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HOMOMORPHISM IN t-Q-INTUITIONISTIC L-FUZZY SUB RINGS

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Abstract: In this paper, the notion of t-Q-intuitionistic L-fuzzy sub rings (normal sub rings and ideals) are defined and the homomorphic behavior of t - Q -intuitionistic L-fuzzy sub rings (normal sub rings and ideals) and inverse images has been obtained. Some new results are obtained based on this notion.

AMS Subject Classification: 30C45
Key Words: t-Q-intuitionistic L-fuzzy subset, t-Q-intuitionistic L-fuzzy sub rings, t-Q-intuitionistic L-fuzzy normal sub ring, t-Q-intuitionistic L-fuzzy left (right) ideals, homomorphism

1. Introduction

The concept of intuitionistic L-fuzzy subset was introduced by Atanassov in [5, 6] as a generalization of Zadeh's fuzzy sets. In the case of intuitionistic fuzzy sets there were several attempts to define intuitionistic fuzzy rings, see [4, 9]. A. Solairaju and R. Nagarajan in [1, 2] introduced and defined a new algebraic structure called Q-fuzzy sub rings. On the other hand Palaniappan, Arjanan and Palanovelrajan in [8] define intuitionistic L-fuzzy sub rings. Wang liu and Yin in [4] defined intuitionistic fuzzy ideals with thresholds (α, β)

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of rings. Sharma in [9] introduced the notion of translates of intuitionistic fuzzy sub rings. The notion of t-intuitionistic fuzzy quotient group has been introduced by sharma in [10]. Here in this paper, we introduce the notion of t-Q-intuitionistic L-fuzzy subset and then define t-Q-intuitionistic L-fuzzy sub rings (normal sub rings and ideals) of a ring R and establish some new results.

2. Preliminary

Definition 2.1. Let X be a non empty set, and $L = (L, \leq)$ be a lattice with least element 0, and greatest element 1 and Q be a non empty set. A Q-L-fuzzy subset μ of X is a function $\mu : X \times Q \to L$.

Definition 2.2. Let $L = (L, \leq)$ be a complete lattice with an evaluative order reversing operation $N : L \to L$ and Q be a non empty set. A Q-intuitionistic L-fuzzy subset (QILFS) μ in X is defined as an object of the form $\mu = \{< (x, q), \lambda_\mu(x, q), \delta_\mu(x, q) >; x \in X \text{ and } q \in Q\}$ where $\lambda_\mu : X \times Q \to L$ and $\delta_\mu : X \times Q \to L$ define the degree of member ship, and the degree of non member ship of the element $x \in X$, respectively, and for every $x \in X$ and $q \in Q$ satisfying $\lambda_\mu(x, q) \leq N(\delta_\mu(x, q))$.

Definition 2.3. Let R be a ring. AQ-intuitionistic L-fuzzy subset μ of R is said to be a Q-intuitionistic L-fuzzy sub ring (QILFSR) of R if it satisfies the following axioms:

1. $\lambda_\mu(x - y, q) \geq \min\{\lambda_\mu(x, q),\ \lambda_\mu(y, q)\}$;
2. $\lambda_\mu(xy, q) \geq \min\{\lambda_\mu(x, q),\ \lambda_\mu(y, q)\}$;
3. $\delta_\mu(x - y, q) \leq \max\{\delta_\mu(x, q),\ \delta_\mu(y, q)\}$;
4. $\delta_\mu(xy, q) \leq \max\{\delta_\mu(x, q),\ \delta_\mu(y, q)\}$.

Definition 2.4. Let R be a ring. AQ-intuitionistic L-fuzzy sub ring μ of R is said to be a Q-intuitionistic L-fuzzy normal sub ring (QILFNSR) of R if it satisfies:

1. $\lambda_\mu(xy, q) = \lambda_\mu(yx, q)$;
2. $\delta_\mu(xy, q) = \delta_\mu(yx, q)$;

For all $x, y \in R$ and $q \in Q$.

