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ON FUZZY NEARLY C-α-COMPACTNESS IN FUZZY TOPOLOGICAL SPACES

R.V.M. Rangarajan[1], A. Vadivel[2], M. Chandrasekharan[3]

[1]Department of Mathematics
K. S. R. College of Engineering
Tiruchengode, INDIA
[2]Mathematics Section (FEAT)
Annamalai University
Annamalainagar, Tamil Nadu, 608 002, INDIA
[3]Science and Humanities
Nanda Engineering College
Erode, 638052, INDIA

Abstract: In this paper the concept of fuzzy nearly C-α-compactness is introduced in fuzzy topological spaces and fuzzy bitopological spaces. Several characterizations and some interesting properties of these are discussed. The properties of fuzzy almost continuous and fuzzy almost open functions are also discussed.

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Key Words: fuzzy nearly C-α-compact, fuzzy almost α-continuous, fuzzy almost α-open (α-closed), pairwise fuzzy nearly C-α-compact, pairwise fuzzy almost α-continuous and pairwise fuzzy almost α-open

1. Introduction

It is seen from the literature that a class of mathematicians has so far taken keen interest in generalising the concept of compactness to fuzzy topological spaces by different approaches (for example see [4], [7], [8], [9]). C. L. Chang [4] introduced and developed the concept of fuzzy topological spaces based on

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the concept of a fuzzy set introduced by Zadeh in [10]. Since then, various important notions in the classical topology such as compactness have been extended to fuzzy topological spaces. The concept of nearly C-compactness in general topology was introduced and studied in [8].

In what follows by X we shall mean a fuzzy topological space (or an fts, for short) as prescribed by Chang [4], whereas by a fuzzy set in a non-void set X is meant, as usual a function λ from X to the unit closed interval $[0, 1](= I$, say) i.e., $\lambda \in I^X$. By x_t we shall mean a fuzzy set whose value at the singleton support $\{x\}$ is t and 0 otherwise. For a fuzzy set λ in X, $1-\lambda$ denotes the fuzzy complement of λ and is defined by $(1-\lambda)(x) = 1-\lambda(x)$, for each $x \in X$. The notations $Cl(\lambda)$ and $Int(\lambda)$ will stand for the closure and interior respectively of the fuzzy set λ in the fts X [1]. For any family $\{\lambda_i : i \in \Lambda\}$ of fuzzy sets in X, the union $\bigvee_{i\in\Lambda} \lambda_i$ and $\bigwedge_{i\in\Lambda} \lambda_i$ are defined as $(\bigvee_{i\in\Lambda} \lambda_i)(x) = \sup_{i\in\Lambda} \lambda_i(x)$ and $(\bigwedge_{i\in\Lambda} \lambda_i)(x) = \inf_{i\in\Lambda} \lambda_i(x)$ for each $x \in X$ [10], where Λ is an arbitrary index set. The characteristic function of a subset A of X is denoted by χ_A and is defined as

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

Let $U = \{\lambda_\alpha\}_{\alpha\in\Delta}$ be a family of members from T. Then U is called a cover of X if $\bigvee_{\alpha\in\Delta} \lambda_\alpha = 1$ and a subfamily of U having a similar property is called a subcover of U. A fuzzy topological space $(X,\ T)$ is said to be fuzzy compact [9] if every cover of X by members of T has a finite subcover. Further x_α shall denote a fuzzy point [8] with a support x and value $\alpha(0 < \alpha \leq 1)$. For a fuzzy set λ in X, we write $x_\alpha \in \lambda$ provided $\alpha \leq \lambda(x)$.

The purpose of this paper is to introduce and study the concept of nearly C-α-compactness in fuzzy setting. In Section 2 we give preliminaries, Section 3 deals with the concept of fuzzy nearly C-α-compactness and some of the characterizations in fuzzy topological spaces, and Section 4 deals with the properties of fuzzy almost α-continuous, fuzzy almost α-open functions. Section 5 deals with the concept of fuzzy nearly C-α-compactness in fuzzy bitopological spaces.

