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A NOTE ON SOFT-PREOPEN SETS

Mrudla Ravindran[1], Manju[1], Gnanambal Ilango[2]
[1,2]Department of Mathematics
C.M.S. College of Science & Commerce
Coimbatore, 641049, Tamil Nadu, INDIA
[2]Department of Mathematics
Government Arts College
Coimbatore, 641018, Tamil Nadu, INDIA

Abstract: Intersection of two soft-preopen sets need not be a soft-preopen set. In this paper, a collection of soft sets which on intersection with soft-preopen gives a soft-preopen is considered. The soft-topology generated by such a collection is discussed and certain properties of this soft-topology are analyzed.

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Key Words: soft topological space, soft-α set, soft-semi open set, soft-preopen set, soft-semi preopen set

1. Introduction

In 1999, the Russian researcher Molodstov [6] introduced the concept of a soft set, and started to develop the basics of the corresponding theory as a new approach for modelling uncertainties. He pointed out several directions for the applications of soft sets, such as game theory, Riemann integration, theory of measurement, smoothness of functions and so on. He also showed that soft set theory is free from the parameterization inadequacy syndrome of other theories developed for vagueness.

Maji et al [16] presented some new definitions on soft sets and discussed in detail the application of soft set theory in decision making problem. In 2011, Shabir and Naz[14] initiated the soft topological spaces. Sabir Hussian

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and Bashir Ahmad[17] discussed about the some properties of soft topological spaces. J.Mahanta and P.K.Das[12] introduced and gave a details of soft semi open, soft semi closed, soft-semi continuity, soft-compactness, soft-semi connectedness and soft-semi separation axioms. Andrijevic[1] gave many results on preopen sets in general topology. Mrudula Ravindran and Gnanambal Ilango[8] introduced soft preopen sets and proved some properties of soft preopen sets. Dimitrije Andrijevic[4] discussed about a collection of sets which on intersection with pre-open sets are preopen, in general topology.

2. Preliminaries

A soft topology is denoted by (X,A,τ) or simply by $\tilde{X}$. The soft closure of (F,A) and soft-interior of (F,A) with respect to $\tilde{\tau}$ are denoted by $\tilde{s}cl(F,A)$ and $s\tilde{int}(F,A)$ respectively.

Definition 1. A soft-subset (F,A) of (X,A,τ) is called

(i) a soft-α set if $(F,A)\tilde{\subset}\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A)))$

(ii) a soft-semi open set if $(F,A)\tilde{\subset}\tilde{s}cl(\tilde{s}int(F,A))$

(iii) a soft-preopen set if $(F,A)\tilde{\subset}\tilde{s}int(\tilde{s}cl(F,A))$

(iv) a soft-semi preopen set if (F,A)$\tilde{\subset}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))$

we denote the families of these soft-sets by $\tilde{\tau}_\alpha, \tilde{S}SO(\tilde{X}), \tilde{S}PO(\tilde{X})$,
$\tilde{S}SPO(\tilde{X})$ respectively.

Theorem 2. *(X,A,τ) is a soft topological space in which every soft-preopen set is soft open if and only if (X,A,τ) is sub maximal.*

Theorem 3. *(X,A,τ) is a soft topological space in which every soft subset is soft preopen if and only if every soft open set in(X,A,τ) is soft closed.*

Proof. Let (F,A) be open and closed. Then

$$\begin{aligned}(F,A) &= \tilde{s}int(F,A) = \tilde{s}cl(F,A).\\ \tilde{s}cl(F,A) &= \tilde{s}int(\tilde{s}cl(F,A)) \Rightarrow \tilde{s}cl(F,A)\tilde{\subset}\tilde{s}int(\tilde{s}cl(F,A))\\ &\Rightarrow (F,A)\tilde{\subset}\tilde{s}int(\tilde{s}cl(F,A)).\end{aligned}$$

Every soft open is soft-preopen. Every soft subset is soft preopen. Conversely, let(F,A) be a soft open set, $(F,A)\tilde{\subseteq}\tilde{s}int(\tilde{s}cl(F,A))$ and $\tilde{s}cl(F,A)\tilde{\subseteq}\tilde{s}int(\tilde{s}cl(F,A))$

given every soft set is soft preopen.
But $\tilde{s}int(\tilde{s}cl(F,A))\tilde{\subseteq}\tilde{s}cl(F,A) \Rightarrow \tilde{s}cl(F,A) = \tilde{s}int(\tilde{s}cl(F,A))$
$\Rightarrow$ Every open set is closed. □

