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SOMEWHAT e-$\mathcal{I}$-CONTINUOUS AND SOMEWHAT e-$\mathcal{I}$-OPEN FUNCTIONS VIA IDEALS

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Abstract: In this paper, new classes of functions are introduced and studied by making use of e-$\mathcal{I}$-open sets and e-$\mathcal{I}$-closed sets. Relationship between the new classes and other classes of functions are established besides giving examples, counterexamples, properties and characterizations.

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1. Introduction and Preliminaries

The subject of ideals in topological spaces has been studied by Kuratowski [13] and Vaidyanathaswamy [19]. Jankovic and Hamlett [12] investigated further

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properties of ideal topological spaces. In 1992, Jankovic and Hamlett [11] introduced the notion of I-open sets in ideal topological spaces. Abd El-Monsef et al. [1] investigated I-open sets and I-continuous functions. In this paper, using the notion of e-$\mathcal{I}$-open sets defined in [2] see also [3], [4], the concepts of somewhat e-$\mathcal{I}$-continuous functions and somewhat e-$\mathcal{I}$-open functions are introduced and studied. Some characterizations and properties for somewhat e-$\mathcal{I}$-continuity and somewhat e-$\mathcal{I}$-openness are obtained besides giving examples and counterexamples.

An ideal $\mathcal{I}$ on a topological space (X,τ) is a nonempty collection of subsets of X which satisfies the following conditions: $A \in \mathcal{I}$ *and* $B \subset A$ *implies* $B \in \mathcal{I}$; $A \in \mathcal{I}$ *and* $B \in \mathcal{I}$ *implies* $A \cup B \in \mathcal{I}$. A topological space (X,τ) with an ideal $\mathcal{I}$ is called an ideal topological space and is denoted by $(X,\tau,\mathcal{I})$. Given an ideal topological space $(X,\tau,\mathcal{I})$ on X and if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^{\star} : \wp(X) \to \wp(X)$, called the local function [18, 12] of A with respect to τ and $\mathcal{I}$, is defined as follows: for $A \subseteq X$,

$$A^{\star}(\mathcal{I},\tau) = \{x \in X \mid U \cap A \notin \mathcal{I} \text{for every } U \in \tau(x)\}$$

where $\tau(x) = \{U \in \tau \mid x \in U\}$. It is known in [12] that $Cl^{\star}(A) = A \cup A^{\star}(\mathcal{I},\tau)$ is a Kuratowski closure operator. When there is no chance for confusion, we will simply write $A^{\star}$ for $A^{\star}(\mathcal{I},\tau)$. $X^{\star}$ is often a proper subset of X. A subset A of an ideal topological space $(X,\tau,\mathcal{I})$ is said to be R-I-open (resp. R-I-closed) [20] if $A = Int(Cl^{\star}(A))$ $(resp. A = Cl^{\star}(Int(A))$. A point $x \in X$ is called a $\delta - \mathcal{I}$-cluster point of A if $Int(Cl^{\star}(U)) \cap A \neq \emptyset$ for each open set U containing x. The family of all δ-$\mathcal{I}$-cluster points of A is called the δ-$\mathcal{I}$-closure of A and is denoted by $\delta Cl_I(A)$. The δ-$\mathcal{I}$-interior of A is the union of all R-I-open sets of X contained in A and is denoted by $\delta Int_I(A)$. A is said to be δ-$\mathcal{I}$-closed if $\delta Cl_I(A) = A$ [20].

Definition 1. A subset A of an ideal topological space $(X,\tau,\mathcal{I})$ is said to be:

1. I-open [1] if $A \subset Int(A^{\star})$.

2. $semi^{\star}$-$\mathcal{I}$-open [10] if $A \subset Cl(\delta Int_I(A))$.

3. e-$\mathcal{I}$-open if [2] $A \subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A))$.

Remark 2. In the following diagram we denote by arrows the implications between the open sets and the above three relations. It is known that (1) $\mathcal{I}$-openness and openness are independent [1, 6], (2) every $\mathcal{I}$-open set is $semi^{\star}$-$\mathcal{I}$-

open [9], and (3) every open set is e-$\mathcal{I}$-open[2].

$$\begin{array}{ccc} R\text{-}\mathcal{I}\text{-open} & \longrightarrow & semi^{*}\text{-}\mathcal{I}\text{-open} \\ & & \downarrow \\ \text{open} & \longrightarrow & e\text{-}\mathcal{I}\text{-open} \end{array}$$

Definition 3. [8] A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be somewhat-continuous provided that if for $U \in \sigma$ and $f^{-1}(U) \neq \emptyset$ there exists an open set V in τ such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$.