Definition 2.5. Let μ be a QILFS of a ring R. And let $t \in [0,1]$, then the QIFS μ^t of R is called the t-Q -intuitionistic fuzzy subset (tQILFS) of R with respect to QILFS μ and is defined as $\mu^t = (\lambda_{\mu^t}, \delta_{\mu^t})$, where

$$\lambda_{\mu^t}(x,q) = \min\{\lambda_\mu(x,q), t\}$$

and

$$\delta_{\mu^t}(x,q) = \max\{\delta_\mu(x,q), 1-t\},$$

forall $x \in R$.

Definition 2.6. Let X, Y be two non empty sets and $\Phi : X \to Y$ be a mapping. Let μ and γ be two tQILFS of X and Y respectively. Then the image of μ under the map Φ is denoted by $\Phi(\mu)$ and is defined as $\Phi(\mu^t)(y,q) = (\lambda_\Phi(\mu^t)(y,q), \delta_\Phi(\mu^t)(y,q))$ where

$$\lambda_\Phi(\mu^t)(y,q) = \begin{cases} \sup\{\lambda_{\mu^t}(x,q)\}, & x \in \Phi^{-1}(y), \\ 0, & \text{otherwise,} \end{cases}$$

$$\lambda_\Phi(\mu^t)(y,q) = \begin{cases} \inf\{\delta_{\mu^t}(x,q)\}, & x \in \Phi^{-1}(y), \\ 1, & \text{otherwise,} \end{cases}$$

also the pre-image of γ^t under Φ is denoted by $\Phi^{-1}(\gamma^t)$ and is defined as

$$\Phi^{-1}(\gamma^t)(x,q) = (\lambda_{\Phi^{-1}}(\gamma^t)(x,q), \delta_{\Phi^{-1}}(\gamma^t)(x,q)),$$

where

$$\lambda_{\Phi^{-1}}(\gamma^t)(x,q) = \lambda_{\gamma^t}(\Phi(x),q)$$

and

$$\delta_{\Phi^{-1}}(\gamma^t)(x,q) = \delta_{\gamma^t}(\Phi(x),q).$$

This means that $\Phi^{-1}(\gamma^t)(x,q) = (\lambda_{\gamma^t}(\Phi(x),q), \delta_{\gamma^t}(\Phi(x),q))$.

Proposition 2.7. *Let $\Phi : X \to Y$ be a mapping. Let μ and γ be two tQILFS of X and Y respectively. Then $\Phi^{-1}(\gamma^t) = (\Phi^{-1}(\gamma))^t$ and $\Phi(\mu^t) = (\Phi(\mu))^t$ for all $t \in [0,1]$.*

Proof. We have

$$\begin{aligned} \Phi^{-1}(\gamma^t)(x,q) =& \gamma^t(\Phi(x),q) = (\lambda_{\gamma^t}(\Phi(x),q), \delta_{\gamma^t}(\Phi(x),q) \\ =& \{\min\{\lambda_\gamma(\Phi(x),q), t\}, \max\{\delta_\gamma(\Phi(x),q), 1-t\}\} \end{aligned}$$

$$=\{\min\{\lambda_{\Phi^{-1}(\gamma)}(x,q),t\},\max\{\delta_{\Phi^{-1}(\gamma)}(x,q),1-t\}\}$$
$$=(\lambda_{(\Phi^{-1}(\gamma))^t}(x,q),\delta_{(\Phi^{-1}(\gamma))^t}(x,q))=\left(\Phi^{-1}(\gamma)\right)^t(x,q).$$

Therefore

$$\begin{aligned}\Phi^{-1}(\gamma^t)=\left(\Phi^{-1}(\gamma)\right)^t\ \Phi(\mu^t)(y)\\ &=(\sup\{\lambda_{\mu^t}(x,q);\Phi(x)=y\},\inf\{\delta_{\mu^t}(x,q);\Phi(x)=y\})\\ &=(\sup\{\min\{\lambda_\mu(x,q),t\};\Phi(x)=y\},\inf\{\max\{\delta_\mu(x,q),1-t\};\Phi(x)=y\})\\ &=(\min\{sup\{\lambda_\mu(x,q);\Phi(x)=y\},t\},\max\{\inf\{\delta_\mu(x,q);\Phi(x)=y\},1-t\})\\ &=(\min\{(\lambda_{\Phi(\mu)}(y,q),t\},\max\{\delta_{\Phi(\mu)}(y,q),1-t\})\\ &=(\lambda_{(\Phi(\mu))^t}(y,q),\delta_{(\Phi(\mu))^t}(y,q))=(\Phi(\mu))^t(y,q).\end{aligned}$$