2. Preliminaries

Most of the concepts, notations and definitions which we have used in this paper are standard by now. But for the sake of compactness, we recall some definitions and results used in the sequel.

Definition 1. A fuzzy filter base on X is a non-empty collection $\mathcal{F}$ of fuzzy sets on X satisfying the conditions

(i) $0 \notin \Im$; where 0 stands for empty fuzzy set;

(ii) $\lambda_1, \lambda_2 \in \Im \Rightarrow \lambda_1 \wedge \lambda_2 \in \Im$;

(iii) $\lambda < \mu \in \Im \Rightarrow \lambda \in \Im$.

Definition 2. A fuzzy set λ in a fts $(X,\ T)$ is called

(i) fuzzy regular open set [1] if $Int(Cl(\lambda)) = \lambda$ and a fuzzy regular closed set if $Cl(Int(\lambda)) = \lambda$,

(ii) a fuzzy α-open set [2] if $\lambda \leq Int(Cl(Int(\lambda)))$ and a fuzzy α-closed set if $Cl(Int(Cl(\lambda))) \leq \lambda$.

Definition 3. A fuzzy topological space (X, T) is said to be fuzzy α-regular if for every fuzzy closed set λ and a point $x \notin \lambda$, there exists disjoint fuzzy α-open sets γ and μ such that $x \in \gamma$ and $\lambda < \mu$. A fuzzy set δ is fuzzy α-regular open if $Int_\alpha(Cl_\alpha(\delta)) = \delta$.

Definition 4. [8] A topological space (X, T) is said to be nearly C-compact if for any ordinary subset A of X, $A \neq X$ such that χ_A (the characteristic function of $A \subset X$) is a proper regular closed set and for each open cover of $\{\lambda_\alpha; \alpha \in \Delta\}$ of χ_A there exists a finite subfamily $\{\lambda_{\alpha_1}, \lambda_{\alpha_2}, ..., \lambda_{\alpha_n}\}$ such that $\chi_A \subset \bigcup\limits_{i=1}^{n} Cl(\lambda_{\alpha_i})$.

Definition 5. [7] A fuzzy topological space (X, T) is said to be fuzzy nearly C-compact if for any ordinary subset A of X, $A \neq X$ such that χ_A (the characteristic function of $A \subset X$) is a proper fuzzy regular closed set and for each fuzzy open cover of $\{\lambda_\alpha : \alpha \in \Delta\}$ of χ_A there exists a finite subfamily $\{\lambda_{\alpha_1}, \lambda_{\alpha_2}, \ldots, \lambda_{\alpha_n}\}$ such that $\chi_A \leq \bigvee\limits_{i=1}^{n} Cl(\lambda_{\alpha_i})$.

3. Fuzzy Nearly C-α-Compactness in Fuzzy Topological Spaces

Definition 6. [9] A topological space is said to be nearly C-α-compact if given a regular closed set A and an α-open cover $\mathcal{U} = \{O_i | i \in \wedge\}$ of A there exists a finite subfamily $\{O_i; i = 1, 2, ..., n\}$ of $\mathcal{U}$ such that $A \subset \bigcup\limits_{i=1}^{n} Cl_\alpha(O_i)$.

Definition 7. Let (X, T) be a fuzzy topological space. (X, T) is said to be fuzzy nearly C-α-compact if for any ordinary subset A of X, $A \neq X$ such that χ_A (the characteristic function of $A \subset X$) is a proper fuzzy regular closed set and for each fuzzy α-open cover of $\{\lambda_\alpha; \alpha \in \Delta\}$ of χ_A there exists a finite subfamily $\{\lambda_{\alpha_1}, \lambda_{\alpha_2}, ..., \lambda_{\alpha_n}\}$ such that $\chi_A \leq \bigvee_{i=1}^{n} Cl_\alpha(\lambda_{\alpha_i})$.

