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ITERATIVE METHOD WITH HIGHER-ORDER CONVERGENCE FOR SCALAR EQUATIONS

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Abstract: We establish new higher-order iterative methods for the solution of scalar equations by using the decomposition technique mainly due to Adomian, see [2].

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1. Introduction

During last many years, much attention has been given to establish several iterative methods for solving nonlinear equations, see [1, 3, 4, 5, 6, 8, 9, 11, 12, 13, 14] and the references therein. These methods can be classified as one-step, two-step and three-step methods. Two-step methods have been suggested by combining the well-known Newton method with other one-step im-

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plicit methods. In [5] Chun has proposed and studied several one-step and two-step iterative methods with higher-order convergence by using the decomposition technique of Adomian [2]. To overcome this draw back, Noor and Noor [12] suggested and analyzed a family of multi-step methods for solving nonlinear equations using a different type of decomposition technique mainly due to Daftardar-Gejji and Jafari [7], which does not involve the high-order differentials of the function.

Our problem, to recall, is solving equations in one variable. We are given a function f, and would like to find at least one solution to the equation $f(x) = 0$. Note that, a prior, we do not put any restrictions on the function f; we do need to be able to evaluate the function: otherwise, we cannot even check that a given solution $x = \alpha$ is true, that is, $f(\alpha) = 0$. In reality, the mere ability to be able to evaluate the function does not suffice. We need to assume some kind of "good behavior". The more we assume, the more potential we have, on the one hand, to develop fast algorithms for finding the root. At the same time, the more we assume, the fewer functions are going to satisfy our assumptions! This is a fundamental paradigm in Numerical Analysis.

During the last many years, the numerical techniques for solving nonlinear equations has been successfully applied (see for example [2, 3, 4] and the references there in).

We know that one of the fundamental algorithm for solving nonlinear equations is so-called fixed point iteration method (FPM) [10].

In order to use fixed point iteration method, we need the following information:

1. We need to know that there is a solution to the equation.

2. We need to know approximately where the solution is (that is, an approximation to the solution).

It is well known that the fixed point iteration method has the first order convergence.

We establish new higher-order iterative methods for the solution of scalar equations by using the decomposition technique mainly due to Adomian [2]. The methods are performing very well in comparison to the fixed point method and the methods discussed in [4].

2. Preliminaries

We need the following results: In the fixed point iteration method for solving the nonlinear equation $f(x) = 0$, the equation is usually rewritten as

$$x = g(x), \tag{2.1}$$

where

(i) there exists $[a, b]$ such that $g(x) \in [a, b]$ for all $x \in [a, b]$,

(ii) there exists $[a, b]$ such that $|g'(x)| \leq L < 1$ for all $x \in [a, b]$.

Considering the following iteration scheme:

$$x_{n+1} = g(x_n), \quad n = 0, 1, 2, \cdots \tag{2.2}$$

and starting with a suitable initial approximation x_0, we build up a sequence of approximations, say $\{x_n\}$, for the solution of the nonlinear equation, say α. The scheme will converge to the root α, provided that

(i) the initial approximation x_0 is chosen in the interval $[a, b]$,

(ii) g has a continuous derivative on (a, b),

(iii) $|g'(x)| < 1$ for all $x \in [a, b]$,

(iv) $a \leq g(x) \leq b$ for all $x \in [a, b]$ (see [10]).

The order of convergence for the sequence of approximations derived from an iteration method is defined in the literature, as

Definition 2.1. [16] Let $\{x_n\}$ converge to α. If there exist an integer constant p, and real positive constant C such that

$$\lim_{n\to\infty} \left| \frac{x_{n+1} - \alpha}{(x_n - \alpha)^p} \right| = C,$$

then p is called the *order* and C the constant of convergence.

To determine the order of convergence of the sequence $\{x_n\}$, let us consider the Taylor expansion of $g(x_n)$

$$\begin{aligned} g(x_n) = g(x) + \frac{g'(x)}{1!}(x_n - x) + \frac{g''(x)}{2!}(x_n - x)^2 + \cdots \\ + \frac{g^{(k)}(x)}{k!}(x_n - x)^k + \cdots .. \end{aligned} \tag{2.3}$$

Using (2.1) and (2.2) in (2.3) we have

$$\begin{aligned} x_{n+1} - x = g'(x)(x_n - x) + \frac{g''(x)}{2!}(x_n - x)^2 + \cdots \\ + \frac{g^{(k)}(x)}{k!}(x_n - x)^k + \cdots , \end{aligned}$$

and we can state the following result [10]:

Lemma 2.2. [3] *Suppose that* $g \in C^p[a,b]$. *If* $g^{(k)}(x) = 0$ *for* $k = 1, 2, \ldots, p-1$ *and* $g^{(p)}(x) \neq 0$, *then the sequence* $\{x_n\}$ *is of order* p.