Hence we get $\Phi(\mu^t)=(\Phi(\mu))^t$. □

Definition 2.8. Let μ be a QILFS of a ring R. And let $t\in[0,1]$, then μ is called t-Q -intuitionistic L -fuzzy sub ring (tQILFSR) of R if is QILFSR of R. This means that μ^t satisfies the following conditions:

1. $\lambda_{\mu^t}\ (x-y,q)\geq\min\{\lambda_{\mu^t}(x,q),\lambda_{\mu^t}(y,q)\}$;
2. $\lambda_{\mu^t}(xy,q)\geq\min\{\lambda_{\mu^t}(x,q),\lambda_{\mu^t}(y,q)\}$;
3. $\delta_{\mu^t}(x-y,q)\leq\max\{\delta_{\mu^t}(x,q),\delta_{\mu^t}(y,q)\}$;
4. $\delta_{\mu^t}(xy,q)\leq\max\{\delta_{\mu^t}(x,q),\delta_{\mu^t}(y,q)\}$;

for all $x,y\in R$ and $q\in Q$.

Theorem 2.9. *If μ is QILFNSR of a ring R, then μ is also tQILFNSR of a ring R.*

Proof. Let $x,y\in R$ be any elements, then

$$\lambda_{\mu^t}(xy,q)=\min\{\lambda_\mu(xy,q),t\}=\min\{\lambda_\mu(yx,q),t\}=\lambda_{\mu^t}(yx,q).$$

Similarly

$$\delta_{\mu^t}(xy,q)=\max\{\delta_\mu(xy,q),1-t\}=\max\{\delta_\mu(yx,q),1-t\}=\delta_{\mu^t}(yx,q).$$

Therefore μ is also tQILFNSR of R. □

Definition 2.10. Let μ be a QILFS of a ring R. And let $t \in [0,1]$, then μ is called t-Q -intuitionistic L -fuzzy left ideal (tQILFLI) of R. If:

1. $\lambda_{\mu^t}(x-y,q) \geq \min\{\lambda_{\mu^t}(x,q), \lambda_{\mu^t}(y,q)\}$;
2. $\lambda_{\mu^t}(xy,q) \geq \lambda_{\mu^t}(y,q)$;
3. $\delta_{\mu^t}(x-y,q) \leq \max\{\delta_{\mu^t}(x,q), \delta_{\mu^t}(y,q)\}$;
4. $\delta_{\mu^t}(xy,q) \leq \delta_{\mu^t}(y,q)$ for all $x, y \in R$ and $q \in Q$.

Definition 2.11. Let μ be a QILFS of a ring R. And let $t \in [0,1]$, then μ is called t-Q -intuitionistic L -fuzzy right ideal (tQILFRI) of R. If:

1. $\lambda_{\mu^t}(x-y,q) \geq \min\{\lambda_{\mu^t}(x,q), \lambda_{\mu^t}(y,q)\}$;
2. $\lambda_{\mu^t}(xy,q) \geq \lambda_{\mu^t}(x,q)$;
3. $\delta_{\mu^t}(x-y,q) \leq \max\{\delta_{\mu^t}(x,q), \delta_{\mu^t}(y,q)\}$;
4. $\delta_{\mu^t}(xy,q) \leq \delta_{\mu^t}(x,q)$ for all $x, y \in R$ and $q \in Q$.

Proposition 2.12. *If μ is QILFLI of a ring R, then μ is also tQILFLI of a ring R.*

Proof. By Definition 2.8 and Definition 2.11, we need to prove that

$$\lambda_{\mu^t}(xy,q) \geq \lambda_{\mu^t}(y,q) \text{ and } \delta_{\mu^t}(xy,q) \leq \delta_{\mu^t}(y,q),$$

for all $x, y \in R$.