From the above definition it is clear that fuzzy compactness implies fuzzy nearly C-α-compactness. However, the converse is not true as the following example shows:

Example 1. *Let $X = \{a, b\}$, $T = \{0, 1, f_n\}$ where $f_n : X = \{a, b\} \to [0, 1]$ is such that $f_n(X) = 1 - \frac{1}{n}$, $\forall x \in X$. The only possible non-empty subsets of X are $A_1 = \{a\}$ and $A_2 = \{b\}$. Further, since $ClInt\chi_{A_1} = 0 \neq \chi_{A_1}$ and $ClInt\chi_{A_2} = 0 \neq \chi_{A_2}$, it follows that χ_{A_1} and χ_{A_2} are not fuzzy regular closed. So vacously (X, T) is fuzzy nearly C-α-compact. Now we claim that (X, T) is not fuzzy compact. Indeed, $\bigvee_{n=1}^{\infty} f_n = 1$ shows that $\{f_n\}_{n=1}^{\infty}$ is a fuzzy open cover of 1_X but for every infinite integer, say n_0, we have $\bigvee_{n=1}^{n_0} f_n < 1$ and therefore $\{f_n\}_{n=1}^{\infty}$ has no finite subcover for 1_X. That is, (X, T) is not fuzzy compact.*

Proposition 1. *In a fuzzy topological space (X, T) the following assertions are equivalent.*
(1) X is fuzzy nearly C-α-compact.
(2) For each subset $A \subset X$ such that χ_A is proper fuzzy regular closed and for each fuzzy α-regular open cover $\mathcal{U} = \{\lambda_\alpha\}_{\alpha \in \Delta}$ of χ_A there exists a finite subfamily $\{\lambda_{\alpha_1}, \lambda_{\alpha_2}, \ldots, \lambda_{\alpha_n}\}$ of $\mathcal{U}$ such that $\chi_A \leq \bigvee_{i=1}^{n} Cl_\alpha(\lambda_{\alpha_i})$.
(3) For each subset $A \subset X$ such that χ_A is a proper fuzzy regular closed set and for each family $\Im = \{\mu_\alpha\}_{\alpha \in \Delta}$ of non-zero fuzzy regular closed sets such that $\left(\bigwedge_{\alpha \in \Delta} \mu_\alpha\right) \wedge \chi_A = 0$, there exists a finite subfamily $\{\mu_{\alpha_1}, \mu_{\alpha_2}, \ldots, \mu_{\alpha_n}\}$ of $\Im$ such that $\left\{\bigwedge_{i=1}^{n} Int_\alpha(\mu_{\alpha_i})\right\} \wedge \chi_A = 0$.
(4) For any subset $A \subset X$ such that χ_A is a proper fuzzy regular closed set and for each family $\Im = \{\mu_\alpha\}_{\alpha \in \Delta}$ of fuzzy regular closed sets, if for each finite subfamily $\{\mu_{\alpha_1}, \mu_{\alpha_2}, \ldots, \mu_{\alpha_n}\}$ of $\Im$ we have $\left[\bigwedge_{i=1}^{n} Int_\alpha(\mu_{\alpha_i})\right] \wedge \chi_A \neq 0$, then $\left(\bigwedge_{\alpha \in \Delta} \mu_\alpha\right) \wedge \chi_A \neq 0$.

Proof. (1)$\Rightarrow$ (2) follows easily from Definition 7.

(2)$\Rightarrow$ (1) Suppose (2) holds. Let A be any subset of X such that χ_A is proper fuzzy regular closed. Let $\mathcal{U} = \{\lambda_\alpha\}_{\alpha\in\Delta}$ be a fuzzy α-open cover of χ_A. Then $\{Int_\alpha(Cl_\alpha\lambda_\alpha)\}_{\alpha\in\Delta}$ will be a fuzzy α-regular open cover of χ_A. Then by (2), there exists a finite subfamily $\{Int_\alpha(Cl_\alpha(\lambda_{\alpha_i}))\}_{i=1}^n$ such that $\chi_A \leq \bigvee_{i=1}^{n} Cl_\alpha\{Int_\alpha(Cl_\alpha(\lambda_{\alpha_i}))\} \leq \bigvee_{i=1}^{n} Cl_\alpha(\lambda_{\alpha_i})$.

