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ON L-FUZZY (K, E)-SOFT QUASI-UNIFORM SPACES

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Abstract: In this paper, we investigated the topological properties of L-fuzzy (K, E)-soft quasi-uniformities in stsc-quantales. We obtain L-fuzzy (K, E)-soft topology and L-fuzzy (K, E)-soft neighborhood spaces induced by L-fuzzy (K, E)-soft quasi-uniformity. Moreover, we study the relations among L-fuzzy (K, E)-soft topology, L-fuzzy (K, E)-soft neighborhood system and L-fuzzy (K, E)-soft quasi-uniformity.

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1. Introduction

In 1999 Molodtsov [15] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. Maji et al. [13,14] gave the first practical application of soft sets in decision making problems. Many researchers have contributed towards the algebraic structure

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of soft set theory [1,3,7]. In 2011, Shabir and Naz [23] initiated the study of soft topological spaces. They defined soft topology on the collection of soft sets over X and established their several properties. Aygünoglu et.al [4] introduced the concept of soft topology in the sense of Šostak [24].

Höhle [9,10] introduced L-fuzzy topologies with al- gebraic structure L(cqm, quantales, MV -algebra). It has developed in many directions [8,11,17,18]. Ramadan et al. [19,20] define L-fuzzy (K, E)-soft topology and L-fuzzy (K, E)-soft neighborhood system in a stsc-quantale lattice L and investigated the relation between them.

In this paper, we investigated the topological properties of L-fuzzy (K, E)-soft quasi-uniformities in stsc-quantales. We obtain L-fuzzy (K, E)-soft topology and L-fuzzy (K, E)-soft neighborhood spaces induced by L-fuzzy (K, E)-soft quasi-uniformity. Moreover, we study the relations among L-fuzzy (K, E)-soft topology, L-fuzzy (K, E)-soft neighborhood system and L-fuzzy (K, E)-soft quasi-uniformity.

2. Preliminaries

Let $L = (L, \leq, \vee, \wedge, 0, 1)$ be a completely distributive lattice with the least element 0 and the greatest element 1 in L.

Definition 2.1. [8,10,22] A complete lattice $(L, \leq, \odot)$ is called a strictly two-sided commutative quantale (stsc-quantale, for short) iff it satisfies the following properties.

(L1) $(L, \odot)$ is a commutative semigroup,

(L2) $x = x \odot 1$, for each $x \in L$ and 1 is the universal upper bound,

(L3) $\odot$ is distributive over arbitrary joins, i.e. $(\bigvee_i x_i) \odot y = \bigvee_i (x_i \odot y)$.

There exists a binary operation $\rightarrow$ satisfying the following condition

$$x \rightarrow y = \bigvee \{z \in L | x \odot z \leq y\}.$$

In this paper, we always assume that $(L, \leq, \odot, \rightarrow, ^*)$ is a stsc-quantales with an order reversing involution * which is defined by $x^* = x \rightarrow 0$ unless otherwise specified.

Remark 2.2. Every completely distributive lattice $(L, \leq, \wedge, \vee, ^*)$ with order reversing involution * is a stsc-quantale $(L, \leq, \odot, ^*)$ with a strong negation * where $a^* = 1 - a$.

Lemma 2.3. (see [8,22]) For each $x, y, z, x_i, y_i, w \in L$, we have the following properties:

(1) $1 \to x = x$, $0 \odot x = 0$,

(2) If $y \le z$, then $x \odot y \le x \odot z$, $x \to y \le x \to z$ and $z \to x \le y \to x$,

(3) $x \le y$ iff $x \to y = 1$,

(4) $(\bigwedge_i y_i)^* = \bigvee_i y_i^*$, $(\bigvee_i y_i)^* = \bigwedge_i y_i^*$,

(5) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$,

(6) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$,

(7) $x \to (\bigvee_i y_i) \ge \bigvee_i (x \to y_i)$,

(8) $(\bigwedge_i x_i) \to y \ge \bigvee_i (x_i \to y)$,

(9) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$,

(10) $x \odot y = (x \to y^*)^*$,

(11) $(x \to y) \odot (z \to w) \le (x \odot z) \to (y \odot w)$,

(12) $x \to y \le (x \odot z) \to (y \odot z)$ and $(x \to y) \odot (y \to z) \le x \to z$,

(13) $x \to y = y^* \to x^*$, $y \to z \le (x \to y) \to (x \to z)$,

(14) $\bigvee_{i\in\Gamma} x_i \to \bigvee_{i\in\Gamma} y_i \ge \bigwedge_{i\in\Gamma}(x_i \to y_i)$ and $\bigwedge_{i\in\Gamma} x_i \to \bigwedge_{i\in\Gamma} y_i \ge \bigwedge_{i\in\Gamma}(x_i \to y_i)$.

Throughout this paper, X refers to an initial universe, E and K are the sets of all parameters for X, and L^X is the set of all L-fuzzy sets on X.

Definition 2.4. (see [4,7]) A map f is called an L- fuzzy soft set on X, where f is a mapping from E into L^X, i.e., $f_e := f(e)$ is an L- fuzzy set on X, for each $e \in E$. The family of all L- fuzzy soft sets on X is denoted by $(L^X)^E$. Let f and g be two L-fuzzy soft sets on X.

(1) f is an L-fuzzy soft subset of g and we write $f \sqsubseteq g$ if $f_e \le g_e$, for each $e \in E$. f and g are equal if $f \sqsubseteq g$ and $g \sqsubseteq f$.