Definition 4. A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is said to be somewhat-$\mathcal{I}$-continuous (resp. somewhat $semi^{\star}$-$\mathcal{I}$-continuous) if for any $U \in \sigma$ such that $f^{-1}(U) \neq \emptyset$ there exists an I-open (resp. $semi^{\star}$-$\mathcal{I}$-open) set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$.

Definition 5. [8] A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be somewhat-open provided that if $U \in \tau$ and $U \neq \emptyset$, then there exists an open set V in σ such that $V \neq \emptyset$ and $V \subset f(U)$.

Definition 6. A function $f : (X, \tau) \longrightarrow (Y, \sigma, \mathcal{I})$ is said to be somewhat-I-open (resp. somewhat $semi^{\star}$-$\mathcal{I}$-open) provided that if $U \in \tau$ and $U \neq \emptyset$, then there exists an $\mathcal{I}$-open (resp. $semi^{\star}$-$\mathcal{I}$-open) set V in Y such that $V \neq \emptyset$ and $V \subset f(U)$.

2. Somewhat e-$\mathcal{I}$-Continuous Functions

Definition 7. Let $(X, \tau, \mathcal{I})$ be an ideal topological space and (Y, σ) be any topological space. A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is said to be somewhat e-$\mathcal{I}$-continuous provided that if $U \in \sigma$ and $f^{-1}(U) \neq \emptyset$, then there exists an e-$\mathcal{I}$-open set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$.

Theorem 8. *If $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is somewhat continuous, then it is somewhat e-$\mathcal{I}$-continuous.*

Proof. Trivial □

Theorem 9. *Every somewhat $semi^{\star}$-$\mathcal{I}$-continuous function is somewhat e-$\mathcal{I}$-continuous.*

Proof. Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ be a somewhat $semi^{\star}$-$\mathcal{I}$-continuous function. Let U be any open set in Y such that $f^{-1}(U) \neq \emptyset$. Since f is somewhat *semi**-$\mathcal{I}$-continuous, there exists a *semi**-$\mathcal{I}$-open set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$. Since every *semi**-$\mathcal{I}$-open set is e-$\mathcal{I}$-open, there exists an e-$\mathcal{I}$-open set V such that $V \neq \emptyset$ and $V \subset f^{-1}(U)$, which implies that f is somewhat e-$\mathcal{I}$-continuous. □

Remark 10. The converses of the above theorem, need not be true in general as shown by the following example.

Example 11. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$, $\mathcal{I} = \{\emptyset, \{a\}\}$, $\sigma = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$. Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ be an identity function. Then f is somewhat e-$\mathcal{I}$-continuous but it is neither somewhat continuous nor somewhat *semi**-$\mathcal{I}$-continuous. Since the inverse image of $\{c\}$ in (Y, σ) is an e-$\mathcal{I}$-open but it is neither open nor *semi**-$\mathcal{I}$-open.

Theorem 12. *If $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is a somewhat e-$\mathcal{I}$-continuous surjection and $g : (Y, \sigma) \longrightarrow (Z, \varsigma)$ is somewhat continuous, then $g \circ f : (X, \tau, \mathcal{I}) \longrightarrow (Z, \varsigma)$ e-$\mathcal{I}$-continuous.*

Proof. Let W be any open set of (Z, ς) and $(g \circ f)^{-1}(W) \neq \emptyset$. Then $g^{-1}(W) \neq \emptyset$. Since g is somewhat continuous, there exists $V \in \sigma$ such that $\emptyset \neq V \subset g^{-1}(W)$. Since f is surjective, $\emptyset \neq f^{-1}(V) \subset f^{-1}(g^{-1}(W))$. Since f is somewhat e-$\mathcal{I}$-continuous, There exists an e-$\mathcal{I}$-open set U in (X, τ) such that $\emptyset \neq U \subset f^{-1}(V)$. Therefore, we have $\emptyset \neq U \subset (g \circ f)^{-1}(W))$. This show that $g \circ f$ is somewhat e-$\mathcal{I}$-continuous. □

