International Journal of Pure and Applied Mathematics
**Volume 110 No. 4 2016, 623-643**
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
**url:** http://www.ijpam.eu
**doi:** 10.12732/ijpam.v110i4.5
ijpam.eu

# SOME COMMON RANDOM FIXED POINT THEOREMS OF COMPATIBLE MAPPINGS OF VARIOUS TYPES FOR RANDOM MAPPINGS IN MULTIPLICATIVE METRIC SPACES

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**Abstract:** In this paper first, we introduce the notions of compatible mappings of various types in framework of random fixed points in multiplicative metric spaces and using these notions we prove common random fixed point theorems for these mappings.

**AMS Subject Classification:** 47H10, 54H25

**Key Words:** multiplicative metric spaces, random fixed points, compatible mappings of type $(A)$, of type $(B)$, of type $(C)$, of type $(P)$, random mappings

## 1. Introduction and Preliminaries

It is well know that the set of positive real numbers $\mathbb{R}_+$ is not complete according to the usual metric. To overcome this problem, in 2008, Bashirov et al. [1]

Received: August 31, 2016
Revised: October 5, 2016
Published: November 9, 2016



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studied the multiplicative calculus and defined a new distance so called multiplicative distance. By using this idea, Özavsar and Çevikel [3] introduced the concept of multiplicative metric spaces and studied some topological properties in such spaces.

**Definition 1.1.** ([1]) Let $X$ be a nonempty set. A multiplicative metric is a mapping $d : X \times X \to \mathbb{R}_+$ satisfying the following conditions:

(i) $d(x, y) \geq 1$ for all $x, y \in X$ and $d(x, y) = 1$ if and only if $x = y$;

(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(iii) $d(x, y) \leq d(x, z) \cdot d(z, y)$ for all $x, y, z \in X$ (multiplicative triangle inequality).

Then the mapping $d$ together with $X$, that is, $(X, d)$ is a multiplicative metric space.

**Example 1.2.** ([3]) Let $\mathbb{R}^n_+$ be the collection of all $n$-tuples of positive real numbers. Let $d^* : \mathbb{R}^n_+ \times \mathbb{R}^n_+ \to \mathbb{R}$ be defined as follows:

$$d^*(x, y) = \left|\frac{x_1}{y_1}\right|^* \cdot \left|\frac{x_2}{y_2}\right|^* \cdots \left|\frac{x_n}{y_n}\right|^*,$$

where $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n) \in \mathbb{R}^n_+$ and $|\cdot|^* : \mathbb{R}_+ \to \mathbb{R}_+$ is defined by

$$|a|^* = \begin{cases} a & \text{if } a \geq 1, \\ \frac{1}{a} & \text{if } a < 1. \end{cases}$$

Then it is obvious that all conditions of a multiplicative metric are satisfied. Therefore $(\mathbb{R}^n_+, d^*)$ is a multiplicative metric space.

**Remark 1.3.** ([6]) We note that multiplicative metrics and metric spaces are independent.

One can refer to [3] for detailed multiplicative metric topology.

**Definition 1.4.** Let $(X, d)$ be a multiplicative metric space. Then a sequence $\{x_n\}$ in $X$ is said to be

(1) a *multiplicative convergent* to $x$ if for every multiplicative open ball $B_\epsilon(x) = \{y \mid d(x, y) < \epsilon\}$, $\epsilon > 1$, there exists $N \in \mathbb{N}$ such that $x_n \in B_\epsilon(x)$ for all $n \geq N$, that is, $d(x_n, x) \to 1$ as $n \to \infty$.

(2) a *multiplicative Cauchy sequence* if for all $\epsilon > 1$, there exists $N \in \mathbb{N}$ such that $d(x_n, x_m) < \epsilon$ for all $m, n \geq N$, that is, $d(x_n, x_m) \to 1$ as $n, m \to \infty$.

(3) We call a multiplicative metric space *complete* if every multiplicative Cauchy sequence in it is multiplicative convergent to $x \in X$.

In 2012, Özavsar and Çevikel [3] gave the concept of multiplicative contraction mapping and proved some fixed point theorem of such mappings on a complete multiplicative metric spaces.

**Definition 1.5.** Let $f$ be a mapping of a multiplicative metric space $(X, d)$ into itself. Then $f$ is said to be a *multiplicative contraction* if there exists a real number $\lambda \in [0, 1)$ such that

$$d(fx, fy) \leq d^{\lambda}(x, y) \quad \text{for all } x, y \in X.$$

## 2. Relationships and Properties of Compatible Mappings and its Variants for Random Mappings

Now we introduce the following concepts.