This proves (2) $\Rightarrow$ (1).

The proof for (2)$\Rightarrow$ (3), (3)$\Rightarrow$ (2), (3)$\Rightarrow$ (4) and (4)$\Rightarrow$ (1) are in [7]. □

Proposition 2. *For any fuzzy topological space (X,T), the following assertions are equivalent.*
(1) X is fuzzy nearly C-α-compact.
(2) If $A \subset X$ is such that χ_A is a proper fuzzy regular closed and $\Im$ is a family of fuzzy regular closed sets of X such that $\chi_A \leq \left(1 - \bigwedge_{\lambda\in\Im} \lambda\right)$, then there exists a finite number of elements of $\Im$, say $\lambda_1, \lambda_2, ..., \lambda_n$, such that $\chi_A \leq 1 - \bigwedge_{i=1}^{n} Int_\alpha\lambda_i$.

Proof. (1) $\Rightarrow$ (2) Suppose X is fuzzy nearly C-α-compact. Let $A \subset X$ be such that χ_A is proper fuzzy regular closed set. Let $\Im$ be a family of fuzzy regular closed sets of X such that $\chi_A \leq \left(1 - \bigwedge_{\lambda\in\Im} \lambda\right) = \bigvee_{\lambda\in\Im}(1-\lambda)$. Clearly $\mathcal{U} = \{1-\lambda\}_{\lambda\in\Im}$ is a fuzzy regular open cover of χ_A. Hence by assumption (1) there exists a finite number of elements (say) $\lambda_1, \lambda_2, ..., \lambda_n$ such that $\chi_A \leq \bigvee_{i=1}^{n} Cl_\alpha(\lambda_i)$. Therefore $\bigwedge_{i=1}^{n} Int_\alpha\lambda_i = 1 - \bigvee_{i=1}^{n} Cl_\alpha\lambda_i \leq 1 - \chi_A$. That is, $\chi_A \leq 1 - \bigwedge_{i=1}^{n} Int_\alpha\lambda_i$. This proves (1) $\Rightarrow$ (2).

(2)$\Rightarrow$ (1) Let $A \subset X$ be such that χ_A is a proper fuzzy regular closed set in X. Let $\Im$ be a family of fuzzy regular open sets of X such that $\chi_A \leq \bigvee_{\lambda\in\Im} \lambda$. Put $\mathcal{U} = \{1-\lambda\}_{\lambda\in\Im}$. Then $\mathcal{U}$ is clearly a family of fuzzy regular closed sets of X such that $\chi_A \leq \bigvee_{\lambda\in\Im} \lambda = \bigvee_{\lambda\in\Im}[1-(1-\lambda)] = 1 - \bigwedge_{\lambda\in\Im}(1-\lambda)$. Hence by assumption (2) there exists a finite number of elements, say $1-\lambda_1, 1-\lambda_2, ..., 1-\lambda_n$, such that $\chi_A \leq 1 - \bigwedge_{i=1}^{n} Int_\alpha(1-\lambda_i) = \bigvee_{i=1}^{n}(1 - Int_\alpha(1-\lambda_i)) = \bigvee_{i=1}^{n} Cl_\alpha\lambda_i$. This proves (2)$\Rightarrow$ (1). □

4. Properties of Fuzzy Almost α-Continuous and Fuzzy Almost α-Open Functions

Definition 8. Let (X,T) and (Y,S) be any two fuzzy topological spaces. A mapping f : $(X,T) \to (Y,S)$ is said to be fuzzy almost α-continuous if the inverse image of every fuzzy regular open (closed) set is fuzzy α-open (α-closed).

Definition 9. Let (X,T) and (Y,S) be any two fuzzy topological spaces. A mapping f : $(X,T) \to (Y,S)$ is said to be fuzzy almost α-open (α-closed) if the image of every fuzzy regular open (closed) set is fuzzy α-open (α-closed).