Theorem 13. *Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ and $g : (Y, \sigma) \longrightarrow (Z, \zeta)$ be any two functions. If f is somewhat e-$\mathcal{I}$-continuous and g is continuous, then $g \circ f$ is somewhat e-$\mathcal{I}$-continuous.*

Proof. Let $U \in \zeta$ and $(g \circ f)^{-1}(U) \neq \emptyset$. Then $g^{-1}(U) \neq \emptyset$. Since $U \in \zeta$ and g is continuous $g^{-1}(U) \in \sigma$. Since $f^{-1}(g^{-1}(U)) \neq \emptyset$ and f is somewhat e-$\mathcal{I}$-continuous, there exists an e-$\mathcal{I}$-open set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$. Then $g \circ f$ is somewhat e-$\mathcal{I}$-continuous. □

Remark 14. In Theorem 13, if f is a continuous function and g is a somewhat e-$\mathcal{I}$-continuous function, then it is not necessarily true that $g \circ f$ is somewhat e-$\mathcal{I}$-continuous. Since every continuous function is somewhat e-$\mathcal{I}$-continuous, the composition of somewhat e-$\mathcal{I}$-continuous functions need not be somewhat e-$\mathcal{I}$-continuous. The following example serves this purpose.

Example 15. Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$, $\mathcal{I} = \{\phi, \{b\}\}$, $\sigma = \{\phi, X, \{a\}, \{b, c\}\}$, $\zeta = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ and $\mathcal{J} = \{\phi, \{a\}\}$. Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{I})$ and $g : (Y, \sigma, \mathcal{J}) \longrightarrow (Z, \zeta)$ be the identity functions. Then clearly f is continuous and g is somewhat e-$\mathcal{I}$-continuous but $g \circ f$ is not somewhat e-$\mathcal{I}$-continuous. Since $\{c\} \in \zeta$ and $(g \circ f)^{-1}(U) = (g \circ f)^{-1}(\{c\}) = \{c\}$ not somewhat e-$\mathcal{I}$-open, $g \circ f$ is not somewhat e-$\mathcal{I}$-continuous.

Definition 16. A subset S of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be e-$\mathcal{I}$-dense if $Cl_e^{\star}(S) = X$. In other words if there is no proper e-$\mathcal{I}$-closed set M in X such that $S \subset M \subset X$.

Theorem 17. *Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ be a surjective function. Then the following are equivalent:*

1. *f is somewhat e-$\mathcal{I}$-continuous;*
2. *If M is a closed subset of Y such that $f^{-1}(M) \neq X$, then there is a proper e-$\mathcal{I}$-closed subset D of X such that $D \supset f^{-1}(M)$;*
3. *If S is an e-$\mathcal{I}$-dense subset of X, then $f(S)$ is a dense subset of Y.*

Proof. (1)⇒(2): Let M be a closed subset of Y such that $f^{-1}(M) \neq X$. Then $Y - M$ is an open set in Y such that $f^{-1}(Y - M) = X - f^{-1}(M) \neq \emptyset$. By hypothesis (1) there exists an e-$\mathcal{I}$-open set V in X such that $V \neq \emptyset$ and $V \subset f^{-1}(Y - M) = X - f^{-1}(M)$. This means that $X - V \supset f^{-1}(M)$ and $X - V = D$ is a proper e-$\mathcal{I}$-closed set in X. This proves (2).

(2)⇒(3): Let S be an e-$\mathcal{I}$-dense set in X. Suppose that $f(S)$ is not dense in Y. Then there exists a proper closed set M in Y such that $f(S) \subset M \subset Y$. Clearly $f^{-1}(M) \neq X$. Hence by (2) there exists a proper e-$\mathcal{I}$-closed set D such that $S \subset f^{-1}(M) \subset D \subset X$. This contradicts fact that S is e-$\mathcal{I}$-dense in X.

(3)⇒(2): Suppose that (2) is not true. This means there exists a closed set M in Y such that $f^{-1}(M) \neq X$. But there is no proper e-$\mathcal{I}$-closed set D in X such that $f^{-1}(M) \subset D$. This means that $f^{-1}(M)$ is e-$\mathcal{I}$-dense in X. But by (3) $f(f^{-1}(M)) = M$ must be dense in Y, which is contradiction to the choice of M.