**Definition 2.1.** Let $(X, d)$ be a multiplicative metric space and $F : \mathbb{R} \times X \to X$ be a mapping, where $X$ is a nonempty set. Then a mapping $g : \mathbb{R} \to X$ is said to be a *random fixed point* of the mapping $F$ if $F(t, g(t)) = g(t)$ for all $t \in \mathbb{R}$.

**Example 2.2.** Let $X = \mathbb{R}$. Define $T : \mathbb{R} \times X \to X$ by $T(t, x) = \frac{t+5x}{6}$ and a mapping $g : \mathbb{R} \to X$ defined by $g(t) = t$ for every $t \in \mathbb{R}$. Then $T$ has a unique random fixed point in $X$

Next we introduce the notions of compatible mappings and its variants for random mappings in multiplicative metric spaces as follows:

**Definition 2.3.** Let $(X, d)$ be a multiplicative metric space. Two mappings $A$ and $B : \mathbb{R} \times X \to X$ are called

(1) *compatible* for each $t \in \mathbb{R}$ ([4]) if

$$\lim_{n\to\infty} d(A(t, B(t, g_n(t))), B(t, A(t, g_n(t)))) = 1,$$

whenever $\lim_{n\to\infty} A(t, g_n(t)) = \lim_{n\to\infty} B(t, g_n(t))$, $t \in \mathbb{R}$, where $\{g_n\}$ is a sequence of mappings;

(2) *compatible* of type $(A)$ for each $t \in \mathbb{R}$ if

$$\lim_{n\to\infty} d(A(t, B(t, g_n(t))), B(t, B(t, g_n(t)))) = 1$$

and

$$\lim_{n\to\infty} d(B(t, A(t, g_n(t))), A(t, A(t, g_n(t)))) = 1,$$

whenever $\lim_{n\to\infty} A(t, g_n(t)) = \lim_{n\to\infty} B(t, g_n(t))$, $t \in \mathbb{R}$, where $\{g_n\}$ is a sequence of mappings;

(3) *compatible* of type $(B)$ for each $t \in \mathbb{R}$ if

$$\begin{aligned}&\lim_{n\to\infty} d(A(t, B(t, g_n(t))), B(t, B(t, g_n(t))))\\ &\le \Big[\lim_{n\to\infty} d(A(t, B(t, g_n(t))), A(t, g(t)))\\ &\qquad \cdot \lim_{n\to\infty} d(A(t, g(t)), A(t, A(t, g_n(t))))\Big]^{1/2}\end{aligned}$$

and

$$\begin{aligned}&\lim_{n\to\infty} d(B(t, A(t, g_n(t))), A(t, A(t, g_n(t))))\\ &\le \Big[\lim_{n\to\infty} d(B(t, A(t, g_n(t))), B(t, g(t))))\\ &\qquad \cdot \lim_{n\to\infty} d(B(t, g(t)), B(t, B(t, g_n(t))))\Big]^{1/2},\end{aligned}$$

whenever $\lim_{n\to\infty} A(t, g_n(t)) = \lim_{n\to\infty} B(t, g_n(t)) = g(t)$, $t \in \mathbb{R}$, where $\{g_n\}$ is a sequence of mappings;

(4) *compatible* of type $(C)$ for each $t \in \mathbb{R}$ if

$$\begin{aligned}&\lim_{n\to\infty} d(A(t, B(t, g_n(t))), B(t, B(t, g_n(t))))\\ &\le \Big[\lim_{n\to\infty} d(A(t, B(t, g_n(t))), A(t, g(t)))\\ &\qquad \cdot \lim_{n\to\infty} d(A(t, g(t)), A(t, A(t, g_n(t))))\\ &\qquad \cdot \lim_{n\to\infty} d(A(t, g(t)), B(t, B(t, g_n(t))))\Big]^{1/3}\end{aligned}$$

and

$$\begin{aligned}&\lim_{n\to\infty} d(B(t, A(t, g_n(t))), A(t, A(t, g_n(t))))\\ &\le \Big[\lim_{n\to\infty} d(B(t, A(t, g_n(t))), B(t, g(t)))\\ &\qquad \cdot \lim_{n\to\infty} d(B(t, g(t)), B(t, B(t, g_n(t))))\\ &\qquad \cdot \lim_{n\to\infty} d(B(t, g(t)), A(t, A(t, g_n(t))))\Big]^{1/3},\end{aligned}$$

whenever $\lim_{n\to\infty} A(t, g_n(t)) = \lim_{n\to\infty} B(t, g_n(t)) = g(t)$, $t \in \mathbb{R}$, where $\{g_n\}$ is a sequence of mappings;

(5) *compatible* of type $(P)$ for each $t \in \mathbb{R}$ if

$$\lim_{n\to\infty} d(A(t, A(t, g_n(t))), B(t, B(t, g_n(t)))) = 1,$$

whenever $\lim_{n\to\infty} A(t, g_n(t)) = \lim_{n\to\infty} B(t, g_n(t))$, $t \in \mathbb{R}$, where $\{g_n\}$ is a sequence of mappings.