3. New iterative methods

In this section we establish some new iterative methods for the solution of scalar equations by using the decomposition technique mainly due to Adomian [2].

Consider the nonlinear equation

$$f(x) = 0, \quad x \in \mathbb{R}. \tag{3.1}$$

We assume that α is a simple root of (3.1) and γ is an initial guess sufficiently close to α.

By using Taylor's expansion, we get

$$\begin{aligned} x &= g(x) \\ &= g(\gamma) + (x-\gamma) g'(\gamma) + G(x), \end{aligned} \tag{3.2}$$

where $G(x)$ is the auxiliary function is defined as

$$G(x) = g(x) - g(\gamma) - (x-\gamma) g'(\gamma). \tag{3.3}$$

Now (3.2) can be rewritten as:

$$\begin{aligned} x &= \frac{g(\gamma) - \gamma g'(\gamma)}{1 - g'(\gamma)} + \frac{G(x)}{1 - g'(\gamma)} \\ &= c + N(x), \end{aligned} \tag{3.4}$$

where

$$c = \frac{g(\gamma) - \gamma g'(\gamma)}{1 - g'(\gamma)}, \tag{3.5}$$

and

$$N(x) = \frac{G(x)}{1 - g'(\gamma)}. \tag{3.6}$$

We can construct a sequence of higher-order iterative methods by applying the Adomain decomposition method [2].

Adomain decomposition method is based on looking for a solution having the infinite series

$$x = \sum_{n=0}^{\infty} x_i, \tag{3.7}$$

and the nonlinear function is decomposed as

$$N(x) = \sum_{n=0}^{\infty} A_n, \tag{3.8}$$

where A_n are called as the Adomian polynomials depending on $x_0, x_1, x_2, \cdots$ given by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{n}\lambda^i u_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, \cdots, \tag{3.9}$$

or equivalently

$$\begin{aligned} A_0 &= N(x_0), \\ A_1 &= x_1 N'(x_0), \\ A_2 &= x_2 N'(x_0) + \frac{1}{2}x_1^2 N''(x_0), \\ &\cdots. \end{aligned} \tag{3.10}$$

From equations (3.4) and (3.8) we have

$$\sum_{n=0}^{\infty} x_n = c + \sum_{n=0}^{\infty} A_n. \tag{3.11}$$

Thus the iterative sequence is given by

$$x_0 = c, \tag{3.12}$$

and

$$x_{n+1} = A_n, \quad n = 0, 1, 2, \ldots. \tag{3.13}$$

An elementary calculations shows that

$$\begin{aligned} A_0 &= N\left(x_0\right) \\ &= \frac{G\left(x_0\right)}{1 - g'\left(\gamma\right)} \\ &= \frac{g\left(x_0\right) - x_0}{1 - g'\left(\gamma\right)}, \end{aligned} \tag{3.14}$$

and

$$\begin{aligned} N'\left(x\right) &= \frac{G'\left(x\right)}{1 - g'\left(\gamma\right)} \\ &= \frac{g'\left(x\right) - g'\left(\gamma\right)}{1 - g'\left(\gamma\right)}. \end{aligned} \tag{3.15}$$

We have

$$N'(x_0) = \frac{g'(x_0) - g'(\gamma)}{1 - g'(\gamma)}, \tag{3.16}$$

and

$$N''(x_0) = \frac{g''(x_0)}{1 - g'(\gamma)}. \tag{3.17}$$

Consider

$$\begin{aligned} A_1 &= x_1 N'(x_0) \\ &= A_0 N'(x_0) \\ &= \frac{(g(x_0) - x_0)(g'(x_0) - g'(\gamma))}{(1 - g'(\gamma))^2}, \end{aligned} \tag{3.18}$$

and

$$\begin{aligned} A_2 &= x_2 N'(x_0) + \frac{1}{2} x_1^2 N''(x_0) \\ &= A_1 N'(x_0) + \frac{1}{2} A_0^2 N''(x_0) \\ &= \frac{(g(x_0) - x_0)(g'(x_0) - g'(\gamma))^2}{(1 - g'(\gamma))^3} + \frac{1}{2} \frac{(g(x_0) - x_0)^2 (g''(x_0))}{(1 - g'(\gamma))^2}. \end{aligned} \tag{3.19}$$

Note that x is approximated by

$$\begin{aligned} X_m &= x_0 + x_1 + \cdots + x_m \\ &= x_0 + A_0 + A_1 + \cdots + A_{m-1}, \end{aligned} \tag{3.20}$$

where $\lim_{m \to \infty} X_m = x$.

For $m = 0$,

$$x = X_0 = x_0 = c = \frac{g(\gamma) - \gamma g'(\gamma)}{1 - g'(\gamma)}. \tag{3.21}$$

This allows us to suggest the following one-step iterative method for solving the nonlinear equation (3.1) .