(2)⇒(1): Let $U \in \sigma$ and $f^{-1}(U) \neq \emptyset$. Then $Y - U$ is closed and $f^{-1}(Y - U) = X - f^{-1}(U) \neq X$. By hypothesis of (2) there exists a proper e-$\mathcal{I}$-closed set D of X such that $D \supset f^{-1}(Y - U)$. This implies that $X - D \subset f^{-1}(U)$ and $X - D$ is e-$\mathcal{I}$-open and $X - D \neq \emptyset$. □

Theorem 18. *Let $(X, \tau, \mathcal{I})$ be any ideal topological space and (Y, σ) any topological space. If A is an open set in X and $f : (A, \tau/A, \mathcal{I}/A) \to (Y, \sigma)$ is a somewhat e-$\mathcal{I}$-continuous function such that $f(A)$ is dense in Y, then any extension $F : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ of f is somewhat e-$\mathcal{I}$-continuous.*

Proof. Let U be any open set in (Y, σ) such that $F^{-1}(U) \neq \emptyset$. Since $f(A) \subset Y$ is dense in Y and $U \cap f(A) \neq \emptyset$, it follows that $F^{-1}(U) \cap A \neq \emptyset$. That is $f^{-1}(U) \cap A \neq \emptyset$. Hence by hypothesis on f, there exists an e-$\mathcal{I}$-open set V in A such that $V \neq \emptyset$ and $V \subset f^{-1}(U) \subset F^{-1}(U)$ which implies F is somewhat e-$\mathcal{I}$-continuous. □

Theorem 19. *Let $(X, \tau, \mathcal{I})$ and $(Y, \sigma, \mathcal{J})$ be any two ideal topological spaces, $X = A \cup B$ where A and B are open subsets of X and $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be a function such that f/A and f/B are somewhat e-$\mathcal{I}$-continuous. Then f is somewhat e-$\mathcal{I}$-continuous.*

Proof. Let U be any open set in $(Y, \sigma, \mathcal{J})$ such that $f^{-1}(U) \neq \emptyset$. Then $(f/A)^{-1}(U) \neq \emptyset$ or $(f/B)^{-1}(U) \neq \emptyset$ or both $(f/A)^{-1}(U) \neq \emptyset$ and $(f/B)^{-1}(U) \neq \emptyset$.

Case (1) Suppose $(f/A)^{-1}(U) \neq \emptyset$.

Since f/A is somewhat e-$\mathcal{I}$-continuous, there exists an e-$\mathcal{I}$-open set V in A such that $V \neq \emptyset$ and $V \subset (f/A)^{-1}(U) \subset f^{-1}(U)$. Since V is e-$\mathcal{I}$-open in A and A is open in X, V is e-$\mathcal{I}$-open in X. Thus f is somewhat e-$\mathcal{I}$-continuous.

Case (2) the proof is similar with Case (1).

Case (3) Suppose $(f/A)^{-1}(U) \neq \emptyset$ and $(f/B)^{-1}(U) \neq \emptyset$.

This follows from both the Cases (1) and (2). Thus f is somewhat e-$\mathcal{I}$-continuous. □

Definition 20. An ideal topological space $(X, \tau, \mathcal{I})$ is said to be e-$\mathcal{I}$-separable if there exists a countable subset B of X which is e-$\mathcal{I}$-dense in X.

Theorem 21. *If $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is a somewhat e-$\mathcal{I}$-continuous surjective function and X is e-$\mathcal{I}$-separable, then Y is separable.*

Proof. Let $f : X \to Y$ be a somewhat e-$\mathcal{I}$-continuous surjection such that X is e-$\mathcal{I}$-separable. Then by definition there exists a countable subset B of X which is e-$\mathcal{I}$-dense in X. Then by Theorem 17, $f(B)$ is dense in Y. Since B is countable, $f(B)$ is also a countable set which is dense in Y, which indicates that Y is separable. □

3. e-$\mathcal{I}$-Weakly Equivalent Topologies

Definition 22. Let X be a set and τ and σ be topologies for X. Then τ is said to be weakly equivalent to σ [8] provided that if $U \in \tau$ and $U \neq \emptyset$, then there is an open set V in (X, σ) such that $V \neq \emptyset$ and $V \subset U$ and if $U \in \sigma$ and $U \neq \emptyset$, then there is an open set V in (X, τ) such that $V \neq \emptyset$ and $V \subset U$.