**Definition 2.4.** ([5]) Let $(X, d)$ be a multiplicative metric space and $A$ and $B : \mathbb{R} \times X \to X$ be mappings. Then the mapping $A$ are called *jointly continuous* for each $t \in \mathbb{R}$ if

$$\lim_{n\to\infty} A(t, A(t, g_n(t))) = \lim_{n\to\infty} A(t, B(t, g_n(t))) = A(t, g(t)),$$

whenever $\lim_{n\to\infty} A(t, g_n(t)) = \lim_{n\to\infty} B(t, g_n(t)) = g(t) \in X$, $t \in \mathbb{R}$, where $\{g_n\}$ is a sequence of mappings.

Finally we give the relationships and properties of compatible mappings and its variants for random mappings.

**Proposition 2.5.** *Let $(X, d)$ be a multiplicative metric space and $A$ and $B : \mathbb{R}\times X \to X$ be mappings. Assume that $A$ and $B$ are compatible mappings of type $(A)$. If one of $A$ and $B$ is jointly continuous, then $A$ and $B$ are compatible.*

*Proof.* Since $A$ and $B$ are compatible of type $(A)$, so

$$\lim_{n\to\infty} d(A(t, B(t, g_n(t))), B(t, B(t, g_n(t)))) = 1$$

and

$$\lim_{n\to\infty} d(A(t, A(t, g_n(t))), B(t, A(t, g_n(t)))) = 1,$$

whenever $\lim_{n\to\infty} A(t, g_n(t)) = \lim_{n\to\infty} B(t, g_n(t)) = g(t)$, $t \in \mathbb{R}$, where $\{g_n\}$ is a sequences of mappings.

Suppose that $A$ is jointly continuous. Then

$$\lim_{n\to\infty} A(t, A(t, g_n(t))) = \lim_{n\to\infty} A(t, B(t, g_n(t))) = A(t, g(t)), \quad t \in \mathbb{R}.$$

Now we get

$$\lim_{n\to\infty} d(A(t, B(t, g_n(t))), B(t, A(t, g_n(t)))) = 1,$$

that is, $A$ and $B$ are compatible mappings.

Similarly, if $B$ is jointly continuous, then $A$ and $B$ are compatible mappings. □

**Proposition 2.6.** *Every pair of compatible mappings of type $(A)$ is compatible of type $(B)$.*

*Proof.* Suppose that $A$ and $B$ are compatible mappings of type $(A)$. Then we have

$$\begin{aligned} 1 &= \lim_{n\to\infty} d(A(t,B(t,g_n(t))),B(t,B(t,g_n(t)))) \\ &\le \Big[\lim_{n\to\infty} d(A(t,B(t,g_n(t))),A(t,g(t))) \\ &\qquad \cdot \lim_{n\to\infty} d(A(t,g(t)),A(t,A(t,g_n(t))))\Big]^{1/2} \end{aligned}$$

and

$$\begin{aligned} 1 &= \lim_{n\to\infty} d(B(t,A(t,g_n(t))),A(t,A(t,g_n(t)))) \\ &\le \Big[\lim_{n\to\infty} d(B(t,A(t,g_n(t))),B(t,g(t))) \\ &\qquad \cdot \lim_{n\to\infty} d(B(t,g(t)),B(t,B(t,g_n(t))))\Big]^{1/2} \end{aligned}$$

as derived. □

**Proposition 2.7.** *Let $(X,d)$ be a multiplicative metric space and $A$ and $B:\mathbb{R}\times X\to X$ be jointly continuous. If $A$ and $B$ are compatible of type $(B)$, then they are compatible of type $(A)$.*

*Proof.* Let $\{g_n\}$ be a sequence in $X$ such that

$$\lim_{n\to\infty} A(t,g_n(t)) = \lim_{n\to\infty} B(t,g_n(t)) = g(t), \quad t\in\mathbb{R}.$$

Since $A$ and $B$ are jointly continuous, we have

$$\begin{aligned} &\lim_{n\to\infty} d(A(t,B(t,g_n(t))),B(t,B(t,g_n(t)))) \\ &\le \Big[\lim_{n\to\infty} d(A(t,B(t,g_n(t))),A(t,g(t))) \\ &\qquad \cdot \lim_{n\to\infty} d(A(t,g(t)),A(t,A(t,g_n(t))))\Big]^{1/2} \\ &= 1 \end{aligned}$$

and

$$\begin{aligned} &\lim_{n\to\infty} d(B(t,A(t,g_n(t))),A(t,A(t,g_n(t)))) \\ &\le \Big[\lim_{n\to\infty} d(B(t,A(t,g_n(t))),B(t,g(t))) \\ &\qquad \cdot \lim_{n\to\infty} d(B(t,g(t)),B(t,B(t,g_n(t))))\Big]^{1/2} \\ &= 1. \end{aligned}$$