Algorithm 3.1. *For a given x_0 compute the approximate solution x_{n+1} by the iterative scheme*

$$x_{n+1} = \frac{g(x_n) - x_n g'(x_n)}{1 - g'(x_n)}, \quad g'(x_n) \neq 1, \quad n = 0, 1, 2, \ldots,$$

which is mainly due to Kang et al. [15].

For $m = 1$,

$$\begin{aligned} x = X_1 &= x_0 + x_1 \\ &= c + A_0 \\ &= \frac{g(\gamma) - \gamma g'(\gamma)}{1 - g'(\gamma)} + \frac{g(x_0) - x_0}{1 - g'(\gamma)}. \end{aligned} \tag{3.22}$$

This allows us to suggest the following one-step iterative method for solving the nonlinear equation (3.1) .

Algorithm 3.2. *For a given x_0 compute the approximate solution x_{n+1} by the iterative scheme*

Predictor-step:

$$y_n = \frac{g(x_n) - x_n g'(x_n)}{1 - g'(x_n)},$$

Corrector-step:

$$x_{n+1} = \frac{g(x_n) - x_n g'(x_n)}{1 - g'(x_n)} + \frac{g(y_n) - y_n}{1 - g'(x_n)}.$$

For $m = 2$,

$$\begin{aligned} x \approx X_2 &= x_0 + x_1 + x_2 \\ &= c + A_0 + A_1 \\ &= \frac{g(\gamma) - \gamma g'(\gamma)}{1 - g'(\gamma)} + \frac{g(x_0) - x_0}{1 - g'(\gamma)} + \frac{(g(x_0) - x_0)(g'(x_0) - g'(\gamma))}{(1 - g'(\gamma))^2}, \end{aligned} \tag{3.23}$$

which provides the following iteration scheme:

Algorithm 3.3. *For a given x_0 compute the approximate solution x_{n+1} by the iterative scheme*

Predictor-step:

$$y_n = \frac{g(x_n) - x_n g'(x_n)}{1 - g'(x_n)},$$

Corrector-step:

$$x_{n+1} = \frac{g(x_n) - x_n g'(x_n)}{1 - g'(x_n)} + \frac{g(y_n) - y_n}{1 - g'(x_n)} + \frac{(g(y_n) - y_n)(g'(y_n) - g'(x_n))}{(1 - g'(x_n))^2}.$$

For $m = 3$,

$$\begin{aligned} x = X_2 &= x_0 + x_1 + x_2 + x_3 \\ &= c + A_0 + A_1 + A_2 \\ &= \frac{g(\gamma) - \gamma g'(\gamma)}{1 - g'(\gamma)} + \frac{g(x_0) - x_0}{1 - g'(\gamma)} + \frac{(g(x_0) - x_0)(g'(x_0) - g'(\gamma))}{(1 - g'(\gamma))^2} \\ &\quad + \frac{(g(x_0) - x_0)(g'(x_0) - g'(\gamma))^2}{(1 - g'(\gamma))^3} + \frac{1}{2}\frac{(g(x_0) - x_0)^2 (g''(x_0))}{(1 - g'(\gamma))^2}. \end{aligned} \tag{3.24}$$

This allows us to suggest the following one-step iterative method for solving the nonlinear equation (3.1).

Algorithm 3.4. *For a given x_0 compute the approximate solution x_{n+1} by the iterative scheme*

Predictor-step:

$$y_n = \frac{g(x_n) - x_n g'(x_n)}{1 - g'(x_n)},$$

Corrector-step:

$$\begin{aligned} x_{n+1} = &\frac{g(x_n) - x_n g'(x_n)}{1 - g'(x_n)} + \frac{g(y_n) - y_n}{1 - g'(x_n)} + \frac{(g(y_n) - y_n)(g'(y_n) - g'(x_n))}{(1 - g'(x_n))^2} \\ &+ \frac{(g(y_n) - y_n)(g'(y_n) - g'(x_n))^2}{(1 - g'(x_n))^3} + \frac{1}{2}\frac{(g(y_n) - y_n)^2 (g''(y_n))}{(1 - g'(x_n))^2}. \end{aligned}$$

4. Convergence analysis

In this section, we discuss the convergence analysis of Algorithms 3.2, 3.3 and 3.4.

