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ON SOFT LIFTS AND SOFT TRANSFORMATIONS INDUCED BY SOFT SETS

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Abstract: We introduced the notion of monotonic operation $u_F : P(X) \to P(X)$ induced by $A_F, F^{\leftarrow}$ in any soft set (F, X) $(A \subseteq X)$ in [13]. In this paper, we also introduce the notions of soft lift and soft transformation induced by $F^{\leftarrow}$. We investigate properties for the notions. In particular, Theorem 3.14 and Theorem 3.15 are obtained.

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1. Introduction

We know that fuzzy sets [21], vague sets [9] and rough sets [18] are tools applied to avoid difficulties for dealing with uncertainties in economics, engineering, environmental science, sociology and computer science. In 1999, Molodtsov introduced the concept of soft set [17] to deal complicated problems and uncertainties as the following: The soft set is an approximate description of an object precisely consisting of two parts, namely predicate and approximate value set. In [14], Maji et al. introduced several operators for soft set theory: equality of

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two soft sets, subset and superset of soft set, complement of a soft set, null soft set, and absolute soft set. For the soft set theory, Ali et al. [2] proposed new operations on soft sets, and in [4], the new operations were investigated with the notions defined in [2, 14]. Recently, the properties and applications on the soft set theory have been studied widely and applied in the various branches of mathematics [1, 3, 4, 5, 6, 7, 8, 10, 11, 15, 16, 17, 20]. In particular, the combining the soft set theory and any other notions of mathematics have been studied steadily. In [12, 16], we combined topological structures and soft set theory. In this paper, we are going to combine a monotonic operation and soft set theory. We introduced the notion of monotonic operation $u_F : P(X) \to P(X)$ induced by $A_F, F^{\leftarrow}$ [12, 16] in a given soft set (F, X) $(A \subseteq X)$ where U is a common universe set and X is a fixed subset of parameters, and studied basic properties of the operation. In this paper, we also introduce the notions of soft lift and soft transformation induced by $F^{\leftarrow}$. We investigate properties for the notions and the relations among soft lifts, soft transformations and the operation u_F. Finally, we have Theorem 3.14 and Theorem 3.15.

2. Preliminaries

Let U be an initial universe set (simply, universe set) and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. We will call E *the set of parameters* with respect to U.

Definition 2.1 ([17]). A pair (F, A) is called a *soft set* over U if $A \subset E$ and $F : A \to P(U)$, where $P(U)$ is the set of all subsets of U.

Definition 2.2 ([14]). Let U be a universe set and E be a set of parameters. Let (F, A) and (G, B) be soft sets over a common universe set U and $A, B \subseteq E$. Then (F, A) is a *subset* of (G, B), denoted by $(F, A)\widetilde{\subseteq}(G, B)$, if

(i) $A \subset B$; (ii) for all $e \in A$, $F(e) \subseteq G(e)$.
(F, A) equals (G, B), denoted by $(F, A) = (G, B)$, if $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$.

Definition 2.3 ([14]). A soft set (F, A) over U is said to be a *null soft set* denoted by Φ, if $\forall e \in A, F(e) = \emptyset$.

Definition 2.4 ([14]). A soft set (F, A) over U is said to be an *absolute soft set* denoted by $\tilde{A}$, if $\forall e \in A, F(e) = U$.

Definition 2.5 ([16]). Let (F, X) be a soft set over a universe set U. For $A \subseteq X$, we define $F(A) = \cup\{F(a) : a \in A\}$.

Definition 2.6 ([16]). Let (F, X) be a soft set over a universe set U. For $A \subseteq X \subseteq E$ and $S \subseteq U$,

$$A_F = \{a \in A : F(a) = \emptyset\}; \quad A^F = \{a \in A : F(a) \neq \emptyset\};$$

$$F^{\leftarrow}(S) = \{a \in X : F(a) \subseteq S \text{ and } F(a) \neq \emptyset\}.$$

Lemma 2.7 ([16]). *Let (F, X) be a soft set over a universe set U. Then for $A, B \subseteq X$,*

(i) $A \subseteq B$ implies $F(A) \subseteq F(B)$; (ii) $F(A \cup B) = F(A) \cup F(B)$;

(iii) $F(A \cap B) \subseteq F(A) \cap F(B)$.

Theorem 2.8 ([12, 16]). *Let (F, X) be a soft set over a universe set U. Then for $A, B \subseteq X$,*

(i) $A = A_F \cup A^F$; (ii) $F(A) = F(A^F)$; (iii) $A^F \subseteq F^{\leftarrow}(F(A))$;

(iv) if $A \subseteq B$, then $A_F \subseteq B_F$, $A^F \subseteq B^F$ and $F(A) \subseteq F(B)$;

(v) $F^{\leftarrow}(F(F^{\leftarrow}(F(A))) = F^{\leftarrow}(F(A))$.