For a soft subset (F, A) of $\tilde{X}$, let $\tilde{s}cl_\alpha(F,A)$ and $\tilde{s}int_\alpha(F,A)$ stands for the closure of (F,A) and interior of (F, A) with respect to (X, A, τ_α). The complement of a soft-α set is called a soft-α closed set. Similarly soft-semi closed, soft-pre closed and soft-semi pre closed sets are defined. If (F, A) is a soft subset of $\tilde{X}$, then the soft pre closure (resp. soft semi closure) of (F, A), denoted by $\tilde{s}pcl(F,A)(resp.\tilde{s}scl(F,A))$, is the intersection of all soft pre closed(resp. soft semi closed) subsets of $\tilde{X}$ containing (F, A). The soft pre-interior (resp. semi-interior) of (F, A) denoted by $\tilde{s}pint(F,A)(resp.\tilde{s}sint(F,A))$, is the union of all soft preopen (resp. semi open) sets contained in (F, A).

Theorem 4. *If $\tilde{U}$ is soft open and (F, A) is soft preopen, then $\tilde{U}\tilde{\cap}(F,A)$ is soft preopen.*

Theorem 5 (8)**.** *Let (F, A) be a soft subset of a soft topological space $\tilde{X}$. Then*

(a) (F, A) is soft-preclosed if and only if $\tilde{s}cl(\tilde{s}int(F,A))\tilde{\subset}(F,A)$.

(b) (F, A) is soft-semi preclosed if and only if $\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A)))\tilde{\subset}(F,A)$.

Proof. (a) (F, A) is soft-preclosed $\Rightarrow (F,A)^c$ is soft-preopen.

$$\Rightarrow (F,A)^c\tilde{\subset}\tilde{s}int(\tilde{s}cl((F,A)^c))$$
$$\Rightarrow (F,A)^c\tilde{\subset}[\tilde{s}cl(\tilde{s}int(F,A))]^c$$
$$\Rightarrow (F,A)\tilde{\supset}[\tilde{s}cl(\tilde{s}int(F,A))]$$

conversely,

$$\Rightarrow \tilde{s}cl(\tilde{s}int(F,A))\tilde{\subset}(F,A)$$
$$\Rightarrow [\tilde{s}cl(\tilde{s}int(F,A))]^c\tilde{\supset}(F,A)^c$$
$$\Rightarrow \tilde{s}int(\tilde{s}cl(F,A)^c)\tilde{\supset}(F,A)^c$$

$\Rightarrow$(F, A)is soft-preclosed.
(b) (F, A)is soft-semi preclosed $\Rightarrow (F,A)^c$ is soft-semi preopen.

$$\Rightarrow (F,A)^c\tilde{\subset}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)^c))$$

$$\Rightarrow (F,A)^c \tilde{\subset} [\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A)))]^c$$
$$\Rightarrow (F,A) \tilde{\supset} [\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A)))]$$

conversely,

$$\Rightarrow \tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A))) \tilde{\subset} (F,A)$$
$$\Rightarrow [\tilde{s}int(\tilde{s}cl(F,A))]^c \tilde{\supset} (F,A)^c$$
$$\Rightarrow \tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)^c)) \tilde{\supset} (F,A)^c$$

(F,A) is soft-semi preclosed. □

Theorem 6. *Let (F, A) be a soft subset of a soft topological space $\tilde{X}$. Then*

(a) $\tilde{s}scl(F,A) = (F,A) \tilde{\bigcup} \tilde{s}int(\tilde{s}cl(F,A))$

(b) $\tilde{s}sint(F,A) = (F,A) \tilde{\bigcap} \tilde{s}cl(\tilde{s}int(F,A))$

(c) $\tilde{s}cl_\alpha(F,A) = (F,A) \tilde{\bigcup} \tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))$

(d) $\tilde{s}int_\alpha(F,A) = (F,A) \tilde{\bigcap} \tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A)))$

(e) $\tilde{s}pint(F,A) = (F,A) \tilde{\bigcup} \tilde{s}cl(\tilde{s}int(F,A))$

(f) $\tilde{s}pint(F,A) = (F,A) \tilde{\bigcap} \tilde{s}int(\tilde{s}cl(F,A))$

Proof. (a)

$$\tilde{s}int(\tilde{s}cl((F,A) \tilde{\bigcup} \tilde{s}int(\tilde{s}cl(F,A)))) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(F,A) \tilde{\bigcup} \tilde{s}int(\tilde{s}cl(F,A)))$$
$$= \tilde{s}int(\tilde{s}cl(F,A)) \tilde{\subseteq} (F,A) \tilde{\bigcup} \tilde{s}int(\tilde{s}cl(F,A))$$