(2) The intersection of f and g is an L- fuzzy soft set $h = f \sqcap g$, where $h_e = f_e \wedge g_e$, for each $e \in E$.

(3) The union of f and g is an L- fuzzy soft set $h = f \sqcup g$, where $h_e = f_e \vee g_e$, for each $e \in E$.

(4) An L- fuzzy soft set $h = f \odot g$ is defined as $h_e = f_e \odot g_e$, for each $e \in E$.

(5) The complement of an L- fuzzy soft sets on X is denoted by f^*, where $f^* : E \to L^X$ is a mapping given by $f^*_e = (f_e)^*$, for each $e \in E$.

(6) 0_X is defined if $(0_X)_e(x) = 0$, for each $e \in E$, $x \in X$.

(6) 1_X is defined if $(1_X)_e(x) = 1$, for each $e \in E$, $x \in X$.

Definition 2.5. (see [4]) Let $\varphi : X \to Y$ and $\psi : E \to K$ be two mappings, where E and K are parameters sets for the crisp sets X and Y, respectively. Then $\varphi_\psi : (X, E) \to (Y, K)$ is called a fuzzy soft mapping. Let f and g be two fuzzy soft sets over X and Y, respectively and let φ_ψ be a fuzzy soft mapping from (X, E) into (Y, K).

(1) The image of f under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi(f)$ is the fuzzy soft set on Y defined by

$$\varphi(f)_b(y) = \bigvee_{\varphi(x)=y} \Big(\bigvee_{\psi(e)=b} f_e(x) \Big).$$

(2) The pre-image of g under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi^{-1}(g)$ is the fuzzy soft set on X defined by

$$\varphi_\psi^{-1}(g)_e(x) = g_{\psi(e)}(\varphi(x)), \quad \forall e \in E, \forall x \in X.$$

Definition 2.6. (see [19]) A mapping $\mathcal{T} : K \to L^{(L^X)^E}$ (where $\mathcal{T}_k := \mathcal{T}(k) : (L^X)^E \to L$ is a mapping for each $k \in K$) is called an L-fuzzy (K, E)-soft topology on X if it satisfies the following conditions for each $k \in K$.

(O1) $\mathcal{T}_k(0_X) = \mathcal{T}_k(1_X) = 1$,

(O2) $\mathcal{T}_k(f \odot g) \geq \mathcal{T}_k(f) \odot \mathcal{T}_k(g)$ $\forall\, f, g \in (L^X)^E$,

(O3) $\mathcal{T}_k(\bigsqcup_i f_i) \geq \bigwedge_{i \in I} \mathcal{T}_k(f_i)$ $\forall\, f_i \in (L^X)^E$, $i \in I$.

The pair $(X, \mathcal{T})$ is called an L-fuzzy (K, E)-soft topological space.
An L-fuzzy (K, E)-soft topology is called enriched if

(SR) $\mathcal{T}_k(\alpha \odot f) \geq \mathcal{T}_k(f)$ for all $f \in (L^X)^E$ and $\alpha \in L$.

Let $(X, \mathcal{T}^1)$ be an L-fuzzy (K_1, E_1)-soft topological space and $(Y, \mathcal{T}^2)$ be an L-fuzzy (K_2, E_2)-soft topological space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and

$\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from $(X, \mathcal{T}^1)$ into $(Y, \mathcal{T}^2)$ is called L-fuzzy soft continuous if

$$\mathcal{T}^2_{\eta(k)}(f) \leq \mathcal{T}^1_k(\varphi^{-1}_{\psi,\eta}(f)) \ \ \forall\ f \in (L^Y)^{E_2}, k \in K_1.$$

Lemma 2.7. (see [18]) *Define a binary mapping $S : (L^X)^E \times (L^X)^E \to L$ by*

$$S(f, g) = \bigwedge_{x\in X} \bigwedge_{e\in E} (f_e(x) \to g_e(x)) \ \ \forall\ f, g \in (L^X)^E, \ \ \forall\ e \in E.$$

Then $\forall f, g, h, m, n \in (L^X)^E$ the following statements hold:

(1) $f \sqsubseteq g$ iff $S(f, g) = 1$.

(2) If $f \sqsubseteq g$, then $S(h, f) \leq S(h, g)$ and $S(f, h) \geq S(g, h)$.

(3) $S(f, h) \odot S(h, g) \leq S(f, g)$. Moreover, $\bigvee_{h\in(L^X)^E}(S(f, h) \odot S(h, g)) = S(f, g)$.

(4) $S(f, g) \odot S(m, n) \leq S(f \odot m, g \odot n)$.

(5) If $\varphi_\psi : (X, E) \to (Y, F)$ is a fuzzy soft mapping, then $S(p, q) \leq S(\varphi^{-1}_\psi(p), \varphi^{-1}_\psi(q))$, for each $p, q \in (L^Y)^F$.

Definition 2.8. [19] An L- fuzzy (K, E)-soft quasi- uniformity is a mapping $\mathcal{U} : K \to L^{(L^{X\times X})^E}$ which satisfies the following conditions.

(SU1) There exists $u \in (L^{X\times X})^E$ such that $\mathcal{U}_k(u) = 1$.

(SU2) If $v \sqsubseteq u$, then $\mathcal{U}_k(v) \leq \mathcal{U}_k(u)$.

(SU3) For every $u, v \in (L^{X\times X})^E, \mathcal{U}_k(u \odot v) \geq \mathcal{U}_k(u) \odot \mathcal{U}_k(v)$.