Definition 23. Let (X, τ) and (X, σ) be topological spaces with same ideal $\mathcal{I}$. Then τ is said to be e-$\mathcal{I}$-weakly equivalent to σ provided that if $U \in \tau$ and $U \neq \emptyset$, then there is an e-$\mathcal{I}$-open set V in $(X, \sigma, \mathcal{I})$ such that $V \neq \emptyset$ and $V \subset U$ and if $U \in \sigma$ and $U \neq \emptyset$, then there is an e-$\mathcal{I}$-open set V in $(X, \tau, \mathcal{I})$ such that $V \neq \emptyset$ and $V \subset U$.

Theorem 24. *Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma_1)$ be a somewhat e-$\mathcal{I}$-continuous surjective function and let σ_2 be a topology for Y. If σ_2 is weakly equivalent to σ_1, then the function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma_2)$ is somewhat e-$\mathcal{I}$-continuous.*

Proof. Since σ_2 is weakly equivalent to σ_1, the identity function $i : (Y, \sigma_1) \longrightarrow (Y, \sigma_2)$ is somewhat-continuous. Therefore, by Theorem 12,

$$f = f \circ i : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma_2)$$

is somewhat e-$\mathcal{I}$-continuous.

□

Theorem 25. *Let $f : (X, \tau_1, \mathcal{I}) \longrightarrow (Y, \sigma)$ be a somewhat-continuous function and let τ_2 be a topology for X. If τ_2 is e-$\mathcal{I}$-weakly equivalent to τ_1, then the function $f : (X, \tau_2, \mathcal{I}) \longrightarrow (Y, \sigma)$ is somewhat e-$\mathcal{I}$-continuous.*

Proof. Since τ_2 is e-$\mathcal{I}$-weakly equivalent to τ_1, the identity function $i : (X, \tau_2, \mathcal{I}) \longrightarrow (X, \tau_1, \mathcal{I})$ is somewhat e-$\mathcal{I}$-continuous. Therefore, by Theorem 12, $f = f \circ .i : (X, \tau_2, \mathcal{I}) \longrightarrow (Y, \sigma)$ is somewhat e-$\mathcal{I}$-continuous. □

4. Somewhat e-$\mathcal{I}$-Open Functions

Definition 26. A function $f : (X, \tau) \longrightarrow (Y, \sigma, \mathcal{I})$ is said to be somewhat e-$\mathcal{I}$-open provided that for $U \in \tau$ and $U \neq \emptyset$ there exists an e-$\mathcal{I}$-open set V in Y such that $V \neq \emptyset$ and $V \subset f(U)$.

Theorem 27. *If a function $f : (X, \tau) \longrightarrow (Y, \sigma, \mathcal{I})$ is somewhat open, then it is somewhat e-$\mathcal{I}$-open.*

Theorem 28. *Every somewhat semi*$^{\square}$*-*$\mathcal{I}$*-open function is somewhat e-*$\mathcal{I}$*-open.*

Proof. Let $f : (X, \tau) \longrightarrow (Y, \sigma, \mathcal{I})$ be a somewhat *semi*$^{\square}$-$\mathcal{I}$-open function. Let $U \in \tau$ and $U \neq \emptyset$. Since f is somewhat *semi*$^{\square}$-$\mathcal{I}$-open, there exists a *semi*$^{\square}$-$\mathcal{I}$-open set V in X such that $V \neq \emptyset$ and $V \subset f(U)$. Since every *semi*$^{\square}$-$\mathcal{I}$-open set is e-$\mathcal{I}$-open, there exist an e-$\mathcal{I}$-open set V such that $V \neq \emptyset$ and $V \subset f(U)$, which implies that f is somewhat e-$\mathcal{I}$-open. □

Remark 29. The converses of the above theorems, need not be true in general as shown by the following example.

Example 30. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$, $\mathcal{I} = \{\emptyset, \{a\}\}$, and $\sigma = \{\emptyset, X, \{a\}, \{b, c\}\}$. then the identity function $f : (X, \tau) \longrightarrow (Y, \sigma, \mathcal{I})$ is somewhat e-$\mathcal{I}$-open but it is neither somewhat *semi*$^{\square}$-$\mathcal{I}$-open nor somewhat open. The image of $\{c\} \in \tau$ is e-$\mathcal{I}$-open but it is neither *semi*$^{\square}$-$\mathcal{I}$-open nor somewhat open.