$\Rightarrow (F,A) \tilde{\bigcup} \tilde{s}int(\tilde{s}cl(F,A))$ is soft semi closed
$\Rightarrow \tilde{s}scl(F,A) \tilde{\subseteq} (F,A) \tilde{\bigcup} \tilde{s}int(\tilde{s}cl(F,A))$
conversely, $\tilde{s}scl(F,A)$ is soft semi closed

$$\tilde{s}int(\tilde{s}cl(F,A)) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(\tilde{s}scl(F,A)))$$
$$\tilde{\subseteq} \tilde{s}scl(F,A) \Rightarrow (F,A) \tilde{\bigcup} \tilde{s}int(\tilde{s}cl(F,A)) \tilde{\subseteq} \tilde{s}scl(F,A)$$
$$\Rightarrow \tilde{s}scl(F,A) = (F,A) \tilde{\bigcup} \tilde{s}int(\tilde{s}cl(F,A))$$

(b)

$$\begin{aligned}
&\tilde{s}cl(\tilde{s}int((F,A)\tilde{\bigcap}\tilde{s}cl(\tilde{s}int(F,A))))\\
&\tilde{\supseteq}\tilde{s}cl(\tilde{s}int((F,A)\tilde{\bigcap}\tilde{s}cl(\tilde{s}int(F,A)))\\
&=\tilde{s}cl(\tilde{s}int(F,A))\\
&\tilde{\supseteq}(F,A)\tilde{\bigcap}\tilde{s}cl(\tilde{s}int(F,A))
\end{aligned}$$

$\Rightarrow (F,A)\tilde{\bigcap}\tilde{s}cl(\tilde{s}int(F,A))$ is soft semiopen.
$\Rightarrow \tilde{s}\tilde{s}int(F,A)\tilde{\supseteq}(F,A)\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(F,A))$
Conversely, $\tilde{s}\tilde{s}int(F,A)$ is soft semi open$\Rightarrow$

$$\begin{aligned}
&\tilde{s}cl(\tilde{s}int(F,A))\tilde{\supseteq}\tilde{s}cl(\tilde{s}int(\tilde{s}\tilde{s}int(F,A))\\
&\tilde{\supseteq}\tilde{s}\tilde{s}int(F,A)\Rightarrow(F,A)\tilde{\bigcap}\tilde{s}cl(\tilde{s}int(F,A))\\
&\tilde{\supseteq}\tilde{s}\tilde{s}int(F,A)\\
&\Rightarrow \tilde{s}\tilde{s}int(F,A)=(F,A)\tilde{\bigcap}\tilde{s}cl(\tilde{s}int(F,A)))
\end{aligned}$$

(c)

$$\begin{aligned}
&\tilde{s}cl(\tilde{s}int(\tilde{s}cl((F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(F,A))))\\
&\tilde{\subseteq}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(F,A)))\\
&=\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))\\
&\tilde{\subseteq}(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))\Rightarrow(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))
\end{aligned}$$

is soft α-closed.

$$\Rightarrow \tilde{s}cl_\alpha(F,A)\tilde{\subseteq}(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))$$

conversely, $\tilde{s}cl_\alpha(F,A)$ is soft α-closed.

$$\begin{aligned}
&\Rightarrow \tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))\\
&\tilde{\subseteq}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(\tilde{s}cl_\alpha(F,A))))\\
&\tilde{\subseteq}\tilde{s}cl_\alpha(F,A)\Rightarrow(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))
\end{aligned}$$

$$\tilde{\subseteq}\tilde{s}cl_{\alpha}(F,A) \Rightarrow \tilde{s}cl_{\alpha}(F,A) = (F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))$$

(d)

$$\begin{aligned}
&\tilde{s}int(\tilde{s}cl(\tilde{s}int((F,A)\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(F,A)))))\\
&\tilde{\supseteq}\tilde{s}int(\tilde{s}cl(\tilde{s}int((F,A)\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(F,A))))\\
&= \tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A)))\\
&\tilde{\supseteq}(F,A)\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(\tilde{s}int((F,A))))\\
&\Rightarrow (F,A)\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(\tilde{s}int((F,A))))
\end{aligned}$$

is soft α-closed.

$$\Rightarrow \tilde{s}int_{\alpha}(F,A)\tilde{\supseteq}(F,A)\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A)))$$

conversely, $\tilde{s}int_{\alpha}(F,A)$is soft α-open.

$$\begin{aligned}
\Rightarrow \tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A)))\\
\tilde{\supseteq}\tilde{s}int(\tilde{s}cl(\tilde{s}int(\tilde{s}int_{\alpha}(F,A))\\
\tilde{\supseteq}\tilde{s}int_{\alpha}(F,A) \Rightarrow (F,A)\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A)))\\
\tilde{\supseteq}\tilde{s}int_{\alpha}(F,A) \Rightarrow \tilde{s}int_{\alpha}(F,A) = (F,A)\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A)))
\end{aligned}$$