(SU4) If $\mathcal{U}_k(u) \neq 0$ then $1_\triangle \sqsubseteq u$ where, for each $e \in E$,

$$(1_\triangle)_e(x, y) = \begin{cases} 1, & \text{if } x = y, \\ 0, & \text{if } x \neq y. \end{cases}$$

(SU5) $\mathcal{U}_k(u) \leq \bigvee\{\mathcal{U}_k(v) \mid v \circ v \sqsubseteq u\}$, where

$$v_e \circ w_e(x, z) = \bigvee_{y\in X} v_e(x, y) \odot w_e(y, z),$$

The pair $(X,\mathcal{U})$ is called an L-fuzzy (K, E)-soft quasi-uniform space.

An L-fuzzy (K, E)-soft quasi-uniform space $(X,\mathcal{U})$ is said to be an L-fuzzy (K, E)-soft uniform space if

(U) $\mathcal{U}_k(u) \leq \mathcal{U}_k(u^s)$, where $(u^s)_e(x, y) = u_e(y, x)$ for each $k \in K$ and $u \in (L^{X\times X})^E$.

An L-fuzzy (K, E)-soft quasi-uniformity $\mathcal{U}$ on X is said to be stratified if

(SR) $\mathcal{U}_k(\alpha \odot u) \geq \alpha \odot \mathcal{U}_k(u), \quad \forall\ u \in (L^{X\times X})^E, \alpha\imath L$.

Let $(X,\mathcal{U}^1)$ be an L-fuzzy (K_1, E_1)-soft quasi-uniform space and $(Y,\mathcal{U}^2)$ be an L-fuzzy (K_2, E_2)-soft quasi-uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from $(X,\mathcal{U}^1)$ into $(Y,\mathcal{U}^2)$ is called L-fuzzy soft uniformly continuous if

$$\mathcal{U}^2_{\eta(k)}(v) \leq \mathcal{U}^1_k((\varphi \times \varphi)^{-1}_{\psi}(v)) \ \ \forall\ v \in (L^{Y\times Y})^{E_2}, k \in K_1.$$

Remark 2.9. Let $(X,\mathcal{U})$ be an L-fuzzy (K, E)-soft uniform space.

(1) By (SU1) and (SU2), we have $\mathcal{U}_k(1_{X\times X}) = 1$ because $u \sqsubseteq 1_{X\times X}$ for all $u \in (L^{X\times X})^E$.

(2) Since $\mathcal{U}_k(u) \leq \mathcal{U}_k(u^s) \leq \mathcal{U}_k((u^s)^s) = \mathcal{U}_k(u)$, then $\mathcal{U}_k(u) = \mathcal{U}_k(u^s)$.

3. Topological Properties of L-fuzzy (K, E)-Soft Uniform Spaces

Definition 3.1. [18] An L-fuzzy (K, E)-soft neighborhood system on X is a set $N = \{N^x \mid x \in X\}$ of mappings $N^x : K \to L^{(L^X)^E}$ such that for each $k \in K$:

(SN1) $N^x_k(1_X) = 1$ and $N^x_k(0_X) = 0$,

(SN2) $N^x_k(f \odot g) \geq N^x_k(f) \odot N_x(g)$ for each $f, g \in (L^X)^E$,

(SN3) If $f \sqsubseteq g$, then $N^x_k(f) \leq N^x_k(g)$,

(SN4) $N^x_k(f) \leq f_e(x)$ for all $f \in (L^X)^E$. (here $N^x(k) =: N^x_k : (L^X)^E \to L$).

An L-fuzzy (K, E)-soft neighborhood system is called stratified, if

(SR) $N^x_k(\alpha \odot f) \geq \alpha \odot N^x_k(f)$ for all $f \in (L^X)^E$ and $\alpha \in L$.

The pair (X,N) is called an L-fuzzy (K,E)-soft neighborhood space.

Let (X,N) be an L-fuzzy (K_1,E_1)-soft neighborhood space and (Y,M) be an L-fuzzy (K_2,E_2)-soft neighborhood space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from (X,N) into (Y,M) is called L-fuzzy soft continuous at every $x \in X$ if $M^{\phi(x)}_{\eta(k)}(f) \leq N^x_k(\varphi^{-1}_{\psi,\eta}(f)) \quad \forall\ f \in (L^Y)^{E_2}, k \in K_1$.

Theorem 3.2. Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft uniform space. Then the following properties hold:

(1) Define a map $N^{\mathcal{U}} : K \to L^{(L^X)^E}$ by

$$(N^x_k)^{\mathcal{U}}(f) = \bigvee_u \mathcal{U}_k(u) \odot S(u[x],f), \quad \forall\ f \in (L^X)^E,\ x \in X,$$

where $u_e[x](y) = u_e(y,x)$. Then $(X,N^{\mathcal{U}})$ is an L-fuzzy (K,E)-soft neighborhood space.

(2) Define a map $\mathcal{U}^s : K \to L^{(L^{L^{X\times X}})^E}$ by

$$\mathcal{U}^s(u) = \mathcal{U}(u^s), \quad \forall\ u \in (L^{X\times X})^E,\ x \in X,$$

where $u^s_e(x,y) = u_e(y,x)$. Then $(X,N^{\mathcal{U}^s})$ is an L-fuzzy (K,E)-soft neighborhood space.

(3) If $\mathcal{U}$ is stratified, then $N^{\mathcal{U}}$ is also stratified.