Theorem 4.1. *Let $f : D \subset \mathbb{R} \to \mathbb{R}$ for an open interval D and consider the nonlinear equation $f(x) = 0$ (or $x = g(x)$) has a simple root $\alpha \in D$, where $g(x) : D \subset \mathbb{R} \to \mathbb{R}$ be sufficiently smooth in the neighborhood of the root α. Then the order of convergence of the Algorithm 3.2 is at least 3.*

Proof. The functional equations of Algorithm 3.2 is given by

$$M(x) = \frac{-xg'(x) + g(x)}{1 - g'(x)} + \frac{g\left(\frac{-xg'(x)+g(x)}{1-g'(x)}\right) - \frac{-xg'(x)+g(x)}{1-g'(x)}}{1 - g'(x)}.$$

However

$$M'(x) = \frac{1}{(1-g'(x))^3}\Bigg[g''(x)\Bigg\{-2xg'(x) + g(x) + g(x)g'(x) + g'\left(\frac{-xg'(x)+g(x)}{1-g'(x)}\right)x - g'\left(\frac{-xg'(x)+g(x)}{1-g'(x)}\right)g(x) - g\left(\frac{-xg'(x)+g(x)}{1-g'(x)}\right) + g\left(\frac{-xg'(x)+g(x)}{1-g'(x)}\right)g'(x)\Bigg\}\Bigg],$$

$$\begin{aligned} M''(x) = \frac{1}{(-1+g'(x))^5}\Bigg[&2g''\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right)\left[g''(x)\right]^2 xg(x) \\ &- 2g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right)xg'''(x)g'(x) \\ &+ g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right)xg'''(x)\left[g'(x)\right]^2 \\ &- 4g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right)x\left[g''(x)\right]^2 g'(x) \\ &+ 4g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right)\left[g''(x)\right]^2 g(x)g'(x) \\ &+ 2g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right)g'''(x)g(x)g'(x) \\ &- g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right)g'''(x)g(x)[g'(x)]^2 \\ &- 2x[g''(x)]^2[g(x)]^2 - 2x[g''(x)]^2 g'(x) - g'''(x)g(x)g'(x) \\ &- 2xg'''(x)g'(x) + 4xg'''(x)\left[g'(x)\right]^2 - g''(x)g'(x) \\ &+ 3g''(x)\left[g'(x)\right]^2 4\left[g''(x)\right]^2 g(x) + g'''(x)g(x) - 3g''(x)\left[g'(x)\right]^3 \\ &+ g'(x)\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right)g''(x) - 2\left[g''(x)\right]^2 g\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right) \\ &- g'''(x)g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) + g''(x)\left[g'(x)\right]^4 \\ &- g''(x)\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right)\left[g''(x)\right]^2 x^2 \\ &- g''(x)\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right)\left[g''(x)\right]^2 g(x)^2 - 2\left[g''(x)\right]^2 g(x)g'(x) \\ &- g'''(x)g(x)\left[g''(x)\right]^2 + 3g'(x)\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right)g''(x)\left[g'(x)\right]^2 \end{aligned}$$

$$
\begin{aligned}
&- g'(x)\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right) g''(x)\left[g''(x)\right]^3 \\
&+ g'(x)\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right) xg'''(x) \\
&+ 4g'(x)\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right) x\left[g''(x)\right]^2 \\
&- 4g(x)\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right)\left[g''(x)\right]^2 g(x) \\
&- g'(x)\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right) g'''(x)\, g(x) \\
&- 2g'''(x)\, g(x)\left[g'(x)\right]^3 4x\left[g''(x)\right]^2\left[g'(x)\right]^2 \\
&- 2\left[g''(x)\right]^2 g(x)\left[g'(x)\right]^2 g'''(x)\, g(x)\, g'''(x) \\
&+ 4\left[g''(x)\right]^2 g\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right) g(x) \\
&- 2\left[g''(x)\right]^2 g\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right)\left[g''(x)\right]^2 \\
&+ 3g'''(x)\, g\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right) g'(x) \\
&- 3g'''(x)\, g\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right)\left[g'(x)\right]^2 \\
&+ g'''(x)\, g\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right)\left[g'(x)\right]^3 \\
&- 3g'\left(\frac{xg(x)-g(x)}{-1+g'(x)}\right) g''(x)\, g'(x)\bigg],
\end{aligned}
$$

$$
\begin{aligned}
M'''(x) = {} & \frac{1}{(-1+g'(x))^7}\bigg[6g''\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) g''(x)\, xg'''(x)\, g(x)\left[g''(x)\right]^2 \\
&- 12g''g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) g''(x)\, xg'''(x)\, g(x)\, g'(x) - 3\left[g''(x)\right]^2 \\
&+ 6g''(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) g''(x)\, x^2 g'''(x)\, g(x)\, g'(x) \\
&+ 6g''\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) g''(x)\, xg'''(x)\, g(x)\, g''(x) \\
&+ 36g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) g''(x)\, xg'''(x)\, g(x)\,[g(x)]^2 g(x)
\end{aligned}
$$