(e)and(f)are proved in [8]. □

Theorem 7. *For any soft topological space(X,A,τ), if $(F,A)\tilde{\in}\tilde{S}SO(\tilde{X})$ and $(G,A)\tilde{\in}\tilde{S}PO(\tilde{X})(F,A)\tilde{U}(G,A)\tilde{\in}\tilde{S}SPO(\tilde{X})$*

Proof.

$$\begin{aligned}
(F,A)\tilde{U}(G,A)\tilde{\subseteq}\tilde{s}cl(\tilde{s}int(F,A))\tilde{U}\tilde{s}int(\tilde{s}cl(G,A))\\
\tilde{\subseteq}\tilde{s}cl(\tilde{s}int(F,A))\tilde{U}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(G,A))\\
\tilde{\subseteq}\tilde{s}cl[(\tilde{s}int(F,A))\tilde{U}(\tilde{s}int(\tilde{s}cl(G,A))]\\
\tilde{\subseteq}\tilde{s}cl[\tilde{s}int(\tilde{s}cl(F,A))\tilde{U}(\tilde{s}int(\tilde{s}cl(G,A))]\\
\tilde{\subseteq}\tilde{s}cl(\tilde{s}int(\tilde{s}cl((F,A))\tilde{U}(G,A)))\\
\tilde{\subseteq}\tilde{S}SPO(\tilde{X})
\end{aligned}$$

□

Remark 8. 1 Every soft open (resp.closed) set is soft preopen (resp. pre closed) [8].

2 Every soft open (resp.closed) set is softα-open (resp.α-closed)

3 Every soft αopen (resp.α-closed) set is soft preopen (resp.pre closed).

Proof. Let

$$\begin{aligned} & (F,A)\tilde{\in}\alpha OS\tilde{X} \\ & \Rightarrow (F,A)\tilde{\subset}\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A))) \\ since, \tilde{s}int(F,A)\tilde{\subset}\tilde{s}cl(F,A)\tilde{s}cl(\tilde{s}int(F,A)) & \\ & \tilde{\subset}\tilde{s}cl(F,A) \\ & \Rightarrow \tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A))) \\ & \tilde{\subset}\tilde{s}int(\tilde{s}cl(F,A)) \end{aligned}$$

Hence

$$\begin{aligned} (F,A)\tilde{\subset}\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A))) & \\ \tilde{\subset}\tilde{s}int(\tilde{s}cl(F,A)) & \\ \Rightarrow (F,A)\tilde{\subset}\tilde{s}int(\tilde{s}cl(F,A)) & \\ \Rightarrow (F,A)\tilde{\in}\tilde{S}POS(\tilde{X}) & \end{aligned}$$

□

Theorem 9. *Soft topological spaces* (X,A,τ) *and* (X,A,τ_α) *have the same class of soft preopen sets.*

Proof. By the above remark, we obtain (X,A,τ) and (X,A,τ_α) have the same class of soft preopen sets. □

Theorem 10. *Let (F, A) be a soft subset of a soft topological space* $\tilde{X}$*. Then*

(a) $\tilde{s}cl(\tilde{s}pint(F,A)) = \tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))$

(b) $\tilde{s}int(\tilde{s}scl(F,A)) = \tilde{s}int(\tilde{s}cl(F,A))$

Proof. (a)

$$\tilde{s}cl(\tilde{s}pint(F,A)) = \tilde{s}cl((F,A)\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(F,A)))[\text{by theorem 2.5}]$$

$$\tilde{\subset}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))$$

$$\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A))) = \tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A))\tilde{\bigcap}\tilde{s}cl(F,A))$$
$$\tilde{\subset}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A))\tilde{\bigcap}(F,A)$$
$$\tilde{\subset}\tilde{s}cl(\tilde{s}pint(\tilde{s}cl(F,A)))$$

Hence

$$\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A))) = \tilde{s}cl(\tilde{s}pint(F,A))$$

(b)

$$\tilde{s}cl(\tilde{s}sint(F,A)) = \tilde{s}int((F,A)\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(F,A)))$$
$$\tilde{\supset}\tilde{s}int(\tilde{s}cl(F,A))$$

For

$$\tilde{\tau}\tilde{\subset}\tilde{S}SO(\tilde{X}),$$

we have

$$\tilde{s}cl(\tilde{s}sint)(F,A))$$
$$\tilde{\subset}\tilde{s}int(\tilde{s}cl(F,A))$$