But

$$\lambda_{\mu^t}(xy,q) = \min\{\lambda_\mu(xy,q), t\} \geq \lambda_{\mu^t}(x,q) = \min\{\lambda_\mu(y,q), t\} = \lambda_{\mu^t}(y,q).$$

Thus $\lambda_{\mu^t}(xy,q) \geq \lambda_{\mu^t}(y,q)$.

Similarly, we can show that $\delta_{\mu^t}(xy,q) \leq \delta_{\mu^t}(y,q)$.

Hence μ is also tQILFLI of a ring R. □

Corollary 2.13. *If μ is QILFRI of a ring R, then μ is also tQILFRI of a ring R.*

3. Homomorphism and Isomorphism of *t*-*Q*-Intuitionistic *L*-Fuzzy Sub Rings, Ideals and Normal

Theorem 3.1. *Let $\Phi : R_1 \to R_2$ be a ring homomorphism from the ring R_1 into a ring R_2. Let γ be tQILFSR of R_2. Then $\Phi^{-1}(\gamma)$ is tQILFSR of R_1.*

Proof. Let $x, y \in R_1$, since γ be tQILFSR of R_2. Then

$$\Phi^{-1}(\gamma^t)(x-y,q) = (\lambda_{\Phi^{-1}(\gamma^t)}(x-y,q), \delta_{\Phi^{-1}(\gamma^t)}(x-y,q)).$$

$$\lambda_{\Phi^{-1}(\gamma^t)}(x-y,q) = \lambda_{\gamma^t}(\Phi(x-y),q) = \lambda_{\mu^t}(\Phi(x)-\Phi(y),q)$$

$$\geq \min\{\lambda_{\gamma^t}(\Phi(x),q), \lambda_{\gamma^t}(\Phi(y),q)\} = \min\{\lambda_{\Phi^{-1}(\gamma^t)}(x,q), \lambda_{\Phi^{-1}(\gamma^t)}(y,q).$$

Thus

$$\lambda_{\Phi^{-1}(\gamma^t)}(x-y,q) \geq \min\{\lambda_{\Phi^{-1}(\gamma^t)}(x,q), \lambda_{\Phi^{-1}(\gamma^t)}(y,q)\}.$$

Similarly, we can show that

$$\delta_{\Phi^{-1}(\gamma^t)}(x-y,q) \leq \max\{\delta_{\Phi^{-1}(\gamma^t)}(x,q), \delta_{\Phi^{-1}(\gamma^t)}(y,q)\}(y,q)\}.$$

We have

$$\begin{aligned}\lambda_{\Phi^{-1}(\gamma^t)}(xy,q) =& \lambda_{\gamma^t}(\Phi(xy),q)\\ =& \lambda_{\gamma^t}(\Phi(x)\Phi(y),q) \geq \min\{\lambda_{\gamma^t}(\Phi(x),q), \lambda_{\gamma^t}(\Phi(y),q)\}\\ =& \min\{\lambda_{\Phi^{-1}(\gamma^t)}(x,q), \lambda_{\Phi^{-1}(\gamma^t)}(y,q)\}.\end{aligned}$$

Thus

$$\lambda_{\Phi^{-1}(\gamma^t)}(xy,q) \geq \min\{\lambda_{\Phi^{-1}(\gamma^t)}(x,q), \lambda_{\Phi^{-1}(\gamma^t)}(y,q)\}.$$

Also, it is easy to show that

$$\delta_{\Phi^{-1}(\gamma^t)}(xy,q) \leq \max\{\delta_{\Phi^{-1}(\gamma^t)}(x,q), \delta_{\Phi^{-1}(\gamma^t)}(y,q)\}.$$

Therefore $\Phi^{-1}(\gamma^t) = (\Phi^{-1}(\gamma))^t$ is QILFSR of R_1 and hence $\Phi^{-1}(\gamma)$ is tQILFSR of R_1. □

Corollary 3.2. *Let $\Phi : R_1 \to R_2$ be a ring homomorphism from the ring R_1 into a ring R_2. Let γ be tQILFNSR of R_2. Then $\Phi^{-1}(\gamma)$ is tQILFNSR of R_1.*

Proof. Let $x, y \in R_1$, since γ be tQILFSR of R_2 also

$$\Phi^{-1}(\gamma^t)(xy) = (\lambda_{\Phi^{-1}(\gamma^t)}(xy, q), \delta_{\Phi^{-1}(\gamma^t)}(xy, q)).$$