$$
\begin{aligned}
&- 3g''\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) g''(x)\, xg'''(x)\, g(x)\, [g(x)]^2 g'(x) \\
&- 3g''\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) g''(x)\, x^2 g'''(x)\, [g'(x)]^2 \\
&- 36g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg'''(x)\, g'''(x)\, g(x) \\
&+ 36g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg'''(x)\, g''(x)\, [g(x)]^2 \\
&+ 36g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg''(x)\, g'''(x)\, g(x)\, g'(x) \\
&- 12g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg'''(x)\, g''(x)\, [g'(x)]^3 \\
&- 36g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg''(x)\, g'''(x)\, g(x)\, [g(x)]^2 \\
&+ 12g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg''(x)\, g'''(x)\, g(x)\, [g'(x)]^3 \\
&- 6g'''(x)\, g''(x) - 12xg''(x)\, g'''(x) + 9[g''(x)]^2 g'(x) \\
&- 6g''(x)\, g(x)\, [g''(x)]^2 - 2g'''(x)\, g(x)\, g'(x) - 20g'''(x)\, [g'(x)]^3 \\
&+ 10g'''(x)\, [g'(x)]^2 + 18g''(x)\, [g(x)]^3 g(x) + g^{(4)}(x)\, g(x) \\
&- 6[g''(x)]^2 [g'(x)]^3 + 9[g''(x)]^2 [g'(x)]^4 + 20g'''(x)\, [g'(x)]^4 \\
&- 10g'''(x)\, [g'(x)]^5 + 2g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg''' + 12x \\
&+ 6\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg'' - 6[g''(x)]^3 g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) \\
&- g^{(4)}(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) + 2g'''(x)\, [g'(x)]^6 \\
&- 3[g''(x)]^2 [g'(x)]^5 + 6xg'''(x)\, xg(x)\, g''(x)\, g'(x) \\
&+ 18xg'''(x)\, xg''(x)\, [g'(x)]^2 - 30g''(x)\, g'''(x)\, g(x)\, g'(x) \\
&- 3g'''(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g''(x)]^3 x^2 g(x) \\
&+ 3g'''(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g''(x)]^3 x [g(x)]^2 \\
&+ 18g''(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g''(x)]^3 xg(x)
\end{aligned}
$$

$$
\begin{aligned}
&- 3g''\left(x\right)g\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)g''\left(x\right)x^2g\left(x\right)\\
&- 3g''\left(x\right)g\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)g''\left(x\right)g'''\left(x\right)\left[g\left(x\right)\right]^2\\
&+ 9g''\left(x\right)g\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^2xg'\left(x\right)\\
&- 9g''\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^2x\left[g'\left(x\right)\right]^2\\
&- 9g''\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^2g\left(x\right)g'\left(x\right)\\
&+ 9g''\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^2g\left(x\right)\left[g'\left(x\right)\right]^2\\
&+ 3g''\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^2xg\left(x\right)g'\left(x\right)\\
&+ 9g''\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^2xg'\left(x\right)\\
&- 3g''\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^2g\left(x\right)\left[g\left(x\right)\right]^3\\
&+ 9g''\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^3\left[g\left(x\right)\right]^2g'\left(x\right)\\
&- 30xg'''\left(x\right)g''\left(x\right)\left[g'\left(x\right)\right]^3+18g''\left(x\right)g'''\left(x\right)g\left(x\right)\left[g'\left(x\right)\right]^2\\
&+ 6g''\left(x\right)g'''\left(x\right)g\left(x\right)\left[g'\left(x\right)\right]^3+12g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)xg'''\left(x\right)g''\left(x\right)\\
&- 4g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)xg^{(4)}\left(x\right)g\left(x\right)g'\left(x\right)\\
&+ 6g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)xg^{(4)}\left(x\right)g\left(x\right)\left[g'\left(x\right)\right]^2\\
&- 4g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)xg^{(4)}\left(x\right)g\left(x\right)\left[g'\left(x\right)\right]^3\\
&- 36g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)xg''\left(x\right)g\left[g'\left(x\right)\right]^3g'\left(x\right)\\
&- 12g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)xg''\left(x\right)g'''\left(x\right)g\left(x\right)\\
&+ 4g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)xg^{(4)}\left(x\right)g\left(x\right)g'\left(x\right)
\end{aligned}
$$