Therefore, $A$ and $B$ compatible of type $(A)$. This completes the proof. □

**Proposition 2.8.** *Let $(X, d)$ be a multiplicative metric space and $A$ and $B : \mathbb{R} \times X \to X$ be jointly continuous. If $A$ and $B$ are compatible of type $(B)$, then they are compatible.*

*Proof.* Let $\{g_n\}$ be a sequence in $X$ such that

$$\lim_{n\to\infty} A(t, g_n(t)) = \lim_{n\to\infty} B(t, g_n(t)) = g(t), \quad t \in \mathbb{R}.$$

Since $A$ and $B$ are jointly continuous, we have

$$\lim_{n\to\infty} A(t, A(t, g_n(t)) = \lim_{n\to\infty} A(t, B(t, g_n(t)) = A(t, g(t))$$

and

$$\lim_{n\to\infty} B(t, B(t, g_n(t)) = \lim_{n\to\infty} B(t, A(t, g_n(t)) = B(t, g(t)).$$

By multiplicative triangle inequality, we have

$$\begin{aligned} &d(A(t, B(t, g_n(t))), B(t, A(t, g_n(t)))) \\ &\le d(A(t, B(t, g_n(t))), B(t, B(t, g_n(t)))) \\ &\qquad \cdot d(B(t, B(t, g_n(t))), B(t, A(t, g_n(t)))). \end{aligned}$$

Letting $n \to \infty$, we have

$$\begin{aligned} &\lim_{n\to\infty} d(A(t, B(t, g_n(t))), B(t, A(t, g_n(t)))) \\ &\le \Big[\lim_{n\to\infty} d(A(t, B(t, g_n(t))), A(t, g(t))) \\ &\qquad \cdot \lim_{n\to\infty} d(A(t, g(t)), A(t, A(t, g_n(t))))\Big]^{1/2} \\ &\qquad \cdot \lim_{n\to\infty} d(B(t, B(t, g_n(t))), B(t, A(t, g_n(t)))) \\ &= 1. \end{aligned}$$

Therefore, $A$ and $B$ are compatible. This completes the proof. □

**Proposition 2.9.** *Let $(X, d)$ be a multiplicative metric space and $A$ and $B : \mathbb{R} \times X \to X$ be jointly continuous. If $A$ and $B$ are compatible, then they are compatible of type $(B)$.*

*Proof.* Since $A$ and $B$ are compatible so there exists $\{g_n\}$ be a sequence in $X$ such that $\lim_{n\to\infty} A(t, g_n(t)) = \lim_{n\to\infty} B(t, g_n(t)) = g(t)$, $t \in \mathbb{R}$ for which

$$\lim_{n\to\infty} d(A(t, A(t, g_n(t))), B(t, A(t, g_n(t)))) = 1.$$

Since $A$ and $B$ are jointly continuous, we have

$$\lim_{n\to\infty} A(t, A(t, g_n(t))) = \lim_{n\to\infty} A(t, B(t, g_n(t))) = A(t, g(t))$$

and

$$\lim_{n\to\infty} B(t, B(t, g_n(t))) = \lim_{n\to\infty} B(t, A(t, g_n(t))) = B(t, g(t)),$$

so

$$\begin{aligned}\lim_{n\to\infty} A(t, A(t, g_n(t))) &= \lim_{n\to\infty} A(t, B(t, g_n(t))) \\ &= \lim_{n\to\infty} B(t, A(t, g_n(t))) = \lim_{n\to\infty} B(t, B(t, g_n(t))).\end{aligned}$$

Now

$$\begin{aligned}&\lim_{n\to\infty} d(A(t, B(t, g_n(t))), B(t, B(t, g_n(t)))) \\ &\leq \Big[\lim_{n\to\infty} d(A(t, B(t, g_n(t))), A(t, g(t))) \\ &\qquad \cdot \lim_{n\to\infty} d(A(t, g(t)), A(t, A(t, g_n(t))))\Big]^{1/2}\end{aligned}$$

and

$$\begin{aligned}&\lim_{n\to\infty} d(B(t, A(t, g_n(t))), A(t, A(t, g_n(t)))) \\ &\leq \Big[\lim_{n\to\infty} d(B(t, A(t, g_n(t))), B(t, g(t))) \\ &\qquad \cdot \lim_{n\to\infty} d(B(t, g(t)), B(t, B(t, g_n(t))))\Big]^{1/2},\end{aligned}$$

which implies that $A$ and $B$ be compatible of type $(B)$. □

**Proposition 2.10.** *Let $(X, d)$ be a multiplicative metric space and $A$ and $B : \mathbb{R} \times X \to X$ be jointly continuous. Then*

*$(a)$ $A$ and $B$ are compatible if and only if they are compatible of type $(B)$;*

*$(b)$ $A$ and $B$ are compatible of type $(A)$ if and only if they are compatible of type $(B)$.*

*Proof.* $(a)$ One can easily prove it using Propositions 2.8 and 2.9.