Let (F, X) be a soft set over a universe set U. Let us define an operation $u_F : P(X) \to P(X)$ [13] as the following:

$$u_F(A) = A_F \cup F^{\leftarrow}(F(A)) \quad \text{for} \quad A \in P(X).$$

Theorem 2.9 ([13]). *Let (F, X) be a soft set over a universe set U. Then we have the following things: For $A, B \subseteq X$,*

(i) $u_F(\emptyset) = \emptyset$; (ii) $A \subseteq u_F(A)$; (iii) if $A \subseteq B$, then $u_F(A) \subseteq u_F(B)$;

(iv) $u_F(A) = u_F(u_F(A))$.

3. Main Results

In this section, we introduce the notions of soft lifts and soft transformation, and investigate properties for such notions. Let U be a universe set and E be a set of parameters. Let us denote

$$\mathcal{S}S_X = \{(F, X) \mid (F, X) \text{ is a soft set for the fixed subset } X \text{ over a universe set } U\}.$$

Lemma 3.1. *Let $(F, X) \in \mathcal{SS}_X$ and $V_1, V_2 \subseteq U$. Then we have the following things:*

(i) $F^{\leftarrow}(V_1) \cap F^{\leftarrow}(V_2) = F^{\leftarrow}(V_1 \cap V_2)$.

(ii) $F^{\leftarrow}(V_1) \cup F^{\leftarrow}(V_2) \subseteq F^{\leftarrow}(V_1 \cup V_2)$.

Proof. (i) For $a \in X$, $a \in F^{\leftarrow}(V_1) \cap F^{\leftarrow}(V_2)$ iff $a \in F^{\leftarrow}(V_1)$ and $a \in F^{\leftarrow}(V_2)$ iff $F(a) \subseteq V_1$ and $F(a) \subseteq V_2$ for $F(a) \neq \emptyset$ iff $F(a) \subseteq V_1 \cap V_2$ for $F(a) \neq \emptyset$ iff $a \in F^{\leftarrow}(V_1 \cap V_2)$.

(ii) For $a \in X$, $a \in F^{\leftarrow}(V_1) \cup F^{\leftarrow}(V_2) \Rightarrow a \in F^{\leftarrow}(V_1)$ or $a \in F^{\leftarrow}(V_2) \Rightarrow F(a) \subseteq V_1$ or $F(a) \subseteq V_2$ for $F(a) \neq \emptyset \Rightarrow F(a) \subseteq V_1 \cup V_2$ for $F(a) \neq \emptyset \Rightarrow a \in F^{\leftarrow}(V_1 \cup V_2)$. □

Example 3.2. Let $U = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2, e_3, e_4\}$. Consider $X = \{e_1, e_2, e_3\}$ and a soft set (F, X) defined as the following:

$$F(e_1) = \emptyset; F(e_2) = \{x_2\}; \quad F(e_3) = \{x_1, x_3\}.$$

Let $V_1 = \{x_1, x_2\}$ and $V_2 = \{x_2, x_3, x_4\}$. Then $F^{\leftarrow}(V_1 \cup V_2) = \{e_2, e_3\}$. Note that $F(e_3) \not\subseteq V_1, V_2$. So $e_3 \notin F^{\leftarrow}(V_1)$ and $e_3 \notin F^{\leftarrow}(V_2)$. It implies that $F^{\leftarrow}(V_1 \cup V_2) \neq F^{\leftarrow}(V_1) \cup F^{\leftarrow}(V_2)$.

Lemma 3.3. *Let $(F, X) \in \mathcal{SS}_X$ and $x \in X$. Then $x \in F^{\leftarrow}(F(x))$ if and only if $F(x) \neq \emptyset$.*

Proof. Obvious. □

Definition 3.4. Let $(F, X) \in \mathcal{SS}_X$. Let us define a mapping $\varphi_F : X \to P(X)$ by $\varphi_F(x) = F^{\leftarrow}(F(x))$ for each $x \in X$. Then we call the mapping φ_F *a soft lift* over X induced by (F, X). We define $\varphi_F(A) = \cup\{\varphi_F(a) : a \in A\}$.