$$
\begin{aligned}
&- 6g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) g^{(4)}(x)\, g(x)\, [g(x)]^2 \\
&+ 36g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg''(x)\, [g'(x)]^3 g'(x) \\
&- 18g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg''(x)\, [g'(x)]^3 g(x)\, [g(x)]^2 \\
&+ 18g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg''(x)\, [g'(x)]^3 [g(x)]^2 \\
&+ g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg^{(4)}(x)\, g(x)\, g(x)\, [g(x)]^4 \\
&+ 4g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg^{(4)}(x)\, g(x)\, [g(x)]^3 \\
&- g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) xg^{(4)}(x)\, g(x)\, g(x)\, [g(x)]^4 \\
&+ 12xg'''(x)\, g(x)\, g''(x)\, [g(x)]^4 - 6xg''(x)\, g'''(x)\, g(x)\, [g(x)]^4 \\
&+ 24xg''(x)\, g(x)\, g'''(x)\, g(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) g(x) \\
&- 36g''(x)\, g(x)\, g'''(x)\, g(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g(x)]^2 \\
&+ 24g''(x)\, g(x)\, g'''(x)\, g(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g'(x)]^3 \\
&- 6g''(x)\, g(x)\, g'''(x)\, g(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g'(x)]^4 \\
&- 2xg^{(4)}(x)\, g'(x) + 8xg^{(4)}(x)\, [g'(x)]^2 - 12xg^{(4)}(x)\, [g'(x)]^3 \\
&+ 12x[g''(x)]^2 [g(x)]^3 g'(x) + 12g''(x)\, g'''(x)\, g(x) \\
&- 3g^{(4)}(x)\, g(x)\, g'(x) + 2g''''(x)\, g(x)\, [g(x)]^2 \\
&+ g'''\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g''(x)]^3 [g(x)]^3 x^3 \\
&- g'''\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g''(x)]^3 [g(x)]^3 \\
&- 3g''\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g''(x)]^2 x
\end{aligned}
$$

$$
\begin{aligned}
&- 9g''\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^{3}x^{2}\\
&- 9g''\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^{2}\left[g\left(x\right)\right]^{2}\\
&+ 3g''\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^{2}g\left(x\right)\\
&- 30\left[g''\left(x\right)\right]^{3}g\left(x\right)g'\left(x\right)+6\left[g''\left(x\right)\right]^{3}g\left(x\right)\left[g'\left(x\right)\right]^{2}\\
&+ 12\left[g''\left(x\right)\right]^{3}\left[g\left(x\right)\right]^{2}+8xg^{(4)}\left(x\right)\left[g\left(x\right)\right]^{4}+2g^{(4)}\left(x\right)g\left(x\right)\left[g'\left(x\right)\right]^{3}\\
&- 3\left[g^{(4)}\left(x\right)\right]^{3}g\left(x\right)\left[g'\left(x\right)\right]^{4}-24g''\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^{2}g'\left(x\right)\\
&+ 36g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^{2}\left[g'\left(x\right)\right]^{2}\\
&- 10g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)g'''\left(x\right)g'\left(x\right)\\
&- 20g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)g'''\left(x\right)\left[g'\left(x\right)\right]^{3}\\
&+ g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)g'''\left(x\right)\left[g'\left(x\right)\right]^{2}\\
&- 18g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^{3}g\left(x\right)\\
&- g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)g^{(4)}\left(x\right)g\left(x\right)\\
&- 24g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^{2}\left[g\left(x\right)\right]^{3}\\
&+ 6g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\left[g''\left(x\right)\right]^{2}\left[g\left(x\right)\right]^{2}\left[g'\left(x\right)\right]^{4}\\
&+ 10g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)g'''\left(x\right)\left[g'\left(x\right)\right]^{4}\\
&- 2g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)g'''\left(x\right)\left[g'\left(x\right)\right]^{5}+g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)xg^{(4)}\left(x\right)\\
&+ 18g'\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)xg''\left(x\right)\left[g\left(x\right)\right]^{3}-2xg^{(4)}\left(x\right)\left[g'\left(x\right)\right]^{5}\\
&- 12\left[g''\left(x\right)\right]^{3}\left[g'\left(x\right)\right]^{3}-6g''\left(x\right)g'''\left(x\right)g\left(\frac{xg'(x)-g\left(x\right)}{-1+g'(x)}\right)\\
&+ 6\left[g''\left(x\right)\right]^{3}g\left(x\right)\left[g'\left(x\right)\right]^{3}+g^{(4)}\left(x\right)g\left(x\right)\left[g'\left(x\right)\right]^{5}
\end{aligned}
$$

$$
\begin{aligned}
&+18[g''(x)]^3 g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) g'(x) \\
&-18[g(x)]^3 g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g'(x)]^2 \\
&+6[g''(x)]^3 g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g'(x)]^3 \\
&-10g^{(4)}(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g'(x)]^2 \\
&+10g^{(4)}(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g'(x)]^3 \\
&-5g^{(4)}(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g'(x)]^4 \\
&+g^{(4)}(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g'(x)]^5 \\
&+5g^{(4)}(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) g'(x) \\
&-18g''(x)\, g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) [g''(x)]^3 xg(x)\, g'(x)\Bigg].
\end{aligned}
$$

For the exact root α, we obtain $M(\alpha) = \alpha$, $M'(\alpha) = 0 = M''(\alpha)$, and $M'''(\alpha) \neq 0$. Hence the Algorithm 3.2 has third order convergence. □

Theorem 4.2. *Let $f : D \subset \mathbb{R} \to \mathbb{R}$ for an open interval D and consider the nonlinear equation $f(x) = 0$ (or $x = g(x)$) has a simple root $\alpha \in D$, where $g(x) : D \subset \mathbb{R} \to \mathbb{R}$ be sufficiently smooth in the neighborhood of the root α. Then the order of convergence of the Algorithm 3.3 is at least 4.*