$(b)$ One can easily prove it using Propositions 2.6 and 2.7. □

**Proposition 2.11.** *Let $(X, d)$ be a multiplicative metric space and $A$ and $B : \mathbb{R} \times X \to X$ be mappings. Assume that $A$ and $B$ are compatible mappings of type $(B)$. If $A(t, g(t)) = B(t, g(t))$ for some $g(t) \in X$, then*

$$A(t, B(t, g(t))) = A(t, A(t, g(t))) = B(t, B(t, g(t))) = B(t, A(t, g(t))).$$

*Proof.* Suppose that $\{g_n\}$ is a sequence in $X$ defined by $g_n(t) = g(t)$, $n = 1, 2, \ldots$, for some $t \in \mathbb{R}$ and $A(t, g(t)) = B(t, g(t)) = z(t)$, say. Then we have $A(t, g_n(t))$, $B(t, g_n(t)) \to A(t, g(t))$ as $n \to \infty$. Since $A$ and $B$ are compatible of type $(B)$, we have

$$\begin{aligned}
& d(A(t, B(t, g(t))), B(t, B(t, g(t)))) \\
& = \lim_{n\to\infty} d(A(t, B(t, g_n(t))), B(t, B(t, g_n(t)))) \\
& \leq \Big[ \lim_{n\to\infty} d(A(t, B(t, g_n(t))), A(t, A(t, g(t)))) \\
& \qquad \cdot \lim_{n\to\infty} d(A(t, A(t, g(t))), A(t, A(t, g_n(t))))\Big]^{1/2} \\
& = d(A(t, z(t)), A(t, z(t))) \\
& = 1.
\end{aligned}$$

Hence we have $A(t, B(t, g(t)) = B(t, B(t, g(t))$. Therefore, since $A(t, g(t)) = B(t, g(t))$, we have

$$A(t, B(t, g(t))) = A(t, A(t, g(t))) = B(t, B(t, g(t))) = B(t, A(t, g(t))).$$

This completes the proof. □

From Proposition 2.11, we have

**Proposition 2.12.** *Let $(X, d)$ be a multiplicative metric space and $A$ and $B : \mathbb{R} \times X \to X$ be mappings. Assume that $\lim_{n\to\infty} A(t, g_n(t)) = \lim_{n\to\infty} B(t, g_n(t)) = g(t)$, $t \in \mathbb{R}$. Then*

*$(a)$ $\lim_{n\to\infty} B(t, B(t, g_n(t))) = A(t, g(t))$ if $A$ is jointly continuous.*

*$(b)$ $\lim_{n\to\infty} A(t, A(t, g_n(t))) = B(t, g(t))$ if $B$ is jointly continuous.*

*$(c)$ $A(t, B(t, g(t))) = B(t, A(t, g(t)))$ and $A(t, g(t)) = B(t, g(t))$ if $A$ and $B$ are jointly continuous.*

*Proof.* $(a)$ Suppose that $A$ is jointly continuous. Since $\lim_{n\to\infty} A(t, g_n(t)) = \lim_{n\to\infty} B(t, g_n(t)) = g(t)$, $t \in \mathbb{R}$, we have $A(t, A(t, g_n(t)))$, $A(t, B(t, g_n(t))) \to$

$A(t, g(t))$ as $n \to \infty$. Since $A$ and $B$ are compatible of type $(B)$, we have

$$\begin{aligned}
&\lim_{n\to\infty} d(A(t, g(t)), B(t, B(t, g_n(t)))) \\
&= \lim_{n\to\infty} d(A(t, B(t, g_n(t))), B(t, B(t, g_n(t)))) \\
&\leq \Big[ \lim_{n\to\infty} d(A(t, B(t, g_n(t))), A(t, g(t))) \\
&\qquad \cdot \lim_{n\to\infty} d(A(t, g(t)), A(t, A(t, g_n(t))))\Big]^{1/2} \\
&= d(A(t, g(t)), A(t, g(t))) \\
&= 1.
\end{aligned}$$

Therefore, $\lim_{n\to\infty} B(t, B(t, g_n(t))) = A(t, g(t))$. This completes the proof.

$(b)$ The proof of $\lim_{n\to\infty} A(t, A(t, g_n(t))) = B(t, g(t))$ follows by similar arguments as in $(a)$.