Theorem 3.5. *Let $(F, X) \in \mathcal{SS}_X$ and $x \in X$. Then $\varphi_F(\varphi_F(x)) = \varphi_F(x)$.*

Proof. First, we show that $\varphi_F(\varphi_F(x)) \subseteq \varphi_F(x)$. From Lemma 3.1 and Theorem 2.8,

$$\begin{aligned}\varphi_F(\varphi_F(x)) =& \varphi_F(F^{\leftarrow}(F(x))) = \cup_{z \in F^{\leftarrow}(F(x))} \varphi_F(z) \\ =& \cup_{z \in F^{\leftarrow}(F(x))} F^{\leftarrow}(F(z)) \subseteq F^{\leftarrow}(\cup_{z \in F^{\leftarrow}(F(x))} F(z)) \\ =& F^{\leftarrow}(F(F^{\leftarrow}(F(x)))) = F^{\leftarrow}(F(x)) \\ =& \varphi_F(x).\end{aligned}$$

For the other inclusion, let $z \in \varphi_F(x) = F^{\leftarrow}(F(x))$; then $F(z) \neq \emptyset$ and so by Lemma 3.3, $z \in F^{\leftarrow}(F(z))$ and $z \in \varphi_F(z)$ for $z \in F^{\leftarrow}(F(x))$. It implies that $z \in \varphi_F(F^{\leftarrow}(F(x))) = \varphi_F(\varphi_F(x))$. Hence, $\varphi_F(x) \subseteq \varphi_F(\varphi_F(x))$. □

Theorem 3.6. *Let* $(F, X) \in \mathcal{S}S_X$ *and* $\varphi_F : X \to P(X)$ *a soft lifting over* X*. Then for* $A, B \subseteq X$*,*

(i) $\varphi_F(A \cup B) = \varphi_F(A) \cup \varphi_F(B)$*;*

(ii) if $A \subseteq B$*, then* $\varphi_F(A) \subseteq \varphi_F(B)$*;*

(iii) $\varphi_F(A \cap B) \subseteq \varphi_F(A) \cap \varphi_F(B)$*;*

(vi) $\varphi_F(\varphi_F(A)) = \varphi_F(A)$*.*

Proof. (i) Obvious.

(ii) It follows from (i).

(iii) It follows from (ii).

(vi) For $A \subseteq X$, from Theorem 3.5, it follows

$$\varphi_F(\varphi_F(A)) = \varphi_F(\cup_{a \in A} \varphi_F(a)) = \cup_{a \in A} \varphi_F(\varphi_F(a)) = \cup_{a \in A} \varphi_F(a) = \varphi_F(A).$$

So $\varphi_F(\varphi_F(A)) = \varphi_F(A)$. □

Example 3.7. Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ and $E = \{e_1, e_2, e_3, e_4\}$. Consider $X = E$ and a soft set (F, X) defined as the following:

$$F(e_1) = \{x_1, x_2\}; \quad F(e_2) = \{x_2\}; \quad F(e_3) = \{x_2, x_3\}; \quad F(e_4) = \{x_3, x_5\}.$$

Let $A = \{e_1\}$ and $B = \{e_3, e_4\}$. Note that:

$$\varphi_F(A) = F^{\leftarrow}(F(e_1)) = \{e_1, e_2\};$$

$$\varphi_F(B) = F^{\leftarrow}(F(e_3)) \cup F^{\leftarrow}(F(e_4)) = \{e_2, e_3, e_4\};$$

Note that $\varphi_F(A) \cap \varphi_F(B) = \{e_2\}$ and $\varphi_F(A \cap B) = \emptyset$. So $\varphi_F(A \cap B) \neq \varphi_F(A) \cap \varphi_F(B)$.

Theorem 3.8. *Let* $(F, X) \in \mathcal{S}S_X$ *and* $\varphi_F : X \to P(X)$ *a soft lift over* X*. Then for* $A \subseteq X$*,*

(i) $\varphi_F(A_F) = \emptyset$*;*

(ii) $A^F \subseteq \varphi_F(A^F)$*;*

(iii) $\varphi_F(A) \subseteq F^{\leftarrow}F(A)$*;*

(iv) $\varphi_F(A) = \varphi_F(A^F)$*;*

(v) $\varphi_F(F^{\leftarrow}F(A)) = F^{\leftarrow}F(A)$*.*

Proof. (i) For each $a \in A_F$, since $F(a) = \emptyset$, obviously $\varphi_F(A_F) = \cup_{a \in A_F} \varphi_F(a) = \emptyset$.

(ii) For each $x \in A^F$, by Lemma 3.3, $x \in F^{\leftarrow}(F(x)) = \varphi_F(x) \subseteq \varphi_F(A_F)$. So we have (ii).

(iii) From (ii) of Lemma 3.1, $\varphi_F(A) = \cup\{\varphi_F(a) : a \in A\} = \cup_{a\in A} F^{\leftarrow}(F(a)) \subseteq F^{\leftarrow}(\cup_{a\in A} F(a)) = F^{\leftarrow}F(A)$.