Example 2. *Let $X = \{a,\ b,\ c\}$; Define $T_1 = \{0,\ 1,\ \lambda\}$ and $T_2 = \{0,\ 1,\ \mu\}$ where $\lambda(a) = 0,\ \lambda(b) = \frac{2}{3},\ \lambda(c) = \frac{1}{2}$ and $\mu(a) = 1,\ \mu(b) = 0,\ \mu(c) = 0$. Let $f : (X,\ T_1) \to (X,\ T_2)$ be the identity mapping. In $(X,\ T_2)$ the only non-zero fuzzy regular open set is 1 and $f^{-1}(1) = 1$ shows that f is fuzzy almost α-continuous. Now let $g : (X,\ T_2) \to (X,\ T_1)$ be the identity mapping, the only non-zero fuzzy regular open set in $(X,\ T_2)$ is 1 and $f(1) = 1$ implies that f is fuzzy almost α-open.*

Proposition 3. *Let $(X,\ T)$, $(Y,\ S)$ and $(Z,\ R)$ be fuzzy topological spaces. Let $f : X \to Y$ be a fuzzy almost α-open (almost α-closed) map of a space X onto a space Y and let $g : Y \to Z$. If $g \circ f$ is a fuzzy almost α-continuous and fuzzy almost α-open map then g is fuzzy almost α-continuous.*

Proposition 4. *Let $(X,\ T)$, $(Y,\ S)$ and $(Z,\ R)$ be fuzzy topological spaces. Assume that $f : X \to Y$ is fuzzy almost α-continuous and let $g : Y \to Z$ be a fuzzy almost α-continuous and fuzzy almost α-open map. Then $g \circ f$ is fuzzy almost α-continuous.*

Proposition 5. *Let $(X,\ T)$, $(Y,\ S)$ and $(Z,\ R)$ be fuzzy topological spaces. Let $f : X \to Y$ and $g : Y \to Z$ and suppose that $(g \circ f)$ is a fuzzy almost α-open (fuzzy almost α-closed) map. If f is fuzzy almost α-continuous and fuzzy almost α-open surjection, then g is a fuzzy almost α-open (almost α-closed) map.*

Lemma 1. *Let $(X,\ T)$ and $(Y,\ S)$ be fuzzy topological spaces. Let $f : X \to Y$ be any fuzzy almost α-open map. Given any $\lambda \in I^Y$ and any fuzzy regular closed set μ containing $f^{-1}(\lambda)$, there exists a fuzzy α-closed set $\theta \geq \lambda$ such that $f^{-1}(\theta) \leq \mu$.*

Lemma 2. *Let $(X,\ T)$ and $(Y,\ S)$ be fuzzy topological spaces. Let $f : X \to Y$ be a fuzzy almost α-continuous and fuzzy almost α-open map and let λ be any fuzzy α-closed subset of Y. Then*

(1) $Cl_\alpha(f^{-1}(Int_\alpha(\lambda))) = f^{-1}(Cl_\alpha(Int_\alpha\lambda))$.
(2) If λ *is a fuzzy* α*-regular closed subset, then* $Cl_\alpha(f^{-1}(Int_\alpha\lambda)) = f^{-1}(\lambda)$.

Proposition 6. *Let* $(X,\ T)$ *and* $(Y,\ S)$ *be fuzzy topological spaces. Let* $f : X \to Y$ *be a fuzzy almost* α*-continuous and fuzzy almost* α*-open map. If* μ *is a fuzzy* α*-open set in* Y*, then*
(1) $Int_\alpha Cl_\alpha(f^{-1}(Int_\alpha(Cl_\alpha\mu))) = Int_\alpha[Cl_\alpha(f^{-1}(Cl_\alpha\mu))] = Int_\alpha(f^{-1}(Cl_\alpha\mu))$.
(2) $f^{-1}(Int_\alpha(Cl_\alpha\mu)) = Int_\alpha(f^{-1}(Cl_\alpha\mu))$.
(3) If μ *is a fuzzy* α*-regular open set in* Y*, then* $Int_\alpha f^{-1}(f^{-1}(Cl_\alpha\mu)) = f^{-1}(\mu)$.