Therefore,

$$\tilde{s}cl(\tilde{s}sint(F,A)) = \tilde{s}int(\tilde{s}cl(F,A)).$$

□

Theorem 11. *Let (F, A) be a soft-subset of $\tilde{X}$. Then*

(a) $\tilde{s}pcl(\tilde{s}pint(F,A)) = \tilde{s}pint(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(F,A))$

(b) $\tilde{s}scl(\tilde{s}pint(F,A)) = \tilde{s}int(\tilde{s}cl(F,A))$

Proof. (a) Since

$$\tilde{\tau}\tilde{\subset}\tilde{S}PO(\tilde{X})$$

We have,

$$\begin{aligned} \tilde{s}int(F,A) &\tilde{\subset} \tilde{s}pint(F,A) \\ &\tilde{\subset}(F,A) \end{aligned}$$

Therefore,

$$\begin{aligned} \tilde{s}int(\tilde{s}pint(F,A)) =& \tilde{s}int(F,A) \\ \tilde{s}pcl(\tilde{s}pint(F,A)) =& \tilde{s}pint(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(\tilde{s}pint(F,A))) \\ &= \tilde{s}pint(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(F,A)) \end{aligned}$$

(b)

$$\begin{aligned} &\tilde{s}scl(\tilde{s}pint(F,A)) = \tilde{s}pint(F,A)\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(\tilde{s}pint(F,A))) \\ &= ((F,A)\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(F,A)))\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))) \\ &= ((F,A)\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(F,A)))\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(F,A)) \\ &= \tilde{s}int(\tilde{s}cl(F,A)) \end{aligned}$$

□

Theorem 12. *Let (F,A) be a soft-subset of $\tilde{X}$. Then*

(a) $\tilde{s}pcl(\tilde{s}scl(F,A)) = \tilde{s}cl_\alpha(F,A)$

(b) $\tilde{s}scl(\tilde{s}pcl(F,A)) = \tilde{s}pcl(F,A)\tilde{\bigcup}\tilde{s}scl(F,A)$

Proof. (a)

$$\begin{aligned} \tilde{s}pcl(\tilde{s}scl(F,A)) =& \tilde{s}scl(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A)))\text{[by theorem 2.5]} \\ &= (F,A)\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(F,A))\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A))) \\ &= (F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A))) \\ &= \tilde{s}cl_\alpha(F,A) \end{aligned}$$

(b)

$$\tilde{s}scl(\tilde{s}pcl(F,A)) = \tilde{s}pcl(F,A)\tilde{\bigcup}s\tilde{i}nt(\tilde{s}cl(\tilde{s}pcl(F,A)))$$

$$= \tilde{s}pcl(F,A)\tilde{\bigcup}s\tilde{int}(\tilde{s}cl(F,A))$$
$$= \tilde{s}pcl(F,A)\tilde{\bigcup}\tilde{s}scl(F,A)$$

□

3. Soft Topological Space (X, A, τ_γ)

For a soft-topological space (X, A, τ), let $\tilde{\tau_\gamma}$ consists of exactly those soft set (F, A) for which $(F,A)\tilde{\bigcap}(G,B)\tilde{\in}\tilde{S}PO(\tilde{X})$ $\forall (G,B)\tilde{\in}\tilde{S}PO(\tilde{X})$.

Theorem 13. *$\tilde{\tau_\gamma}$ is a soft topology on $\tilde{X}$ such that, $\tilde{\tau_\alpha}\tilde{\subset}\tilde{\tau_\gamma}$.*

Proof. It is obvious that $\tilde{\phi}\tilde{\in}\tilde{\tau_\gamma}$ and $\tilde{X}\tilde{\in}\tilde{\tau_\gamma}$.
Suppose that$\{(F_i,A) : i \in I\}\tilde{\subset}\tilde{\tau_\gamma}$.
Then, $((F_i,A))\tilde{\bigcap}(G,B)\tilde{\in}\tilde{S}PO(\tilde{X})\forall(G,B)\tilde{\in}\tilde{S}PO(\tilde{X})$ and every $i \in I$.
Therefore,

$$(\tilde{\bigcup}(F_i,A)) \cap (G,B) = \tilde{\bigcup}((F_i,A)\tilde{\cap}(G,B))$$

is soft preopen $\forall(G,B)\tilde{\in}\tilde{S}PO(\tilde{X})$, and hence $\tilde{\bigcup}((F_i,A))\tilde{\in}\tilde{\tau_\gamma}$.
Let (H,C) and (J,D) belongs to $\tilde{\tau_\gamma}$. Then

$$((H,C)\tilde{\bigcap}(J,D))\tilde{\bigcap}(G,B)$$
$$= (H,C)\tilde{\bigcap}((J,D)\tilde{\bigcap}(G,B))$$
$$\tilde{\in}\tilde{S}PO(\tilde{X})\forall(G,B)\tilde{\in}\tilde{S}PO(\tilde{X})$$

and hence

$$(H,C)\tilde{\bigcap}(J,D)\tilde{\in}\tilde{\tau_\gamma}$$

Therefore $\tilde{\tau_\gamma}$ is a soft topological space.