(iv) From Theorem 2.8 and (i) of Theorem 3.6, $\varphi_F(A) = \varphi_F(A^F \cup A_F) = \varphi_F(A^F) \cup \varphi_F(A_F) = \varphi_F(A^F)$.

(v) From $F^{\leftarrow}(F(A)) = (F^{\leftarrow}(F(A)))^F$, (ii) and (iii), it follows $F^{\leftarrow}(F(A)) = (F^{\leftarrow}(F(A)))^F \subseteq \varphi_F((F^{\leftarrow}(F(A)))^F) = \varphi_F(F^{\leftarrow}(F(A))) \subseteq F^{\leftarrow}F(F^{\leftarrow}(F(A))) = F^{\leftarrow}(F(A))$. Finally, we have $\varphi_F(F^{\leftarrow}(F(A))) = F^{\leftarrow}(F(A))$.

□

Theorem 3.9. *Let $(F, X) \in \mathcal{SS}_X$ and $(G, Y) \in \mathcal{SS}_Y$, and let φ_F and ψ_G be soft lifts over X and Y, respectively. Then*
(i) $\mu_F \circ \varphi_F = \varphi_F$;
(ii) $\mu_G \circ \psi_G = \psi_G$.

Proof. (i) For $x \in X$, from Theorem 2.8 and $(F^{\leftarrow}(F(x)))_F = \emptyset$, it follows

$$\begin{aligned}(\mu_F \circ \varphi_F)(x) &= \mu_F(\varphi_F(x)) \\ &= \mu_F(F^{\leftarrow}(F(x))) \\ &= F^{\leftarrow}(F(F^{\leftarrow}(F(x))) \cup (F^{\leftarrow}(F(x)))_F \\ &= F^{\leftarrow}(F(x)) \\ &= \varphi_F(x)\end{aligned}$$

(ii) It is similar to the proof of (i).

□

Definition 3.10. Let X and Y be nonempty sets. We call the mapping $T : P(X) \to P(Y)$ *a transformation* if it satisfies the following axioms:
(i) $T(A) = \emptyset$ if and only if $A = \emptyset$.
(ii) For $A_1, A_2 \in P(X)$, $T(A_1 \cup A_2) = T(A_1) \cup T(A_2)$.

Definition 3.11. Let $(F, X) \in \mathcal{SS}_X$ and $(G, Y) \in \mathcal{SS}_Y$, and let φ_F and ψ_G be soft lifts over X and Y, respectively. Let $f : X \to Y$ be a mapping. Then the pair (f, T) is called *a soft transformation* if $T \circ \varphi_F = \psi_G \circ f$.

Theorem 3.12. *Let $(F, X) \in \mathcal{SS}_X$ and $(G, Y) \in \mathcal{SS}_Y$, and let φ_F and ψ_G be soft lifts over X and Y, respectively. Let $f : X \to Y$ be a mapping and T a transformation. Then*

(i) $T \circ \mu_F \circ \varphi_F = T \circ \varphi_F$;
(ii) $\mu_G \circ \psi_G \circ f = \psi_G \circ f$;
(iii) if (f,T) *is a soft transformation, then* $T \circ \mu_F \circ \varphi_F = \mu_G \circ \psi_G \circ f$.

Proof. (i) and (ii) are obtained by Theorem 3.9.

(iii) It is obvious from (i) and (ii).

□

Theorem 3.13. *Let* $(F,X) \in \mathcal{SS}_X$ *and* $(G,Y) \in \mathcal{SS}_Y$*, and let* φ_F *and* ψ_G *be soft lifts over* X *and* Y*, respectively. Let* $f : X \to Y$ *be a function. If* (f,T) *is a soft transformation, then* $\mu_G \circ T \circ \varphi_F = T \circ \varphi_F$.

Proof. For $x \in X$, $(T \circ \varphi_F)(x) = (T \circ \mu_F \circ \varphi_F)(x) = (\mu_G \circ \psi_G \circ f)(x) = (\mu_G \circ T \circ \varphi_F)(x)$. □

Theorem 3.14. *Let* $(F,X) \in \mathcal{SS}_X$ *and* $(G,Y) \in \mathcal{SS}_Y$*, and let* φ_F *and* ψ_G *be soft lifts over* X *and* Y*, respectively. Let* (f,T) *be a soft transformation. Then for* $A \subseteq X$*,*
(i) $(T \circ \varphi_F)(A) = (\psi_G \circ f)(A)$;
(ii) $(T \circ \varphi_F)(A_F) = (\psi_G \circ f)(A_F) = \emptyset$;
(iii) $(T \circ \varphi_F)(A) = (\psi_G \circ f)(A^F)$.