Proposition 7. *The image of a fuzzy nearly* C*-*α*-compact space under a fuzzy almost* α*-continuous and fuzzy almost* α*-open mapping is fuzzy nearly* C*-*α*-compact.*

Proof. Let $f : (X,T) \to (Y,S)$ be a fuzzy almost α-continuous and fuzzy almost α-open mapping from a fuzzy nearly C-α-compact X onto Y. We have to show that Y is also fuzzy nearly C-α-compact. Let A be any subset of Y such that χ_A is fuzzy regular closed in Y. Let $\mathcal{U} = \{\lambda_i\}_{i\in\Delta}$ be a fuzzy regular α-open cover of χ_A in Y. $f^{-1}(\chi_A)$ is a fuzzy regular closed subset of X and $\left\{f^{-1}(\lambda_i)\right\}_{i\in\Delta}$ is a fuzzy regular α-open cover of $f^{-1}(\chi_A)$ in X. Since X is fuzzy nearly C-α-compact, there exists a finite subfamily $\left\{f^{-1}(\lambda_i); i = 1, 2, \dots, n\right\}$ such that

$$f^{-1}(\chi_A) \leq \bigvee_{i=1}^{n} Cl_\alpha\left\{f^{-1}(\lambda_i)\right\} \leq \bigvee_{i=1}^{n}\left\{f^{-1}(Cl_\alpha(\lambda_i))\right\}.$$

That is, $\chi_A \leq \bigvee_{i=1}^{n}\{Cl_\alpha(\lambda_i)\}$. This proves that Y is fuzzy nearly C-α-compact. □

5. Fuzzy Nearly C-α-Compactness in Fuzzy Bitopological Spaces

The concept of fuzzy bitopological spaces was introduced in [5] and subsequently further studied by various authors [3], [6]. In [5] the definition of fuzzy bitopological space was given as follows:

Definition 10. A fuzzy bitopological space is an ordered triple (X, T_1, T_2) where T_1 and T_2 are fuzzy topologies on X.

Definition 11. [7] A bitopological space (X, T_1, T_2) is said to be $(1,2)$-fuzzy nearly C-compact if for every set $A \subset X$ such that χ_A is a proper T_1-fuzzy regular closed set and for every T_2-fuzzy open cover $\mathcal{U}$ of χ_A, there exists a finite subcollection of $\mathcal{U}$, (say) $\lambda_1, \lambda_2, \ldots, \lambda_n$ such that $\chi_A \leq \bigvee_{i=1}^{n} T_2\text{-}Cl(\lambda_i)$. Then (X, T_1, T_2) is said to be pairwise fuzzy nearly C-compact if it is both $(1,2)$-fuzzy nearly C-compact and $(2,1)$-fuzzy nearly C-compact.

Definition 12. A bitopological space (X, T_1, T_2) is said to be $(1,2)$-fuzzy nearly C-α-compact if for every set $A \subset X$ such that χ_A is a proper T_1-fuzzy regular closed set and for every T_2-fuzzy α-open cover $\mathcal{U}$ of χ_A, there exists a finite subcollection of $\mathcal{U}$,(say) λ_1,λ_2,.....,λ_n such that $\chi_A \leq \bigvee_{i=1}^{n} T_2\text{-}Cl_\alpha(\lambda_i)$. Then (X, T_1, T_2) is said to be pairwise fuzzy nearly C-α-compact if it is both $(1,2)$-fuzzy nearly C-α-compact and $(2,1)$-fuzzy nearly C-α-compact.

Definition 13. A mapping $f : (X, T_1, T_2) \to (Y, S_1, S_2)$ is said to be pairwise fuzzy almost α-continuous (pairwise fuzzy almost α-open, pairwise fuzzy α-continuous) if the induced mappings $f : (X, T_1) \to (Y, S_1)$ and $f : (X, T_2) \to (Y, S_2)$ are fuzzy almost α-continuous (fuzzy almost α-open, fuzzy α-continuous).