Proof. (1) (SN1) For $\mathcal{U}_k(u) \neq 0,\ 1_\triangle \sqsubseteq u$. Then

$$\begin{aligned}(N^{\mathcal{U}})^x_k(0_X) &= \bigvee_u \mathcal{U}_k(u) \odot S(u[x],0_X)\\ &\leq \bigvee_u \big(\mathcal{U}_k(u) \odot (u_e(x,x) \to 0)\big)\\ &= \bigvee_u \big(\mathcal{U}_k(u) \odot ((1_\triangle)_e(x,x) \to 0)\big) = 0.\end{aligned}$$

Hence $(N^{\mathcal{U}})^x_k(0_X) = 0$. Also, $(N^{\mathcal{U}})^x_k(1_X) = 1$, because

$$(N^{\mathcal{U}})^x_k(1_X) \geq \mathcal{U}_k(1_{X\times X}) \odot \bigwedge_{e\in E}\bigwedge_{y\in X}((1_{X\times X})_e(x,y) \to (1_X)_e(y)) = 1.$$

(SN2) By Lemma 2.7(4), we have

$$\begin{aligned}
&(N^{\mathcal{U}})^x_k(f) \odot (N^{\mathcal{U}})^x_k(g)\\
&= \big(\bigvee_u \mathcal{U}_k(u) \odot S(u[x], f)\big) \odot \big(\bigvee_u \mathcal{U}_k(v) \odot S(v[x], g)\big)\\
&= \bigvee_{u,v} \mathcal{U}_k(u) \odot \mathcal{U}_k(v) \odot S(u[x], f) \odot S(v[x], g)\\
&\leq \bigvee_{u,v} \mathcal{U}_k(u \odot v) \odot S((u \odot v)[x], f \odot g)\\
&\leq \bigvee_{w} \mathcal{U}_k(w) \odot S(w[x], f \odot g) = (N^{\mathcal{U}})^x_k(f \odot g).
\end{aligned}$$

(SN3) By Lemma 2.7 (2), we have

$$\begin{aligned}
(N^{\mathcal{U}})^x_k(f) &= \bigvee_u \mathcal{U}_k(u) \odot S(u[x], f)\\
&\leq \bigvee_u \mathcal{U}_k(u) \odot S(u[x], g) = (N^{\mathcal{U}})^x_k(g).
\end{aligned}$$

(SN4) For $\mathcal{U}_k(u) \neq 0$, $1_\triangle \sqsubseteq u$.

$$\begin{aligned}
(N^{\mathcal{U}})^x_k(f) &= \bigvee_u \mathcal{U}_k(u) \odot \bigwedge_{e\in E} \bigwedge_{y\in X} (u_e(y,x) \to f_e(y))\\
&\leq \bigvee_u \big\{\mathcal{U}_k(u) \odot (u_e(x,x) \to f_e(x))\big\} \leq f_e(x).
\end{aligned}$$

Hence $(X, N^{\mathcal{U}})$ is an L-fuzzy (K,E)-soft neighborhood space.

(2) (SU1) Since there exists $u \in (L^{X\times X})^E$ such that $\mathcal{U}_k(u) = 1 = \mathcal{U}^s_k(u^s)$.

(SU2)-(SU4) are easily proved.

(SU5) $\mathcal{U}^s_k(u) = \mathcal{U}^s_k(u^s) \leq \bigvee\{\mathcal{U}_k(v^s) \mid v^s \circ v^s \sqsubseteq u^s\} = \bigvee\{\mathcal{U}^s_k(v) \mid v \circ v \sqsubseteq u\}$.
(3)

$$\begin{aligned}
&\alpha \odot (N^{\mathcal{U}})^x_k(f) = \alpha \odot \bigvee_u \mathcal{U}_k(u) \odot S(u[x], f)\\
&= \bigvee_u \alpha \odot \mathcal{U}_k(u) \odot S(\alpha,\alpha) \odot S(u[x], f)
\end{aligned}$$

$$\leq \bigvee_u \mathcal{U}_k(\alpha \odot u) \odot S(\alpha \odot u[x], \alpha \odot f) \leq (N^{\mathcal{U}})^x_k(\alpha \odot f).$$

Theorem 3.3. *Let $(X,\mathcal{U})$ be an L-fuzzy (K_1, E_1)-soft uniform space and $(Y,\mathcal{V})$ be an L-fuzzy (K_2, E_2)-soft uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If $\varphi_{\psi,\eta} : (X,\mathcal{U}) \to (Y,\mathcal{V})$ is L-fuzzy soft uniformly continuous, then $\varphi_{\psi,\eta} : (X, N^{\mathcal{U}}) \to (Y, N^{\mathcal{V}})$ is L-fuzzy soft continuous.*

Proof. We have $\varphi^{-1}_{\psi,\eta}(v_{\psi(e)}[\varphi_{\psi,\eta}(x)]) = (\varphi_{\psi,\eta} \times \varphi_{\psi,\eta})^{-1}(v_{\psi(e)})[x]$ from

$$\begin{aligned}
\varphi^{-1}_{\psi,\eta}(v_{\psi(e)}[\varphi_{\psi,\eta}(x)])(z) =& v_{\psi(e)}[\varphi_{\psi,\eta}(x)](\varphi_{\psi,\eta}(z))\\
=& v_{\psi(e)}(\varphi_{\psi,\eta}(z), \varphi_{\psi,\eta}(x))\\
=& (\varphi_{\psi,\eta} \times \varphi_{\psi,\eta})^{-1}(v_{\psi(e)})(z, x)\\
=& (\varphi_{\psi,\eta} \times \varphi_{\psi,\eta})^{-1}(v)_e[x](z).
\end{aligned}$$