Proof. (i) From the property of transformation, we have

$$\begin{aligned}(T\circ\varphi_F)(A) = T(\varphi_F(A)) = T(\cup_{a\in A}\varphi_F(a)) = \cup_{a\in A}T(\varphi_F(a)) = \cup_{a\in A}\psi_G(f(a)) \\ = \psi_G(\cup_{a\in A}f(a)) = (\psi_G \circ f)(A).\end{aligned}$$

(ii) From (i) of Theorem 3.8, $(\psi_G \circ f)(A_F) = (T \circ \varphi_F)(A_F) = T(\varphi_F(A_F)) = T(\emptyset) = \emptyset$.

(iii) Since $A = A^F \cup A_F$ and $A^F \cap A_F = \emptyset$, by (i) and (ii),

$$\begin{aligned}(T \circ \varphi_F)(A) &= (T \circ \varphi_F)(A^F \cup A_F) \\ &= T(\varphi_F(A^F) \cup \varphi_F(A_F)) = T(\varphi_F(A^F)) = (\psi_G \circ f)(A^F).\end{aligned}$$ □

Theorem 3.15. *Let* $(F,X) \in \mathcal{SS}_X$ *and* $(G,Y) \in \mathcal{SS}_Y$*, and let* φ_F *and* ψ_G *be soft lifts over* X *and* Y*, respectively. If* (f,T) *is a soft transformation, then*

(i) $f(A^F) = (f(A))^G$;

(ii) $f(A_F) = (f(A))_G$.

Proof. (i) Let $y \in f(A^F)$. Then there exists $a \in A^F$ such that $f(a) = y$ and $F(a) \neq \emptyset$. Since (f, T) is a soft transformation and $F(a) \neq \emptyset$, $\psi_G(y) = \psi_G(f(a)) = T(\varphi(a)) = T(F^{\leftarrow}(F(a))) \neq \emptyset$, that is, $\psi_G(y) = G^{\leftarrow}(G(y)) \neq \emptyset$. It implies that $G(y) \neq \emptyset$ and $y \in (f(A^F))^G \subseteq (f(A))^G$, and so $f(A^F) \subseteq (f(A))^G$.

For the other inclusion, let $y \in f(A)^G$. Then $G(y) \neq \emptyset$ and $y \in f(A)$. There exists $a \in A$ such that $f(a) = y$ and $G(f(a)) = G(y) \neq \emptyset$. So by Lemma 3.3, $y = f(a) \in G^{\leftarrow}(G(f(a))) = (\psi_G \circ f)(a) = T(\varphi_F(a)) = T(F^{\leftarrow}(F(a)))$. Since $T(F^{\leftarrow}(F(a))) \neq \emptyset$, $F^{\leftarrow}(F(a)) \neq \emptyset$ and $F(a) \neq \emptyset$. It implies $a \in A^F$ and $y = f(a) \in f(A^F)$. Hence $(f(A))^G \subseteq f(A^F)$.

(ii) Let $y \in f(A_F)$. Then there exists $a \in A_F$ such that $f(a) = y$. It implies that $\psi_G(f(a)) = \psi_G(y)$ and $F(a) = \emptyset$. Since (f, T) is a soft transformation and $F^{\leftarrow}(F(a)) = \emptyset$, $\psi_G(f(a)) = T(\varphi_F(a)) = T(F^{\leftarrow}(F(a))) = \emptyset$. So $\psi_G(y) = G^{\leftarrow}(G(y)) = \emptyset$ and $G(y) = \emptyset$. It implies $y \in (f(A_F))_G \subseteq (f(A))_G$. So $f(A_F) \subseteq (f(A))_G$.

For the other part of the proof, let $y \in f(A)_G$. Then $G(y) = \emptyset$ and $y \in f(A)$. There exists $a \in A$ such that $G(f(a)) = G(y)$ and $G(y) = \emptyset$. Since (f, T) is a soft transformation and $G(y) = \emptyset$, $T(F^{\leftarrow}(F(a))) = T(\varphi_F(a)) = (\psi_G \circ f)(a) = G^{\leftarrow}(G(f(a))) = \emptyset$. Since $T(F^{\leftarrow}(F(a))) = \emptyset$, $F^{\leftarrow}(F(a)) = \emptyset$ and so $F(a) = \emptyset$. It implies $a \in A_F$ and $y = f(a) \in f(A_F)$. Hence $(f(A))_G \subseteq f(A_F)$.

□

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