Proof. The functional equations of Algorithm 3.3 is given by

$$
\begin{aligned}
N(x) = {} & \frac{xg'(x)-g(x)}{-1+g'(x)} + \frac{g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) - \frac{xg'(x)-g(x)}{-1+g'(x)}}{-1+g'(x)} \\
& + \frac{1}{(-1+g'(x))^2}\left[\left\{g\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) - \frac{xg'(x)-g(x)}{-1+g'(x)}\right\}\right. \\
& \left. \times \left\{g'\left(\frac{xg'(x)-g(x)}{-1+g'(x)}\right) - g'(x)\right\}\right].
\end{aligned}
$$

For the exact root α, we obtain $N(\alpha) = \alpha$, $N'(\alpha) = 0 = N''(\alpha) = N'''(\alpha)$, and $N''''(\alpha) \neq 0$. Hence the Algorithm 3 has fourth order convergence. □

Theorem 4.3. *Let $f : D \subset \mathbb{R} \to \mathbb{R}$ for an open interval D and consider the nonlinear equation $f(x) = 0$ (or $x = g(x)$) has a simple root $\alpha \in D$, where $g(x) : D \subset \mathbb{R} \to \mathbb{R}$ be sufficiently smooth in the neighborhood of the root α. Then the order of convergence of the Algorithm 3.4 is at least* 4.

5. Applications

We present some examples to illustrate the efficiency of the developed methods, Algorithm 3.1, Algorithm 3.2, Algorithm 3.3 and Algorithm 3.4 and compare them with the fixed point method.

Example 5.1. Consider the equation $x^3 + 4x^2 + 8x + 8 = 0$. We have $g(x) = -\left(1 + \frac{1}{2}x^2 + \frac{1}{8}x^3\right)$, $g'(x) = -\left(x + \frac{3}{8}x^2\right)$, and $g''(x) = -\left(1 + \frac{3}{4}x\right)$. If we take $x_0 = -1.9$, then the comparison of the methods is shown in the following table:

x_n	FPM	Algorithm 3.1	Algorithm 3.2	Algorithm 3.3	Algorithm 3.4
x_1	−1.9476	−2.0050	−1.9995	−2.0001	−2
x_2	−1.9731	−2	−2	−2	
x_3	−1.9864				
x_4	−1.9932				
x_5	−1.9966				
x_6	−1.9983				
x_7	−1.9991				
x_8	−1.9995				
x_9	−1.9997				
x_{10}	−1.9998				
x_{11}	−1.9999				

Example 5.2. Consider the equation $x + \ln(x - 2) = 0$. We have $g(x) = 2 + e^{-x}$, $g'(x) = -e^{-x}$, and $g''(x) = e^{-x}$. If we take $x_0 = 2.1$, then the comparison of the methods is shown in the following table:

x_n	FPM	Algorithm 3.1	Algorithm 3.2	Algorithm 3.3	Algorithm 3.4
x_1	2.1225	2.12	2.12	2.12	2.12
x_2	2.1197				
x_3	2.1201				
x_4	2.12				

Example 5.3. Consider the equation $e^x - 3x^2 = 0$. We have $g(x) = \sqrt{\frac{e^x}{3}}$, $g'(x) = \frac{e^{\frac{x}{2}}}{2\sqrt{3}}$, and $g''(x) = \frac{e^{\frac{x}{2}}}{4\sqrt{3}}$. If we take $x_0 = 0.8$, then the comparison of the

methods is shown in the following table:

x_n	FPM	Algorithm 3.1	Algorithm 3.2	Algorithm 3.3	Algorithm 3.4
x_1	0.86131	0.90768	0.90991	0.91	0.91001
x_2	0.88812	0.91001	0.91001		
x_3	0.9001				
x_4	0.90551				
x_5	0.90796				
x_6	0.90908				
x_7	0.90959				
x_8	0.90982				
x_9	0.90992				
x_{10}	0.90997				
x_{11}	0.90999				
x_{12}	0.91000				
x_{13}	0.91				

Example 5.4. Consider the equation $\sin x - 5x + 2 = 0$. We have $g(x) = \frac{1}{5}(\sin x + 2)$, $g'(x) = \frac{1}{5}\cos x$, and $g''(x) = -\frac{1}{5}\sin x$. If we take $x_0 = 0.5$, then the comparison of the methods is shown in the following table:

x_n	FPM	Algorithm 3.1	Algorithm 3.2	Algorithm 3.3	Algorithm 3.4
x_1	0.49589	0.49501	0.49501	0.49501	0.49501
x_2	0.49516				
x_3	0.49503				
x_4	0.49501				