Hence, it is enough to show that

$$\lambda_{\Phi^{-1}(\gamma^t)}(xy, q) = (\lambda_{\Phi^{-1}(\gamma^t)}(yx, q) \text{ and } \delta_{\Phi^{-1}(\gamma^t)}(xy, q) = \delta_{\Phi^{-1}(\gamma^t)}(yx, q).$$

But

$$\begin{aligned}\lambda_{\Phi^{-1}(\gamma^t)}(xy, q) =& \lambda_{\gamma^t}(\Phi(xy), q)\\ =& \lambda_{\gamma^t}(\Phi(x)\Phi(y), q)\\ =& \lambda_{\gamma^t}(\Phi(y)\Phi(x), q)\\ =& \lambda_{\gamma^t}(\Phi(xy), q)\\ =& \lambda_{\Phi^{-1}(\gamma^t)}(yx, q).\end{aligned}$$

Moreover

$$\begin{aligned}\delta_{\Phi^{-1}(\gamma^t)}(xy, q) =& \delta_{\gamma^t}(\Phi(xy), q)\\ =& \delta_{\gamma^t}(\Phi(x)\Phi(y), q)\\ =& \delta_{\gamma^t}(\Phi(y)\Phi(x), q)\\ =& \delta_{\gamma^t}(\Phi(xy), q)\\ =& \delta_{\Phi^{-1}(\gamma^t)}(yx, q).\end{aligned}$$

Thus $\Phi^{-1}(\gamma^t) = (\Phi^{-1}(\gamma))^t$ is QILFNSR of R_1 and hence $\Phi^{-1}(\gamma)$ is tQILFNSR of R_1. □

Corollary 3.3. *Let $\Phi : R_1 \to R_2$ be a ring homomorphism from the ring R_1 into a ring R_2. Let γ be tQILFLI of R_2. Then $\Phi^{-1}(\gamma)$ is tQILFLI of R_1.*

Proof. Since γ be tQILFSR of R_2 and let $x, y \in R_1$. Then by Theorem 3.1 we need only to prove

$$\lambda_{\Phi^{-1}(\gamma^t)}(xy, q) \geq \lambda_{\Phi^{-1}(\gamma^t)}(y, q)$$

and

$$\delta_{\Phi^{-1}(\gamma^t)}(xy, q) \leq \delta_{\Phi^{-1}(\gamma^t)}(y, q).$$

But

$$\lambda_{\Phi^{-1}(\gamma^t)}(xy,q) = \lambda_{\gamma^t}(\Phi(xy),q) = \lambda_{\gamma^t}(\Phi(x)\Phi(y),q) \geq \lambda_{\gamma^t}(\Phi(y),q) = \lambda_{\Phi^{-1}(\gamma^t)}(y,q).$$

Thus

$$\lambda_{\Phi^{-1}(\gamma^t)}(xy,q) \geq \lambda_{\Phi^{-1}(\gamma^t)}(y,q).$$

Similarly, we can show that $\delta_{\Phi^{-1}(\gamma^t)}(xy,q) \leq \delta_{\Phi^{-1}(\gamma^t)}(y,q)$.

Therefore $\Phi^{-1}(\gamma^t) = (\Phi^{-1}(\gamma))^t$ t is QILFLI of R_1 and hence $\Phi^{-1}(\gamma)$ is tQILFLI of R_1. □

Corollary 3.4. *Let $\Phi : R_1 \to R_2$ be a ring homomorphism from the ring R_1 into a ring R_2. Let γ be tQILFRI of R_2. Then $\Phi^{-1}(\gamma)$ is tQILFRI of R_1.*

Proof. Straight forward. □

Theorem 3.5. *Let $\Phi : R_1 \to R_2$ be epimorphism from the ring R_1 into a ring R_2 and μ be tQILFSR of R_1. Then $\Phi(\mu)$ is tQILFSR of R_2.*

