journal of Applied Mathematics*, **43** (2013), 233 - 237.

[3] K. Krithivasan and K. Sharada, Fuzzy $\omega$ - automaton , *Inform. Sci.*, **138** (2001), 257 - 281.

[4] W. Kuich and G. Rahonis, Fuzzy regular languages over finite and infinite words, *Fuzzy Sets and Systems* **157** (2006), 1532 - 1549.

[5] E.T. Lee and L.A. Zadeh, Note on fuzzy languages, *Inform. Sci.* **1** (1969), 421 - 434.

[6] J. Mordeson and D. Malik, *Fuzzy Automata and Languages: Theory and Applications*, Chapman and Hall ICRC Boca Raton, New York, Washington, D.C. (2002).

[7] D.E. Muller, Infinite sequences and finite machines, *Proceedings of* $4^{th}$ *IEEE Symp. On Switching Circuit Theory and Logical Design*, (1963) 3 - 16.

[8] M.O. Rabin, Decidability of second - order theories and automata on infinite trees, *Trans. Amer. Math. Soc.* **141** (1969), 1 - 35.

[9] G. Rahonis, Infinite fuzzy computations, *Fuzzy Sets and Systems* **153** (2005), 275 - 288.

[10] G. Rozenberg and A. Salomaa, *Handbook of Formal Languages*, Springer, Heidelberg, Volume 3 (1997).