Example 5.5. Consider the equation $x^3 - 5x^2 - 29 = 0$. We have $g(x) = 5 + \frac{29}{x^2}$, $g'(x) = \frac{-58}{x^3}$, and $g''(x) = \frac{174}{x^4}$. If we take $x_0 = 5$, then the comparison of the methods is shown in the following table:

x_n	FPM	Algorithm 3.1	Algorithm 3.2	Algorithm 3.3	Algorithm 3.4
x_1	6.16	5.7923	5.8415	5.8471	5.8479
x_2	5.7643	5.8478	5.8480	5.8480	5.8480
x_3	5.8728	5.8480			
x_4	5.8408				
x_5	5.8501				
x_6	5.8474				
x_7	5.8481				
x_8	5.8479				
x_9	5.8480				

Example 5.6. Consider the equation $x - \cos x = 0$. We have $g(x) = \cos x$, $g'(x) = -\sin x$, and $g''(x) = -\cos x$. If we take $x_0 = 0.6$, then the comparison

of the methods is shown in the following table:

x_n	FPM	Algorithm 3.1	Algorithm 3.2	Algorithm 3.3	Algorithm 3.4
x_1	0.82534	0.74402	0.73874	0.73912	0.73908
x_2	0.67831	0.73909	0.73909	0.73909	0.73909
x_3	0.77863				
x_4	0.71188				
x_5	0.75714				
x_6	0.7268				
x_7	0.7473				
x_8	0.73353				
x_9	0.74282				
x_{10}	0.73656				
x_{11}	0.74078				
x_{12}	0.73794				
x_{13}	0.73986				
x_{14}	0.73856				
x_{15}	0.73944				
x_{16}	0.73885				
x_{17}	0.73924				
x_{18}	0.73898				
x_{19}	0.73916				
x_{20}	0.73903				

Example 5.7. Consider the equation $\cos x - 3x + 1 = 0$. We have $g(x) = \frac{1}{3}\cos x + \frac{1}{3}$, $g'(x) = -\frac{1}{3}\sin x$, and $g''(x) = -\frac{1}{3}\cos x$. If we take $x_0 = 0.5$, then the comparison of the methods is shown in the following table:

x_n	FPM	Algorithm 3.1	Algorithm 3.2	Algorithm 3.3	Algorithm 3.4
x_1	0.62586	0.60852	0.60706	0.6071	0.6071
x_2	0.60348	0.6071	0.6071		
x_3	0.60779				
x_4	0.60697				
x_5	0.6071				

Example 5.8. Consider the equation $xe^x = 1$. We have $g(x) = e^{-x}$, $g'(x) = -e^{-x}$, and $g''(x) = e^{-x}$. If we take $x_0 = 0.5$, then the comparison of

the methods is shown in the following table:

x_n	FPM	Algorithm 3.1	Algorithm 3.2	Algorithm 3.3	Algorithm 3.4
x_1	0.60653	0.56631	0.56712	0.56714	0.56714
x_2	0.57970	0.56714	0.56714		
x_3	0.57117				
x_4	0.56844				
x_5	0.56756				
x_6	0.56728				
x_7	0.56719				
x_8	0.56716				
x_9	0.56715				
x_{10}	0.56714				
x_{11}	0.54524				
x_{12}	0.56007				
x_{13}	0.56486				
x_{14}	0.56641				
x_{15}	0.56691				
x_{16}	0.56706				
x_{17}	0.56712				
x_{18}	0.56713				
x_{19}	0.56714				

Example 5.9. Consider the equation $\cos x - 2x + 3 = 0$. We have $g(x) = \frac{1}{2}\cos x + \frac{3}{2}$, $g'(x) = -\frac{1}{2}\sin x$, and $g''(x) = -\frac{1}{2}\cos x$. If we take $x_0 = 1.5$, then the comparison of the methods is shown in the following table:

x_n	FPM	Algorithm 3.1	Algorithm 3.2	Algorithm 3.3	Algorithm 3.4
x_1	1.5354	1.5236	1.5236	1.5236	1.5236
x_2	1.5177				
x_3	1.5265				
x_4	1.5221				
x_5	1.5243				
x_6	1.5232				
x_7	1.5238				

Example 5.10. Consider the equation $\sin x - 10x + 10 = 0$. We have $g(x) = 1 + \frac{\sin x}{10}$, $g'(x) = \frac{1}{10}\cos x$, and $g''(x) = -\frac{1}{10}\sin x$. If we take $x_0 = 1$, then the comparison of the methods is shown in the following table:

x_n	FPM	Algorithm 3.1	Algorithm 3.2	Algorithm 3.3	Algorithm 3.4
x_1	1.0841	1.0890	1.0886	1.0886	1.0886
x_2	1.0884	1.0886			
x_3	1.0886				
x_4	1.0886				

6. Conclusions

New higher-order iterative methods for the solution of scalar equations by using the Adomian decomposition technique [2] are established.

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