$(c)$ Suppose that $A$ and $B$ are jointly continuous. Since $B(t, g_n(t)) \to g(t)$ as $n \to \infty$ and $A$ is jointly continuous, by $(a)$, $B(t, B(t, g_n(t))) \to A(t, g(t))$ as $n \to \infty$. On the other hand, $B$ is also jointly continuous and $B(t, B(t, g_n(t))) \to B(t, g(t))$. Thus, we have $A(t, g(t)) = B(t, g(t))$ by the uniqueness of limit and so by Proposition 2.11, $A(t, B(t, g(t))) = B(t, A(t, g(t)))$. This completes the proof. □

**Remark 2.13.** In Proposition 2.11, assume that $A$ and $B$ be compatible mappings of type $(C)$ or of type $(P)$ instead of type $(B)$, the conclusion of Proposition 2.11 still holds.

**Remark 2.14.** In Proposition 2.12, assume that $A$ and $B$ be compatible mappings of type $(C)$ or of type $(P)$ instead of of type $(B)$, the conclusion of Proposition 2.12 still holds.

## 3. Random Fixed Points for Compatible Mappings of Various Types

Now, we give the random fixed point theorem of compatible mappings of type $(A)$ for random mappings.

**Theorem 3.1.** *Let $(X, d)$ be a complete multiplicative metric space and $A, B, S$ and $T : \mathbb{R} \times X \to X$ be mappings satisfying the following conditions;*

$(C_1)$ $\qquad S(t, X) \subset B(t, X) \quad$ *and* $\quad T(t, X) \subset A(t, X);$

$$
(C_2)\qquad \begin{aligned}
&d(S(t,x(t)),T(t,y(t)))\\
&\leq [\max\{d(A(t,x(t)),B(t,y(t))),d(A(t,x(t)),S(t,x(t))),\\
&\quad d(B(t,y(t)),T(t,y(t))),d(S(t,x(t)),B(t,y(t))),\\
&\quad d(A(t,x(t)),T(t,y(t)))\}]^{\lambda}
\end{aligned}
$$

*for all $x, y \in X$ and $t \in \mathbb{R}$, where $\lambda \in (0, 1/2)$;*

$(C_3)$ *one of $A, B, S$ and $T$ is jointly continuous.*

*Assume that the pairs $A, S$ and $B, T$ are compatible of type $(A)$. Then $A, B, S$ and $T$ have a unique common random fixed point.*

*Proof.* Suppose that $A$ is jointly continuous. Since $A$ and $S$ be compatible of type $(A)$, by Proposition 2.5, $A$ and $S$ be compatible so results easily follows from [5, Theorem 3.1].

Similarly, we can complete the proof when $B$ or $S$ or $T$ is jointly continuous. This complete the proof. □

Next, we give the random fixed point theorem of compatible mappings of type $(B)$ for random mappings.

**Theorem 3.2.** *Let $(X, d)$ be a complete multiplicative metric space and $A, B, S$ and $T : \mathbb{R} \times X \to X$ be mappings satisfying the conditions $(C_1)$-$(C_3)$.*

*Assume that the pairs $A, S$ and $B, T$ are compatible of type $(B)$. Then $A, B, S$ and $T$ have a unique common random fixed point.*

*Proof.* Let $g_0 : \mathbb{R} \to X$ be an arbitrary mapping. By $(C_1)$, there exists $g_1 : \mathbb{R} \to X$ such that $B(t, g_1(t)) = S(t, g_0(t))$, $t \in \mathbb{R}$ and for this $g_1 : \mathbb{R} \to X$ we can choose $g_2 : \mathbb{R} \to X$ such that $T(t, g_1(t)) = A(t, g_2(t))$, $t \in \mathbb{R}$, and so on. By the method of induction we can define a sequence $\{y_n(t)\}$, $t \in \mathbb{R}$, of mappings as follows:

$$
\begin{aligned}
y_{2n+1}(t) &= B(t, g_{2n+1}(t)) = S(t, g_{2n}(t)),\\
y_{2n}(t) &= A(t, g_{2n}(t)) = T(t, g_{2n-1}(t)), \quad t \in \mathbb{R},
\end{aligned}
$$

$n = 0, 1, 2, \ldots$. From the proof of [5, Theorem 3.1], $\{y_n(t)\}$ is a multiplicative Cauchy sequence and since $X$ is complete so $\{y_n(t)\}$ converges to a point $z(t) \in X$ as $n \to \infty$. Also subsequences of $\{y_n(t)\}$ also converges to a point $z(t) \in X$, that is

$$
\begin{aligned}
&B(t, g_{2n+1}(t)) \to z(t), \quad S(t, g_{2n}(t)) \to z(t),\\
&A(t, g_{2n}(t)) \to z(t), \quad T(t, g_{2n+1}(t)) \to z(t), \quad t \in \mathbb{R}
\end{aligned}
\tag{3.1}
$$

as $n \to \infty$.