Proposition 8. *Every pairwise fuzzy α-continuous and pairwise fuzzy almost α-open image of a pairwise fuzzy nearly C-α-compact space is pairwise fuzzy nearly C-α-compact.*

Proof. Let $f : (X, T_1, T_2) \to (Y, S_1, S_2)$ be any pairwise fuzzy almost α-continuous and pairwise fuzzy almost α-open onto mapping. Assume (X, T_1, T_2) is pairwise fuzzy nearly C-α-compact. We want to show that (Y, S_1, S_2) is pairwise fuzzy nearly C-α-compact.

Let $A \subset Y$ be such that χ_A is a proper S_1-fuzzy regular closed set and let $\mathcal{U}$ be a S_2-fuzzy α-open cover of χ_A. Since f is fuzzy almost α-continuous and fuzzy almost α-open, $f^{-1}(\chi_A)$ is T_1-fuzzy regular closed and $\{f^{-1}(\mu); \mu \in \mathcal{U}\}$is a T_2-fuzzy α-open cover of $f^{-1}(\chi_A)$. Since (X, T_1, T_2) is pairwise fuzzy nearly C-α-compact, there exists a finite subcollection $\{f^{-1}(\mu_k); k = 1, 2, \ldots, n\}$ such that $f^{-1}(\chi_A) \leq \bigvee_{k=1}^{n} T_2\text{-}Cl_\alpha f^{-1}(\mu_k)$. Hence,

$$\begin{aligned}
\chi_A = ff^{-1}(\chi_A) &\leq \bigvee_{k=1}^{n} f[T_2\text{-}Cl_\alpha f^{-1}(\mu_k)] \\
&\leq \bigvee_{k=1}^{n} S_2\text{-}Cl_\alpha(ff^{-1}(\mu_k)) \\
&\leq \bigvee_{k=1}^{n} S_2\text{-}Cl_\alpha(\mu_k).
\end{aligned}$$

This proves that Y is $(1,2)$-fuzzy nearly C-α-compact. Similarly we can show that Y is also $(2,1)$-fuzzy nearly C-α-compact. Thus we have shown that

(Y, S_1, S_2) is pairwise fuzzy nearly C-α-compact. $\square$

Proposition 9. *Let (X, T_1, T_2) be any pairwise fuzzy nearly C-α-compact space. Then if $A \subset X$ is such that χ_A is proper T_i-fuzzy regular closed and $\Im$ is a family of T_j-fuzzy α-closed subsets of X such that $\bigwedge\{\lambda \wedge \chi_A; \lambda \in \Im\} = 0$, there exists a finite number of elements, say $\{\lambda_1, \lambda_2, ..., \lambda_n\}$ of $\Im$ such that*

$$\bigwedge_{k=1}^{n} \{T_j\text{-}Int_\alpha \lambda_k \wedge \chi_A\} = 0, i \neq j,\ i,\ j = 1,\ 2.$$