Thus, by Lemma 2.7(4,5), we have

$$\begin{aligned}
S(v[\varphi_{\psi,\eta}(x)], f) &\leq S(\varphi^{-1}_{\psi,\eta}(v[\varphi_{\psi,\eta}(x)]), \varphi^{-1}_{\psi,\eta}(f))\\
&= S((\varphi_{\psi,\eta} \times \varphi_{\psi,\eta})^{-1}(v)[x], \varphi^{-1}_{\psi,\eta}(f)).
\end{aligned}$$

$$\begin{aligned}
(N^{\mathcal{V}})^{\varphi_{\psi,\eta}(x)}_{\eta(k)}(f) =& \bigvee_v \mathcal{V}_{\eta(k)}(v) \odot S(v[\varphi_{\psi,\eta}(x)], f)\\
\leq& \bigvee_v \mathcal{V}_{\eta(k)}(v) \odot S((\varphi_{\psi,\eta} \times \varphi_{\psi,\eta})^{-1}(v)[x], \varphi^{-1}_{\psi,\eta}(f))\\
\leq& \bigvee_u \mathcal{U}_k((\varphi_{\psi,\eta} \times \varphi_{\psi,\eta})^{-1}(v)) \odot S((\varphi_{\psi,\eta} \times \varphi_{\psi,\eta})^{-1}(v)[x], \varphi^{-1}_{\psi,\eta}(f))\\
\leq& (N^{\mathcal{U}})^x_k(\varphi^{-1}_{\psi,\eta}(f)).
\end{aligned}$$

Theorem 3.4. *(1) The L-fuzzy (K, E) soft neighborhood system $N^{\mathcal{U}} = \{(N^x_k)^{\mathcal{U}} \mid x \in X, k \in K\}$ can be constructed from the cuts $(\mathcal{U}_k)_\alpha$, $\alpha > 0$, of L-fuzzy (K, E) soft uniformity by using of the equality*

$$(N^x_k)^{\mathcal{U}}(f) = \bigvee_{\alpha \in L} \alpha \odot (N^x_k)^{\mathcal{U}}(f, \alpha),$$

where $(N_k^x)^{\mathcal{U}}(f,\alpha)$ *is defined from*

$$(N_k^x)^{\mathcal{U}}(f,\alpha)=\bigvee_v\{S(u[x],f)\mid \mathcal{U}_k(u)\geq\alpha\}.$$

(2)

$$(N_k^x)^{\mathcal{U}}(f)\leq\bigvee_v\left\{(N_k^x)^{\mathcal{U}}(g)\mid g_e(z)\leq(N_k^z)^{\mathcal{U}}(f,\mathcal{U}(v))\right\}.$$

Proof. (1) It is similar to proof of Theorem 3.4 in [17].

(2) For $u\in(L^{X\times X})^E$ and $f\in(L^X)^E$, we have

$$\begin{aligned}(N_k^x)^{\mathcal{U}}(f)&=\bigvee_u\mathcal{U}_k(u)\odot S(u[x],f)\\&=\bigvee_u\Big\{\mathcal{U}_k(u)\odot\bigwedge_{e\in E}\bigwedge_{y\in X}(u_e(y,x)\to f_e(y))\Big\}\\&\leq\bigvee_v\Big\{\mathcal{U}_k(v)\odot\bigwedge_{e\in E}\bigwedge_{y\in X}((v_e\circ v_e)(y,x)\to f_e(y))\Big\}\\&=\bigvee_v\Big\{\mathcal{U}_k(v)\odot\bigwedge_{e\in E}\bigwedge_{y\in X}((\bigvee_{z\in X}v_e(z,x)\odot v_e(y,z))\to\lambda(y))\Big\}\\&=\bigvee_v\Big\{\mathcal{U}_k(v)\odot\bigwedge_{e\in E}\bigwedge_{y\in X}\bigwedge_{z\in X}((v_e(z,x)\odot v_e(y,z))\to f_e(y))\Big\}\\&=\bigvee_v\Big\{\mathcal{U}_k(v)\odot\bigwedge_{e\in E}\bigwedge_{y\in X}\bigwedge_{z\in X}(v_e(z,x)\to(v_e(y,z)\to f_e(y))\Big\}\\&=\bigvee_v\Big\{\mathcal{U}_k(v)\odot\bigwedge_{e\in E}\bigwedge_{y\in X}\bigwedge_{z\in X}(v_e(z,x)\to(v_e(y,z))\to f_e(y))\Big\}\\&=\bigvee_v\Big\{\mathcal{U}_k(v)\odot\bigwedge_{e\in E}\bigwedge_{z\in X}(v_e(z,x)\to\bigwedge_{y\in X}(v_e(y,z)\to f_e(y))\Big\}.\end{aligned}$$