Now suppose that $S$ is jointly continuous. Then from (3.1), we have

$$S(t, S(t, g_{2n}(t))) \to S(t, z(t)), \quad S(t, A(t, g_{2n}(t))) \to S(t, z(t))$$

as $n \to \infty$. Since $A$ and $S$ are compatible of type $(B)$, it follows from Proposition 2.12 that

$$A(t, A(t, g_{2n}(t))) \to S(t, z(t))$$

as $n \to \infty$.

Now putting $x(t) = A(t, g_{2n}(t))$ and $y(t) = g_{2n+1}(t)$ in $(C_2)$, we have

$$\begin{aligned}
&d(S(t, A(t, g_{2n}(t))), T(t, g_{2n+1}(t))) \\
&\le [\max\{d(A(t, A(t, g_{2n}(t))), B(t, g_{2n+1}(t))), \\
&\quad d(A(t, A(t, g_{2n}(t))), S(t, A(t, g_{2n}(t)))), \\
&\quad d(B(t, g_{2n+1}(t)), T(t, g_{2n+1}(t))), d(S(t, A(t, g_{2n}(t))), B(t, g_{2n+1}(t))), \\
&\quad d(A(t, A(t, g_{2n}(t))), T(t, g_{2n+1}(t)))\}]^{\lambda}.
\end{aligned}$$

Letting $n \to \infty$, we have

$$\begin{aligned}
&d(S(t, z(t)), z(t)) \\
&\le [\max\{d(S(t, z(t)), z(t)), 1, 1, d(S(t, z(t)), z(t)), d(S(t, z(t)), z(t))\}]^{\lambda} \\
&= d^{\lambda}(S(t, z(t)), z(t)).
\end{aligned}$$

This implies that $S(t, z(t)) = z(t)$. Since $z(t) = S(t, z(t)) \in S(t, X) \subset B(t, X)$, there exists $u(t) \in X$ such that $z(t) = B(t, u(t))$, $t \in \mathbb{R}$.

Putting $x(t) = A(t, g_{2n}(t))$ and $y(t) = u(t)$ in $(C_2)$, we have

$$\begin{aligned}
&d(S(t, A(t, g_{2n}(t)), T(t, u(t))) \\
&\le [\max\{d(A(t, A(t, g_{2n}(t))), B(t, u(t))), \\
&\quad d(A(t, A(t, g_{2n}(t))), S(t, A(t, g_{2n}(t)))), \\
&\quad d(B(t, u(t)), T(t, u(t))), d(S(t, A(t, g_{2n}(t))), B(t, u(t))), \\
&\quad d(A(t, A(t, g_{2n}(t))), T(t, u(t)))\}]^{\lambda}.
\end{aligned}$$

Letting $n \to \infty$, we obtain

$$\begin{aligned}
&d(S(t, z(t)), T(t, u(t))) \\
&\le \max\{d(S(t, z(t)), z(t)), d(S(t, z(t)), S(t, z(t))), \\
&\quad d(z(t), T(t, u(t))), d(S(t, z(t)), z(t)), d(S(t, z(t)), T(t, u(t)))\}]^{\lambda} \\
&= d^{\lambda}(z(t), T(t, u(t))).
\end{aligned}$$

This implies that $d(z(t), T(t, u(t)) = 1$ and hence $T(t, u(t)) = z(t)$. Since $B$ and $T$ are compatible of type $(B)$ and $B(t, u(t)) = T(t, u(t)) = z(t)$, by Proposition 2.11, we have

$$B(t, z(t)) = B(t, T(t, u(t)) = T(t, B(t, u(t)) = T(t, z(t)).$$

Putting $x(t) = g_{2n}(t)$ and $y(t) = z(t)$ in $(C_2)$, we have

$$\begin{aligned}
&d(S(t, g_{2n}(t)), T(t, z(t)) \\
&\leq [\max\{d(A(t, g_{2n}(t)), B(t, z(t))), d(A(t, g_{2n}(t)), S(t, g_{2n}(t))), \\
&\quad d(B(t, z(t)), T(t, z(t))), d(S(t, g_{2n}(t)), B(t, z(t))), \\
&\quad d(A(t, g_{2n}(t)), T(t, z(t)))\}]^{\lambda}.
\end{aligned}$$

Letting $n \to \infty$, we obtain

$$\begin{aligned}
&d(z(t), T(t, z(t)) \\
&= [\max\{d(z(t), T(t, z(t)), d(z(t), z(t)), d(T(t, z(t)), T(t, z(t))), \\
&\quad d(z(t), T(t, z(t))), d(z(t), T(t, z(t)))\}]^{\lambda} \\
&= d^{\lambda}(z(t), T(t, z(t))).
\end{aligned}$$

This implies that $d(z(t), T(t, z(t))) = 1$ and hence $T(t, z(t)) = z(t)$. Since $T(t, X) \subset A(t, X)$, so there exists a point $v(t) \in X$ such that $z(t) = T(t, z(t)) = A(t, v(t))$.