$$(F,A)\tilde{\in}\alpha OSS(\tilde{X})$$
$$and\,(G,B)\tilde{\in}\tilde{S}PO(\tilde{X}).$$

$$Then\ (F,A)\tilde{\bigcap}(G,B)\tilde{\subseteq}\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A))\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(G,B))$$
$$\tilde{\subseteq}\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A))\tilde{\bigcap}\tilde{s}cl(G,B))$$
$$\tilde{\subseteq}\tilde{s}int(\tilde{s}cl(F,A)\tilde{\bigcap}\tilde{s}cl(G,B))$$
$$\tilde{\subseteq}\tilde{s}int(\tilde{s}cl(F,A)\tilde{\bigcap}(G,B)))$$

Hence $\tilde{\tau_\alpha}\tilde{\subseteq}\tilde{\tau_\gamma}$.
For a soft subset (F, A) of a soft topological space (X,A,τ), let the closure of (F, A) and the interior of (F, A) are denoted by the $\tilde{s}cl_\gamma(F,A)$ and $\tilde{s}int_\gamma(F,A)$ with respect to the (X,A,τ_γ). The family of all sets which are closed in (X,A,τ_γ) will be denoted by $\tilde{\tau_\gamma}$ □

The following theorems are immediate results of theorem-3.1

Theorem 14. *For any soft topological space (X,A,τ) we have*

$$\tilde{\tau_\gamma}\tilde{\subset}\tilde{S}PO(X,A,\tau).$$

Theorem 15. *The family $\tilde{S}PO(\tilde{X})$ is a soft topology on $\tilde{X}$ if and only if $\tilde{S}PO(\tilde{X})=\tilde{\tau_\gamma}$.*

Theorem 16. *A subset (F, A) of a soft topological space (X,A,τ) is soft closed in (X,A,τ_γ) if and only if $(F,A)\bigcup(G,B)$ is soft preclosed for every preclosed set (G, B).*

Proof. Proof follows from the definition of (X,A,τ_γ) □

Theorem 17. *Let (F, A) be a soft subset of a topological space $\tilde{X}$. Then $\tilde{s}cl_\alpha(F,A)=\tilde{s}cl_\gamma(F,A)\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(F,A))$*

Proof.

$$\tilde{s}int(s\tilde{i}nt(\tilde{s}cl(F,A))/(F,A))$$
$$=(\tilde{s}int(\tilde{s}cl(F,A))/\tilde{s}cl(F,A)$$
$$=\phi$$
$$\Rightarrow \tilde{s}int(\tilde{s}cl(F,A))/(F,A)$$

is soft pre closed and hence,

$$\tilde{s}cl_\gamma(F,A)\tilde{\bigcup}(\tilde{s}int(\tilde{s}cl(F,A))/(F,A))$$

$$= \tilde{s}cl_\gamma(F,A)\tilde{\bigcup}(\tilde{s}int(\tilde{s}cl(F,A))$$

is soft preclosed by theorem-3.4,

$$\tilde{s}scl(F,A) = (F,A)\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(F,A))$$
$$\tilde{\subset}\tilde{s}cl_\gamma(F,A)\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(F,A)) \Rightarrow \tilde{s}pcl(\tilde{s}scl(F,A))$$
$$\tilde{\subset}\tilde{s}cl_\gamma(F,A)\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(F,A)).$$

Applying theorem 2.9 we obtain,

$$\tilde{s}cl_\alpha(F,A)$$
$$\tilde{\subset}\tilde{s}cl_\gamma(F,A)\tilde{\bigcup}s\tilde{in}t(\tilde{s}cl(F,A))$$

$\tilde{\tau_\alpha}\tilde{\subset}\tilde{\tau_\gamma} \Rightarrow \tilde{s}cl_\gamma(F,A)\tilde{\subset}\tilde{s}cl_\alpha(F,A)$

$$\Rightarrow \tilde{s}cl_\gamma(F,A)\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(F,A))\tilde{\subset}\tilde{s}cl_\alpha(F,A) \Rightarrow \tilde{s}cl_\alpha(F,A)$$
$$= \tilde{s}cl_\gamma(F,A)\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(F,A))$$

□

Corollary 18. *For a soft subset (F, A) of a topological space $\tilde{X}$ we have* $\tilde{s}cl_\alpha(F,A) = \tilde{s}cl_\gamma(F,A)\tilde{\bigcup}\tilde{s}scl(F,A)$.