Proof. Let $x, y \in R_2$. Then there exist $a, b \in R_1$ such that $\Phi(a) = x$, $\Phi(b) = y$ we know that a, b need not be unique also μ is tQILFSR of R_1:

$$\begin{aligned}\Phi(\mu^t)(x-y,q) =& (\lambda_{\Phi(\mu^t)}(x-y,q), \delta_{\Phi(\mu^t)}(x-y,q))\\ \lambda_{\Phi(\mu^t)}(x-y,q) =& \lambda_{(\Phi(\mu))^t}(x-y,q) = \min\{\lambda_{\Phi(\mu)}(\Phi(a)-\Phi(b),q), t\}\\ =& \min\{\lambda_{\Phi(\mu)}(\Phi(a-b),q), t\} \geq \min\{\lambda_{\mu}(a-b,q), t\}\\ =& \lambda_{\mu}(a-b,q),\end{aligned}$$

for all $a, b \in R_1$ such that $\Phi(a) = x, \Phi(b) = y$. Moreover, we have

$$\begin{aligned}&= \min\{\sup\{\lambda_{\mu^t}(a,q); \Phi(a) = x\}, \sup\{\{\lambda_{\mu^t}(b,q); \Phi(b) = y\}\}\\ &= \min\{\lambda_{\Phi(\mu^t)}(x,q), \lambda_{\Phi(\mu^t)}(y,q)\}.\end{aligned}$$

Thus $\lambda_{\Phi(\mu^t)}(x-y,q) \geq \min\{\lambda_{\Phi(\mu^t)}(x,q), \lambda_{\Phi(\mu^t)}(y,q)\}$.

Similarly, we can show that

$$\delta_{\Phi(\mu^t)}(x-y,q) \leq \max\{\delta_{\Phi(\mu^t)}(x,q), \delta_{\Phi(\mu^t)}(y,q)\},$$

$$\begin{aligned}\lambda_{\Phi(\mu^t)}(xy,q) =& \lambda_{(\Phi(\mu))^t}(xy,q)\\ =& \min\{\lambda_{\Phi(\mu)}(\Phi(a).\Phi(b),q), t\}\end{aligned}$$

$$= \min\{\lambda_{\Phi(\mu)}(\Phi(ab), q), t\} \geq \min\{\lambda_{\mu}(ab, q), t\} = \lambda_{\mu}(ab, q),$$

for all $a, b \in R_1$ such that $\Phi(a) = x, \Phi(b) = y$.

$$\begin{aligned} &= \min\{\sup\{\lambda_{\mu^t}(a, q); \Phi(a) = x\}, \sup\{\{\lambda_{\mu^t}(b, q); \Phi(b) = y\}\} \\ &= \min\{\lambda_{\Phi(\mu^t)}(x, q), \lambda_{\Phi(\mu^t)}(y, q)\}. \end{aligned}$$

Thus $\lambda_{\Phi(\mu^t)}(xy, q) \geq \min\{\lambda_{\Phi(\mu^t)}(x, q), \lambda_{\Phi(\mu^t)}(y, q)\}$.

Similarly, we can show that $\delta_{\Phi(\mu^t)}(xy, q) \leq \max\{\delta_{\Phi(\mu^t)}(x, q), \delta_{\Phi(\mu^t)}(y, q)\}$.

Thus $\Phi(\mu^t) = (\Phi(\mu))^t$ t is QILFSR of R_2 and hence $\Phi(\mu)$ is tQILFSR of R_2. □

Theorem 3.6. *Let $\Phi : R_1 \to R_2$ be epimorphism from the ring R_1 into a ring $R_2$and μbe tQILFNSR of R_1. Then $\Phi(\mu)$ is tQILFNSR of R_2.*

Proof. Let $x, y \in R_2$. Then there exist $a, b \in R_1$ such that $\Phi(a) = x$, $\Phi(b) = y$ we know that a, b need not be unique also μ is tQILFNSR of R_1.