Proof. Suppose (X, T_1, T_2) is fuzzy pairwise nearly C-α-compact. Let $A \subset X$ be such that χ_A is proper T_i-fuzzy regular closed and $\Im$ is a family of T_j-fuzzy α-closed sets of X such that $\bigwedge\{\lambda \wedge \chi_A; \lambda \in \Im\} = 0$. Now $\bigwedge_{\lambda\in\Im}\{\lambda \wedge \chi_A\} = 0 \Rightarrow \bigwedge_{\lambda\in\Im} \lambda \leq 1 - \chi_A \Rightarrow \chi_A \leq 1 - \bigwedge_{\lambda\in\Im} \lambda = \bigvee\{1 - \lambda; \lambda \in \Im\}$. So $\{1 - \lambda; \lambda \in \Im\}$ is a T_j-fuzzy α-open cover of χ_A which is T_i-fuzzy regular closed and hence by assumption we have a finite collection, say $1 - \lambda_1, 1 - \lambda_2, \ldots, 1 - \lambda_n$ such that $\chi_A \leq \bigvee_{k=1}^{n} T_j\text{-}Cl_\alpha(1 - \lambda_k) = \bigvee_{k=1}^{n}(1 - T_j\text{-}Int_\alpha\lambda_k) = 1 - \bigwedge_{k=1}^{n} T_j\text{-}Int_\alpha\lambda_k$. This implies $\bigwedge_{k=1}^{n} T_j\text{-}Int_\alpha\lambda_k \leq 1 - \chi_A = \chi_{X-A}$. Therefore $\chi_A \bigwedge(\bigwedge_{k=1}^{n} T_j\text{-}Int_\alpha\lambda_k) \leq \chi_A \wedge \chi_{X-A} = 0 \Rightarrow \bigwedge_{k=1}^{n}(T_j\text{-}Int_\alpha\lambda_k \wedge \chi_A) = 0$. This proves the proposition. $\square$

Definition 14. A T_j-fuzzy filter base $\Im$ is said to be T_{ij}-α-regular adherent convergent if every T_i-α-regular open neighborhood of the T_j-α-adherent set of $\Im$ contains an element of $\Im$,$i \neq j, i, j = 1, 2$, where we define the T_j-α-adherent set of $\Im$ to be $\bigwedge\{T_j\text{-}Cl_\alpha\lambda; \lambda \in \Im\}$.

Proposition 10. *If (X, T_1, T_2) is pairwise fuzzy nearly C-α-compact then every T_j-α-open fuzzy filter base is T_{ij}-α-regular adherent convergent $(i \neq j,\ i, j = 1, 2)$.*

Proof. Suppose (X, T_1, T_2) is pairwise fuzzy nearly C-α-compact and let $\Im$ be any T_j-α-open filter base with λ the T_j-α-adherent set of $\Im$. Let σ be any fuzzy T_i-α-regular open neighbourhood of λ. So we have $\lambda = \bigwedge\{T_j\text{-}Cl_\alpha\mu; \mu \in \Im\}$ and $\lambda \leq \sigma$ and $1 - \sigma$ is fuzzy T_i-α-regular closed. Now $1 - \sigma \leq 1 - \lambda = 1 - \bigwedge\{T_j\text{-}Cl_\alpha\mu; \mu \in \Im\} = \bigvee\{1 - T_j\text{-}Cl_\alpha\mu; \mu \in \Im\}$. Therefore $\{1 - T_j\text{-}Cl_\alpha\mu; \mu \in \Im\}$ is a T_j-α-open cover of the fuzzy regular closed set $1 - \sigma$. Since (X, T_1, T_2) is pairwise fuzzy nearly C-α-compact, we can find a subcollection, say $\{1 - T_j\text{-}Cl_\alpha\mu_k; k = 1, 2, ..., n\}$ such that $1 - \sigma \leq \bigvee_{k=1}^{n} T_j\text{-}Cl_\alpha(1 - T_j\text{-}Cl_\alpha\mu_k) = \bigvee_{k=1}^{n}[1 - T_j\text{-}$

$Int_\alpha((T_j\text{-}Cl_\alpha \mu_k))] = 1 - \bigwedge_{k=1}^{n} T_j\text{-}Int_\alpha((T_j\text{-}Cl_\alpha \mu_k)) \Rightarrow \bigwedge_{k=1}^{n} T_j\text{-}Int_\alpha(T_j\text{-}Cl_\alpha \mu_k) \leq \sigma$. Further, $\mu_k \leq T_j\text{-}Int_\alpha(T_j\text{-}Cl_\alpha \mu_k)$ for $k = 1, 2, \ldots, n$. Therefore $\bigwedge_{k=1}^{n} \mu_k \leq T_j\text{-}Int_\alpha T_j\text{-}Cl_\alpha \mu_k$. Hence $\bigwedge_{k=1}^{n} \mu_k \leq \sigma \Rightarrow \sigma \in \Im$. Hence the proposition. □

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