Theorem 19. *For a soft subset (F, A) of a topological space $\tilde{X}$ we have* $\tilde{s}int_\alpha(F,A) = \tilde{s}int_\gamma(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(F,A))$
$= \tilde{s}int_\gamma(F,A)\tilde{\bigcup}\tilde{s}int(F,A)$

Proof. Let

$\tilde{s}cl(\tilde{s}cl(\tilde{s}int(F,A))/(F,A)) = \tilde{s}cl(\tilde{s}int(F,A))/\tilde{s}int(F,A)$
$= \phi$
$\Rightarrow \tilde{s}cl(\tilde{s}int(F,A))/(F,A)$ is soft preopen and hence
$\tilde{s}int_\gamma(F,A)\tilde{\bigcap}\tilde{s}cl(\tilde{s}int(F,A))$is soft preopen by the definition of $\tilde{\tau_\gamma}$.
$\tilde{s}sint(F,A) = (F,A)\tilde{\bigcap}\tilde{s}cl(\tilde{s}int(F,A))$

$$\tilde{\supset}\tilde{s}int_\gamma(F,A)\tilde{\bigcap}\tilde{s}cl(\tilde{s}int(F,A))$$

Hence,

$$\tilde{s}pint(\tilde{s}sint(F,A))\tilde{\supset}\tilde{s}int_\gamma(F,A)\tilde{\bigcap}\tilde{s}cl(\tilde{s}int(F,A))$$

which implies that

$$\tilde{s}int_\alpha(F,A)\tilde{\supset}\tilde{s}int_\gamma(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(F,A))$$

$$[\text{sice}, \tilde{s}pint(\tilde{s}sint(F,A)) = \tilde{s}int_\alpha(F,A)]$$

$$\tilde{\tau}_\alpha\tilde{\subset}\tilde{\tau}_\gamma$$
$$\Rightarrow \tilde{s}int_\gamma(F,A)$$
$$\tilde{\supset}\tilde{s}int_\alpha(F,A)$$

Hence

$$\tilde{s}int_\gamma(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(F,A))$$
$$\tilde{\supset}\tilde{s}int_\alpha(F,A)$$

Therefore,

$$\tilde{s}int_\alpha(F,A) = \tilde{s}int_\gamma(F,A)\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(F,A))$$
$$\tilde{s}int_\gamma(F,A)\tilde{\bigcup}\tilde{s}sint(F,A)[\text{by}2.6]$$

□

Lemma 20. *Let (F, A) be a soft semi-pre closed set such that* $(F,A) = \tilde{s}scl(F,A)$ *and* $\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A)))$
$= \tilde{s}cl(\tilde{s}int(F,A))\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(F,A))$. *Then (F, A) is soft preclosed.*

Proof.

Applying 2.10 we have,

$$\tilde{s}pcl(\tilde{s}scl(F,A))$$
$$= \tilde{s}cl_\alpha(F,A)$$
$$= \tilde{s}scl(F,A) \quad \text{and}$$
$$\tilde{s}pcl(F,A)\tilde{\subset}\tilde{s}scl(F,A)$$

Let

$$X_e\tilde{\in}\tilde{s}cl(\tilde{s}int(F,A))\tilde{\subset}\tilde{s}pcl(F,A)$$

$$\tilde{\subset}\tilde{s}scl(F,A)$$
$$= (F,A)\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(F,A)).$$

If $X_e\tilde{\in}\tilde{s}int(\tilde{s}cl(F,A))$, then

$$X_e\tilde{\in}\tilde{s}cl(\tilde{s}int(F,A))\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(F,A)) = \tilde{s}int(\tilde{s}cl\tilde{s}int(F,A)))$$
$$\tilde{\subset}(F,A)$$
$$\Rightarrow \tilde{s}cl(\tilde{s}int(F,A))\tilde{\subset}(F,A)$$

Hence, (F,A)is soft pre closed.

□

Lemma 21. *Let $\tilde{X}$ be a soft topological space. Then*

$$\tilde{s}cl_\alpha(F,A) = \tilde{s}pcl(F,A)\tilde{\bigcup}\tilde{s}scl(F,A)$$

for every soft subset of (F, A) if and only if

$$\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A))) = \tilde{s}int(\tilde{s}cl(F,A))\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(F,A)),$$

for every (F, A).