$\Phi(\mu^t)(xy, q) = (\lambda_{\Phi(\mu^t)}(xy, q), \delta_{\Phi(\mu^t)})(xy, q))$ by Theorem 3.5, we need only to prove that $\lambda_{\Phi(\mu^t)}(xy, q) = \lambda_{\Phi(\mu^t)}(yx, q)$ and $\delta_{\Phi(\mu^t)}(xy, q) = \delta_{\Phi(\mu^t)}(yx, q)$:

$$\begin{aligned} \lambda_{\Phi(\mu^t)}(xy, q) &= \lambda_{\Phi(\mu^t)}(\Phi(a)\Phi(b), q) \\ &= \lambda_{\Phi(\mu^t)}(\Phi(ab), q) \\ &= \sup\{\lambda_{\Phi(\mu^t)}(xy, q); \Phi(ab) = xy\} \\ &= \sup\{\lambda_{\Phi(\mu^t)}(yx, q); \Phi(ab) = xy\} \\ &= \lambda_{\Phi(\mu^t)}(\Phi(ab), q) \\ &= \lambda_{\Phi(\mu^t)}(\Phi(a)\Phi(b), q) \\ &= \lambda_{\Phi(\mu^t)}(yx, q). \end{aligned}$$

□

Also we can show that $\delta_{\Phi(\mu^t)}(xy, q) = \delta_{\Phi(\mu^t)}(yx, q)$

Theorem 3.7. *Let $\Phi : R_1 \to R_2$ be epimorphism from the ring R_1 into a ring $R_2$and μ be tQILFLI of R_1. Then $\Phi(\mu)$ is tQILFLI of R_2.*

Proof. Let $x, y \in R_2$. Then there exist $a, b \in R_2$, then there exist a unique $a, b \in R_1$ such that $\Phi(a) = x$, $\Phi(b) = y$,

$$(\Phi(\mu))^t(xy, q) = (\lambda_{(\Phi(\mu))^t}(xy, q), \delta_{(\Phi(\mu))^t}(xy, q)).$$

Since μ be tQILFLI of R_1, then by Theorem 3.5 we need only to prove that $\lambda_{(\Phi(\mu))^t}(xy,q) \geq \lambda_{(\Phi(\mu))^t}(y,q)$ and

$$\begin{aligned}\delta_{(\Phi(\mu))^t}(xy,q) \leq &\delta_{(\Phi(\mu))^t}(y,q)\lambda_{(\Phi(\mu))^t}(xy,q)\\ =&\min\{\lambda_{\Phi(\mu)}(\Phi(a).\Phi(b),q),t\}\\ =&\min\{\lambda_{\Phi(\mu^t)}(\Phi(ab),q),t\}\\ =&\min\{\lambda_{\mu}(ab,q),t\}\\ =&\lambda_{\mu^t}(ab,q) \geq \lambda_{\mu^t}(b,q)\\ =&\min\{\lambda_{\mu}(b,q),t\}\\ =&\min\{\lambda_{\Phi(\mu)}(\Phi(b),q),t\}\\ =&\min\{\lambda_{\Phi(\mu)}(y,q),t\}\\ =&\lambda_{(\Phi(\mu))^t}(y,q).\end{aligned}$$

Therefore

$$\lambda_{(\Phi(\mu))^t}(xy,q) \geq \lambda_{(\Phi(\mu))^t}(y,q).$$

Similarly, we can show that

$$\delta_{(\Phi(\mu))^t}(xy,q) \leq \delta_{(\Phi(\mu))^t}(y,q).$$

Hence $(\Phi(\mu))^t$ is QILFLI of R_2 and hence $\Phi(\mu)$ is tQILFLI of R_2. □

Corollary 3.8. *Let $\Phi : R_1 \to R_2$ be epimorphism from the ring R_1 into a ring R_2 and μ be tQILFRI of R_1. Then $\Phi(\mu)$ is tQILFRI of R_2.*

Theorem 3.9. *Let R_1, R_2 be any two rings. The homomorphic image of a tQILFSR of R_1 is a tQILFSR of $\Phi(R_1) = R_2$.*