Theorem 31. *If* $f : (X, \tau) \longrightarrow (Y, \sigma)$ *is an somewhat-open function and* $g : (Y, \sigma) \longrightarrow (Z, \zeta, \mathcal{I})$ *is a somewhat e-*$\mathcal{I}$*-open function, then* $g \circ f : (X, \tau) \longrightarrow (Z, \zeta, \mathcal{I})$ *is somewhat e-*$\mathcal{I}$*-open.*

Proof. Suppose that $U \in \tau$ and $U \neq \emptyset$. Since f is somewhat-open, there exists an open set G of (Y, σ) such that $\emptyset \neq G$ and $G \subset f(U)$. Since g is somewhat e-$\mathcal{I}$-open, there exists an e-$\mathcal{I}$-open set $V \in \zeta$ such that $\emptyset \neq V \subset g(G) \subset (g \circ f)(U)$. This implies that $g \circ f$ is somewhat e-$\mathcal{I}$-open. □

Theorem 32. *Let* $f : (X, \tau) \longrightarrow (Y, \sigma, \mathcal{I})$ *be a bijective function. Then the following are equivalent:*

1. f *is somewhat e-*$\mathcal{I}$*-open;*
2. *If* M *is a closed set of* X *such that* $f(M) \neq Y$*, then there is an e-*$\mathcal{I}$*-closed set* D *of* Y *such that* $D \neq Y$ *and* $D \supset f(M)$.

Proof. (1)⇒(2): Let M be a closed set of X such that $f(M) \neq Y$. Then $X - M$ is an open set in X and $X - M \neq \emptyset$. Since f is somewhat e-$\mathcal{I}$-open, there exists an e-$\mathcal{I}$-open set V in Y such that $V \neq \emptyset$ and $V \subset f(X - M)$. Put $D = Y - V$. Clearly D is e-$\mathcal{I}$-closed in Y and we claim that $D \neq Y$. For if $D = Y$, then $V = \emptyset$ which is a contradiction. Since $V \subset f(X - M)$ and f is bijective, $D = Y - V \supset Y - [f(X - M)] = f(M)$.

(2)⇒(1): Let U be any nonempty open set in X. Put $M = X - U$. Then M is a proper closed set of X and $f(M) \neq Y$. Therefore, by (2) there is an

e-$\mathcal{I}$-closed subset D of Y such that $D \neq Y$ and $f(M) \subset D$. Put $V = Y - D$. Clearly V is an e-$\mathcal{I}$-open set and $V \neq \emptyset$. Further, $V = Y - D \subset Y - f(M) = Y - [Y - f(U)] = f(U)$. □

Theorem 33. *If $f : (X, \tau) \to (Y, \sigma, \mathcal{I})$ is a somewhat e-$\mathcal{I}$-open function and A is an open set of X, then $f/A : (A, \tau/A) \to (Y, \sigma, \mathcal{I})$ is also somewhat e-$\mathcal{I}$-open.*

Proof. Let $U \in \tau/A$ such that $U \neq \emptyset$. Since U is open in A and A is open in (X, τ), U is open in (X, τ). By hypothesis, $f : (X, \tau) \to (Y, \sigma, \mathcal{I})$ is somewhat e-$\mathcal{I}$-open and there exists an e-$\mathcal{I}$-open set V in Y such that $\emptyset \neq V \subset f(U)$. Thus for any open set U in $(A, \tau/A)$ with $U \neq \emptyset$, there exists an e-$\mathcal{I}$-open set V in Y such that $\emptyset \neq V \subset f(U)$ which implies that f/A is somewhat e-$\mathcal{I}$-open. □

Theorem 34. *Let (X, τ) be a topological space, $(Y, \sigma, \mathcal{I})$ be an ideal topological space and $X = A \cup B$, where A and B are open sets of X. If $f : (X, \tau) \to (Y, \sigma, \mathcal{I})$ is a function such that f/A and f/B are somewhat e-$\mathcal{I}$-open, then f is somewhat e-$\mathcal{I}$-open.*

Proof. Let U be any open set of (X, τ) such that $U \neq \emptyset$. Since $X = A \cup B$, there are three Cases (1) $A \cap U \neq \emptyset$, (2) $B \cap U \neq \emptyset$ or (3) both $A \cap U \neq \emptyset$ and $B \cap U \neq \emptyset$.

Case (1). Since $A \cap U \in \tau/A$ and f/A is somewhat e-$\mathcal{I}$-open, there exists an e-$\mathcal{I}$-open set V in Y such that $\emptyset \neq V \subset f(U)$. This shows that f is somewhat e-$\mathcal{I}$-open. The other cases are similarly proved. □

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