Let us set $g_e(z)=\bigwedge_{y\in X}(v_e(y,z)\to f_e(y))$. Then $g_e(z)\leq(N_k^z)^{\mathcal{U}}(f,\mathcal{U}(v))$ for all $z\in X$. Thus:

$$\begin{aligned}(N_k^x)^{\mathcal{U}}(f)&\leq\bigvee_v\Big\{\mathcal{U}_k(v)\odot\bigwedge_{e\in E}\bigwedge_{z\in X}(v_e(z,x)\to g_e(z))\mid g_e(z)\leq(N_k^z)^{\mathcal{U}}(f,\mathcal{U}_k(v))\Big\}\\&\leq\bigvee_v\Big\{\mathcal{U}_k(v)\odot\bigwedge_{z\in X}(v_e(z,x)\to g_e(z))\mid g_e(z)\leq(N_k^z)^{\mathcal{U}}(f,\mathcal{U}_k(v))\Big\}\end{aligned}$$

$$\leq \bigvee_{v} \{(N_k^x)^{\mathcal{U}}(g) \mid g_e(z) \leq (N_k^z)^{\mathcal{U}}(f, \mathcal{U}_k(v))\}.$$

Theorem 3.5. *Let (X, N) be a L-fuzzy (K, E) soft neighborhood space. Define a map $\mathcal{T}^N : K \to L^{(L^X)^E}$ by*

$$\mathcal{T}_k^N(f) = \bigwedge_{x \in X} \bigwedge_{e \in E} (f_e(x) \to (N_k^x)(f)).$$

Then:

(1) $\mathcal{T}^N$ is an L-fuzzy (K, E) soft topology on X,

(2) If N is stratified, then $\mathcal{T}^N$ is an enriched L-fuzzy (K, E) soft topology.

Proof. (SO1)

$$\mathcal{T}_k^N(1_X) = \bigwedge_{x \in X} \bigwedge_{e \in E} ((1_X)_e(x) \to N_k^x(1_X) = 1 \to 1 = 1,$$
$$\mathcal{T}_k^N(0_X) = \bigwedge_{x \in X} \bigwedge_{e \in E} ((0_X)_e(x) \to N_k^x(0_X)) = 0 \to 0.$$

(SO2) By Lemma 2.3 (11), we have

$$\begin{aligned} \mathcal{T}_k^N(f \odot g) &= \bigwedge_{x \in X} \bigwedge_{e \in E} ((f_e \odot g_e)(x) \to N_k^x(f \odot g)) \\ &\geq \bigwedge_{x \in X} \bigwedge_{e \in E} ((f_e \odot g_e)(x) \to (N_k^x(f) \odot N_k^x(g))) \\ &\geq \bigwedge_{x \in X} \bigwedge_{e \in E} (f_e(x) \to N_k^x(f)) \odot \bigwedge_{x \in X} \bigwedge_{e \in E} (g_e(x) \to N_k^x(g)) \\ &= \mathcal{T}_k^N(f) \odot \mathcal{T}_k^N(g). \end{aligned}$$

(SO3)

$$\begin{aligned} \mathcal{T}_k^N(\bigsqcup_i f_i) &= \bigwedge_{x \in X} \bigwedge_{e \in E} ((\bigsqcup_i (f_i)_e)(x) \to N_k^x(\bigsqcup_i f_i)) \\ &\geq \bigwedge_{x \in X} \bigwedge_{e \in E} ((\bigsqcup_i (f_i)_e)(x) \to \bigvee_i N_k^x(f_i))(x)) \end{aligned}$$

(by Lemma 2.3 (14))

$$\geq \bigwedge_i \bigwedge_{x\in X} \bigwedge_{e\in E} ((f_i)_e)(x) \to N_k^x(f_i)) = \bigwedge_i \mathcal{T}_k^N(f_i).$$

(SR) By Lemma 2.3 (12), we have

$$\begin{aligned}
\mathcal{T}_k^N(\alpha \odot f) &= \bigwedge_{x\in X} \bigwedge_{e\in E} ((\alpha \odot f_e)(x) \to N_k^x(\alpha \odot f)) \\
&\geq \bigwedge_{x\in X} \bigwedge_{e\in E} ((\alpha \odot f_e)(x) \to (\alpha \odot N_k^x(f))) \\
&\geq \bigwedge_{x\in X} \bigwedge_{e\in E} (f_e(x) \to N_k^x(f)) = \mathcal{T}_k^N(f).
\end{aligned}$$

Corollary 3.6. *Let Let $(X,\mathcal{U})$ be an L-fuzzy (K,E) soft uniform space and $N^{\mathcal{U}} = \{N_x^{\mathcal{U}} \mid x \in X\}$ be an L-fuzzy (K,E) soft neighborhood system on X. Define a map $\mathcal{T}^{\mathcal{U}} : K \to L^{(L^X)^E}$ by*

$$\mathcal{T}^{\mathcal{U}}(f) = \bigwedge_{x\in X} (f_e(x) \to (N_k^x)^{\mathcal{U}}(f)).$$

Then:

(1) $\mathcal{T}^{\mathcal{U}}$ is an L-fuzzy (K,E) soft topology on X,

(2) If $N^{\mathcal{U}}$ is stratified, then $\mathcal{T}^{\mathcal{U}}$ is an enriched L-fuzzy topology.