Proof. Let (F, A) be an arbitrary soft subset of $\tilde{X}$

$$(F,A) = \tilde{s}pint(F,A)$$
$$\Rightarrow \tilde{s}cl_\alpha(\tilde{s}pint(F,A)) = \tilde{s}pcl(\tilde{s}pint(F,A))\tilde{\bigcup}\tilde{s}scl(\tilde{s}pint(F,A))$$
$$\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A))) = \tilde{s}pint(F,A)\tilde{\bigcup}\tilde{s}cl(s\tilde{i}nt((F,A))\tilde{\bigcup}\tilde{s}int(\tilde{s}cl(F,A))$$
$$= \tilde{s}cl(\tilde{s}int(F,A))\tilde{\bigcup}s\tilde{i}nt(\tilde{s}cl(F,A))\text{[by using theorem 2.8 and 2.9]}$$

□

Theorem 22. *The family $\tilde{S}PO(\tilde{X})$ is a soft-topology on $\tilde{X}$ if and only if the following two conditions are satisfied.*

(1) The intersection of any two soft preopen sets in $\tilde{X}$ is soft semi-preopen.

(2) For any soft subset (F, A) of $\tilde{X}$, we have $\tilde{s}cl_\alpha(F,A) = \tilde{s}pcl(F,A)\tilde{\bigcup}\tilde{s}scl(F,A)$

Proof. suppose that $\tilde{S}PO(\tilde{X})$ is a soft-topology on $\tilde{X}$
Then $\tilde{S}PO(\tilde{X}) = \tilde{\tau_\gamma}[bytheorem3.3]$ and hence $\tilde{s}pcl(F,A) = \tilde{s}cl_\alpha(F,A)$ for every soft subset (F,A).

Applying cor 3.2, we have

$$\tilde{s}cl_\alpha(F,A) = \tilde{s}cl_\gamma(F,A)\tilde{\bigcup}\tilde{s}scl(F,A) = \tilde{s}pcl(F,A)\tilde{\bigcup}\tilde{s}scl(F,A).$$

Therefore the second condition is satisfied. Conversely, suppose that both conditions are satisfied. Let (F, A) and (G, B) be soft preclosed in $\tilde{X}$. Then $[(F,A)\tilde{\bigcup}(G,B)]$ is soft semi-preclosed, using the second condition,

$$\tilde{s}cl_\alpha(F,A) = \tilde{s}pcl(F,A)\tilde{\bigcup}\tilde{s}scl(F,A) = \tilde{s}scl(F,A)$$

and

$$\tilde{s}cl_\alpha(G,B) = \tilde{s}pcl(G,B)\tilde{\bigcup}\tilde{s}scl(G,B) = \tilde{s}scl(G,B)$$

$$\tilde{s}cl_\alpha[(F,A)\tilde{\bigcup}(G,B)] = \tilde{s}cl_\alpha(F,A)\tilde{\bigcup}\tilde{s}cl_\alpha(G,B)$$

$$= \tilde{s}scl(F,A)\tilde{\bigcup}\tilde{s}scl(G,B)\tilde{\subset}\tilde{s}scl[(F,A)\tilde{\bigcup}(G,B)]$$

Hence,

$$\tilde{s}cl_\alpha[(F,A)\tilde{\bigcup}(G,B)] = \tilde{s}scl[(F,A)\tilde{\bigcup}(G,B)]$$

$$\tilde{s}cl(\tilde{s}int(\tilde{s}cl(\tilde{X}/[(F,A)\tilde{\bigcup}(G,B)])))$$

$$= \tilde{s}int(\tilde{s}cl(\tilde{X}/((F,A)\tilde{\bigcup}(G,B)]))\tilde{\bigcup}\tilde{s}cl(\tilde{s}int(\tilde{X}/[(F,A)\tilde{\bigcup}(G,B)))$$

Therefore, [Taking $(F,A) = \tilde{X}/((F,A)\tilde{\bigcup}(G,B))$] and by lemma 3.2

$$\tilde{s}int(\tilde{s}cl(\tilde{s}int(F,A)\tilde{\bigcup}(G,B)))$$

$$= \tilde{s}cl(\tilde{s}int((F,A)\tilde{\bigcup}(G,B)))\tilde{\bigcap}\tilde{s}int(\tilde{s}cl(F,A)\tilde{\bigcup}(G,B))$$

$(F,A)\tilde{\bigcup}(G,B)$ satisfies all the conditions of Lemma-3.1, we have $(F,A)\tilde{\bigcup}(G,B)$ is soft pre closed. Hence, $\tilde{S}PO(\tilde{X})$ is a soft-topology on $\tilde{X}$ □

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