Proof. Let μ be a tQILFSR of R_1. We have to prove that γ is tQILFSR of R_2, now for $\Phi(x), \Phi(y) \in R_2$ and $q \in Q$.
$\lambda_{\gamma^t}(\Phi(x) - \Phi(y), q) = \lambda_{\gamma^t}(\Phi(x - y), q) = \min\{\lambda_{\gamma}(\Phi(x - y), q), t\}$
$\geq \min\{\lambda_{\gamma}(x - y, q), t\} = \min\{\lambda_{\gamma^t}(x, q), \lambda_{\gamma^t}(y, q)\}$.
Thus $\lambda_{\gamma^t}(\Phi(x) - \Phi(y), q) \geq \min\{\lambda_{\gamma^t}(\Phi(x), q), \lambda_{\gamma^t}(\Phi(y), q)\}$
also for $\Phi(x), \Phi(y) \in R_2$ and $q \in Q$.
$\lambda_{\gamma^t}(\Phi(x)\Phi(y), q) = \lambda_{\gamma^t}(\Phi(xy), q) = \min\{\lambda_{\gamma}(\Phi(xy), q), t\}$
$\geq \min\{\lambda_{\gamma}(xy, q), t\} = \min\{\lambda_{\gamma^t}(x, q), \lambda_{\gamma^t}(y, q)\}$
Thus $\lambda_{\gamma^t}(\Phi(x)\Phi(y), q) \geq \min\{\lambda_{\gamma^t}(\Phi(x), q), \lambda_{\gamma^t}(\Phi(y), q)\}$.
Similarly we can prove that $\delta_{\gamma^t}(\Phi(x){-}\Phi(y), q) \leq \max\{\delta_{\gamma^t}(\Phi(x),), \delta_{\gamma^t}(\Phi(y), q)\}$

And $\delta_{\gamma^t}(\Phi(x)\Phi(y),q) \leq \max\{\delta_{\gamma^t}(\Phi(x),),\delta_{\gamma^t}(\Phi(y),q)\}$. Therefore γ is tQILFSR of R_2. □

Corollary 3.10. *Let R_1, R_2 be any two rings. The homomorphic image of a tQILFLI (tQILFRI) of R_1 is a tQILFLI (tQILFRI) of $\Phi(R_1) = R_2$.*

Theorem 3.11. *Let R_1, R_2 be any two rings. The homomorphic image of a tQILFNSR of R_1 is a tQILFNSR of $\Phi(R_1) = R_2$.*

Proof. Since μ is a tQILFSR of R_1. We have to prove that γ is tQILFSR of R_2, now for $\Phi(x),\Phi(y) \in R_2$ and $q \in Q$, clearly γ is tQILFSR of R_2.

Also μ is is tQILFSR of R_1. Now,

$$\begin{aligned}\lambda_{\gamma^t}(\Phi(x)\Phi(y),q) &= \lambda_{\gamma^t}(\Phi(xy),q) \geq \lambda_{\mu^t}(xy,q)\\ &= \lambda_{\mu^t}(yx,q) = \lambda_{\mu^t}(\Phi(yx),q) = \lambda_{\gamma^t}(\Phi(y)\Phi(x),q).\end{aligned}$$

Thus $\lambda_{\gamma^t}(\Phi(x)\Phi(y),q) = \lambda_{\gamma^t}(\Phi(y)\Phi(x),q)$ also for all $\Phi(x),\Phi(y) \in R_2$ and $q \in Q$

$$\begin{aligned}\delta_{\gamma^t}(\Phi(x)\Phi(y),q) &= \delta_{\gamma^t}(\Phi(xy),q) \leq \delta_{\mu^t}(xy,q) = \delta_{\mu^t}(yx,q)\\ &= \delta_{\gamma^t}(\Phi(yx),q) = \delta_{\gamma^t}(\Phi(y)\Phi(x),q).\end{aligned}$$

Thus $\delta_{\gamma^t}(\Phi(x)\Phi(y),q) = \delta_{\gamma^t}(\Phi(y)\Phi(x),q)$. Therefore γ is tQILFNSR of R_2. □

Corollary 3.12. *Let R_1, R_2 be any two rings. The homomorphic preimage of a tQILFSR of $\Phi(R_1) = R_2$ is a tQILFSR of R_1.*

Corollary 3.13. *Let R_1, R_2 be any two rings. The homomorphic preimage of a tQILFNSR of $\Phi(R_1) = R_2$ is a tQILFNSR of R_1.*

Corollary 3.14. *Let R_1, R_2 be any two rings. The homomorphic preimage of a tQILFLI (tQILFRI) of $\Phi(R_1) = R_2$ is a tQILFLI (tQILFRI) of R_1.*

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