Theorem 3.7. *Let $(X,\mathcal{U})$ be an L-fuzzy (K_1,E_1)-soft uniform space and $(Y,\mathcal{V})$ be an L-fuzzy (K_2,E_2)-soft uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. If a map $\varphi_{\psi,\eta} : (X,\mathcal{U}) \to (Y,\mathcal{V})$ is L-fuzzy soft uniformly continuous, then a map $\varphi_{\psi,\eta} : (X,\mathcal{T}^{\mathcal{U}}) \to (Y,\mathcal{T}^{\mathcal{V}})$ is L-fuzzy soft continuous.*

Proof. Obviously

$$\begin{aligned}
&\mathcal{T}_{\eta(k)}^{\mathcal{V}}(f) \to \mathcal{T}_k^{\mathcal{U}}(\varphi_{\psi,\eta}^{-1}(f)) \\
&= \bigwedge_{y\in Y} (f_e(y) \to (N^{\mathcal{V}})_{\eta(k)}^y(f)) \to \bigwedge_{x\in X} (\varphi_{\psi,\eta}^{-1}(f))(x) \to (N^{\mathcal{U}})_k^x(\varphi_{\psi,\eta}^{-1}(f))) \\
&\geq \bigwedge_{x\in X} (\varphi_{\psi,\eta}^{-1}(f)(x) \to (N^{\mathcal{V}})_{\eta(k)}^{\varphi_{\psi,\eta}(x)}(f)) \to \bigwedge_{x\in X} (\varphi_{\psi,\eta}^{-1}(f)(x) \to (N^{\mathcal{U}})_k^x(\varphi_{\psi,\eta}^{-1}(f)))
\end{aligned}$$

$$\geq \bigwedge_{x\in X}\Big((\varphi^{-1}_{\psi,\eta}(f)(x) \to (N^{\mathcal{V}})^{\varphi_{\psi,\eta}(x)}_{\eta(k)}(f)) \to (\varphi^{-1}_{\psi,\eta}(f)(x) \to (N^{\mathcal{U}})^{x}_{k}(\varphi^{-1}_{\psi,\eta}(f)))\Big)$$

(by Lemma 2.3 (13,14))

$$\geq \bigwedge_{x\in X}\Big(((N^{\mathcal{V}})^{\varphi_{\psi,\eta}(x)}_{\eta(k)}(f)) \to (N^{\mathcal{U}})^{x}_{k}(\varphi^{-1}_{\psi,\eta}(f)))\Big).$$

Thus, if $(N^{\varphi_{\psi,\eta}(x)}_{\eta(k)})^{\mathcal{V}}(f) \leq (N^{x}_{k})^{\mathcal{U}}(\varphi^{-1}_{\psi,\eta}(f))$, then $\mathcal{T}^{\mathcal{V}}_{\eta(k)}(f) \leq \mathcal{T}^{\mathcal{U}}_{k}(\varphi^{-1}_{\psi,\eta}(f))$. So, $\varphi_{\psi,\eta}$ is L-fuzzy soft continuous.

Example 3.9. Let $X = \{h_i \mid i = \{1,2,3\}\}$ with h_i=house and $E = \{e,b\}$ with e=expensive, b= beautiful. Define a binary operation $\odot$ on $[0,1]$ by

$$x \odot y = \max\{0, x+y-1\},\ x \to y = \min\{1-x+y, 1\}$$

Then $([0,1], \wedge, \to, 0, 1)$ is a stsc-quantle.

(1) Put $v, v\odot v, w \in ([0,1]^{X\times X})^E$ as

$$v_e = \begin{pmatrix} 1 & 0.6 & 0.5 \\ 0.3 & 1 & 0.5 \\ 0.4 & 0.6 & 1 \end{pmatrix},\ v_b = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.7 & 1 & 0.5 \\ 0.6 & 0.6 & 1 \end{pmatrix}$$

$$(v\odot v)_e = \begin{pmatrix} 1 & 0.2 & 0 \\ 0 & 1 & 0 \\ 0 & 0.2 & 1 \end{pmatrix},\ (v\odot v)_b = \begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 1 & 0 \\ 0.2 & 0.2 & 1 \end{pmatrix}$$

$$w_e = \begin{pmatrix} 1 & 0.4 & 0.5 \\ 0.4 & 1 & 0.5 \\ 0.4 & 0.6 & 1 \end{pmatrix},\ w_b = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.3 & 1 & 0.5 \\ 0.2 & 0.3 & 1 \end{pmatrix}$$

We define $\mathcal{U} : K = \{k_1, k_2\} \to [0,1]^{([0,1]^{X\times X})^E}$ as follows:

$$\mathcal{U}_{k_1}(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.6, & \text{if } v \sqsubseteq u \neq 1_{Y\times Y}, \\ 0.3, & \text{if } v\odot v \sqsubseteq u \not\sqsupseteq v, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{U}_{k_2}(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.5, & \text{if } w \sqsubseteq u \neq 1_{Y\times Y}, \\ 0, & \text{otherwise.} \end{cases}$$

We obtain $\mathcal{U}^s : K = \{k_1, k_2\} \to [0,1]^{([0,1]^{X\times X})^E}$ as follows:

$$\mathcal{U}^s_{k_1}(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.6, & \text{if } v^t \sqsubseteq u \neq 1_{Y\times Y}, \\ 0.3, & \text{if } (v\odot v)^t \sqsubseteq u \not\sqsupseteq v, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{U}^s_{k_2}(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y} \\ 0.5, & \text{if } w^t \sqsubseteq u \neq 1_{Y\times Y}, \\ 0, & \text{otherwise.} \end{cases}$$

Where $v^t_e(x,y) = v_e(y,x), \forall v \in \{v, v\odot v, w\}, e \in E$.

(2) We obtain a $[0,1]$-fuzzy (K,E)-soft neighborhood system on X as: Since $(N^{\mathcal{U}})^x_k(f) = \bigvee_u \mathcal{U}_k(u) \odot S(u[x], f)$, we have

$$\begin{aligned}(N^{\mathcal{U}})^{h_1}_{k_1}(f) &= f_e(h_1)\wedge f_e(h_2)\wedge f_e(h_3)\wedge f_b(h_1)\wedge f_b(h_2)\wedge f_b(h_2) \\ &\vee\Big(0.6\odot(f_e(h_1)\wedge(0.3\to f_e(h_2))\wedge(0.4\to f_e(h_3))) \\ &\wedge(f_b(h_1)\wedge(0.7\to f_b(h_2))\wedge(0.6\to f_b(h_3)))\Big) \\ &\vee\Big(0.3\odot(f_e(h_1)\wedge(f_b(h_1)\wedge(0.4\to f_b(h_2))\wedge(0.2\to f_b(h_3)))\Big),\end{aligned}$$

$$\begin{aligned}(N^{\mathcal{U}^s})^{h_1}_{k_1}(f) &= f_e(h_1)\wedge f_e(h_2)\wedge f_e(h_3)\wedge f_b(h_1)\wedge f_b(h_2)\wedge f_b(h_3) \\ &\vee\Big(0.6\odot(f_e(h_1)\wedge(0.6\to f_e(h_2))\wedge(0.5\to f_e(h_3))) \\ &\wedge(f_b(h_1)\wedge(0.5\to f_b(h_2))\wedge(0.3\to f_b(h_3)))\Big) \\ &\vee\Big(0.3\odot(f_e(h_1)\wedge(0.2\to f_e(h_2))\wedge f_b(h_1))\Big),\end{aligned}$$

$$\begin{aligned}(N^{\mathcal{U}})^{h_1}_{k_2}(f) &= f_e(h_1)\wedge f_e(h_2)\wedge f_e(h_3)\wedge f_b(h_1)\wedge f_b(h_2)\wedge f_b(h_2) \\ &\vee\Big(0.5\odot(f_e(h_1)\wedge(0.4\to f_e(h_2))\wedge(0.4\to f_e(h_3))) \\ &\wedge(f_b(h_1)\wedge(0.5\to f_b(h_2))\wedge(0.3\to f_b(h_3)))\Big),\end{aligned}$$

$$\begin{aligned}(N^{\mathcal{U}^s})^{h_1}_{k_2}(f) &= f_e(h_1)\wedge f_e(h_2)\wedge f_e(h_3)\wedge f_b(h_1)\wedge f_b(h_2)\wedge f_b(h_3) \\ &\vee\Big(0.5\odot(f_e(h_1)\wedge(0.4\to f_e(h_2))\wedge(0.5\to f_e(h_3))) \\ &\wedge(f_b(h_1)\wedge(0.5\to f_b(h_2))\wedge(0.3\to f_b(h_3)))\Big).\end{aligned}$$

Put $f \in (L^X)^E$ as

$$\begin{aligned} &f_e(h_1) = 0.9, \quad f_e(h_2) = 0.1, \quad f_e(h_3) = 0.6, \\ &f_b(h_1) = 0.8, \quad f_b(h_2) = 0.4, \quad f_b(h_3) = 0.5. \end{aligned}$$

Then:

$$
\begin{array}{llll}
(N^{\mathcal{U}})^{h_1}_{k_1}(f)=0.3, & (N^{\mathcal{U}^s})^{h_1}_{k_1}(f)=0.1, & (N^{\mathcal{U}})^{h_2}_{k_1}(f)=0.1, & (N^{\mathcal{U}^s})^{h_2}_{k_1}(f)=0.1,\\
(N^{\mathcal{U}})^{h_3}_{k_1}(f)=0.1, & (N^{\mathcal{U}^s})^{h_3}_{k_1}(f)=0.1, & (N^{\mathcal{U}})^{h_3}_{k_1}(f)=0.1, & (N^{\mathcal{U}^s})^{h_3}_{k_1}(f)=0.1,\\
(N^{\mathcal{U}})^{h_1}_{k_2}(f)=0.2, & (N^{\mathcal{U}^s})^{h_1}_{k_2}(f)=0.2, & (N^{\mathcal{U}})^{h_2}_{k_2}(f)=0.1, & (N^{\mathcal{U}^s})^{h_2}_{k_2}(f)=0.1,\\
(N^{\mathcal{U}})^{h_3}_{k_2}(f)=0.1, & (N^{\mathcal{U}^s})^{h_3}_{k_2}(f)=0.1. & &
\end{array}
$$

Since $\mathcal{T}^{\mathcal{U}}_k(f)=\bigwedge_{x\in X}(\bigvee_{e\in E} f_e(x)\to (N^{\mathcal{U}})^x_k(f))$, we have

$$
\begin{array}{l}
\mathcal{T}^{\mathcal{U}}_{k_1}(f)=(0.9\to 0.3)\wedge(0.4\to 0.1)\wedge(0.6\to 0.1)=0.4,\\
\mathcal{T}^{\mathcal{U}}_{k_2}(f)=(0.9\to 0.2)\wedge(0.4\to 0.1)\wedge(0.6\to 0.1)=0.3,\\
\mathcal{T}^{\mathcal{U}^s}_{k_1}(f)=(0.9\to 0.1)\wedge(0.4\to 0.1)\wedge(0.6\to 0.1)=0.2,\\
\mathcal{T}^{\mathcal{U}^s}_{k_2}(f)=(0.9\to 0.2)\wedge(0.4\to 0.1)\wedge(0.6\to 0.1)=0.3.
\end{array}
$$

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