Putting $x(t) = v(t)$ and $y(t) = z(t)$ in $(C_2)$, we have

$$\begin{aligned}
&d(S(t, v(t)), z(t)) \\
&= d(S(t, v(t)), T(t, z(t))) \\
&\leq [\max\{d(A(t, v(t)), B(t, z(t))), d(A(t, v(t)), S(t, v(t))), \\
&\quad d(B(t, z(t)), T(t, z(t))), d(S(t, v(t)), B(t, z(t))), \\
&\quad d(A(t, v(t)), T(t, z(t)))\}]^{\lambda} \\
&= [\max\{1, d(z(t), S(t, v(t))), 1, d(S(t, v(t)), z(t)), 1\}]^{\lambda}.
\end{aligned}$$

This implies that $S(t, v(t)) = z(t)$. Since $A$ and $S$ are compatible $(B)$ and $S(t, v(t)) = A(t, v(t)) = z(t)$, it follows from Proposition 2.11 that $A(t, z(t)) = A(t, S(t, v(t)) = S(t, A(t, v(t)) = S(t, z(t))$. Hence $A(t, z(t)) = S(t, z(t)) = B(t, z(t)) = T(t, z(t)) = z(t)$. Therefore $z(t)$ is a common random fixed point of $A, B, S$ and $T$.

Similarly, we can complete the proof when $T$ is jointly continuous.

Next, suppose that $A$ is jointly continuous,

$$A(t, S(t, g_{2n}(t))) \to A(t, z(t)), \quad A(t, A(t, g_{2n2}(t))) \to A(t, z(t)).$$

Since $A$ and $S$ are compatible of type $(B)$, it follows from Proposition 2.12 that

$$S(t, S(t, g_{2n}(t))) \to A(t, z(t)) \quad \text{as } n \to \infty.$$

Now, we claim that $z(t) = A(t, z(t))$.

Putting $x(t) = S(t, g_{2n}(t))$ and $y(t) = g_{2n+1}(t)$ in $(C_2)$, we have

$$\begin{aligned} &d(S(t, S(t, g_{2n}(t))), T(t, g_{2n+1}(t))) \\ &\le [\max\{d(A(t, S(t, g_{2n}(t))), B(t, g_{2n+1}(t))), \\ &\quad d(A(t, S(t, g_{2n}(t))), S(t, S(t, g_{2n}(t)))), \\ &\quad d(B(t, g_{2n+1}(t)), T(t, g_{2n+1}(t))), d(S(t, S(t, g_{2n}(t))), B(t, g_{2n+1}(t))), \\ &\quad d(A(t, S(t, g_{2n}(t))), T(t, g_{2n+1}(t)))\}]^{\lambda}. \end{aligned}$$

Letting $n \to \infty$, we have

$$\begin{aligned} &d(A(t, z(t)), z(t)) \\ &\le [\max\{d(A(t, z(t)), z(t)), 1, 1, d(A(t, z(t)), z(t)), d(A(t, z(t)), z(t))\}]^{\lambda}. \end{aligned}$$

This implies that $d(A(t, z(t)), z(t)) = 1$ and hence $A(t, z(t)) = z(t)$.

Next we claim that $z(t) = S(t, z(t))$.

Putting $x(t) = z(t)$ and $y(t) = g_{2n+1}(t)$ in $(C_2)$, we have

$$\begin{aligned} &d(S(t, z(t)), T(t, g_{2n+1}(t))) \\ &\le [\max\{d(A(t, z(t)), B(t, g_{2n+1}(t))), d(A(t, z(t)), S(t, z(t))), \\ &\quad d(B(t, g_{2n+1}(t)), T(t, g_{2n+1}(t))), d(S(t, z(t)), B(t, g_{2n+1}(t))), \\ &\quad d(A(t, z(t)), T(t, g_{2n+1}(t)))\}]^{\lambda}. \end{aligned}$$

Letting $n \to \infty$, we have

$$\begin{aligned} &d(S(t, z(t)), z(t)) \\ &\le [\max\{d(z(t), z(t)), d(z(t), S(t, z(t))), 1, \\ &\quad d(S(t, z(t)), z(t)), d(z(t), z(t))\}]^{\lambda}. \end{aligned}$$

This implies that $d(S(t, z(t)), z(t)) = 1$ and hence $S(t, z(t)) = z(t)$. Since $z(t) = S(t, z(t)) \in S(t, X) \subset B(t, X)$, there exists $w(t) \in X$ such that $z(t) = B(t, w(t))$, $t \in \mathbb{R}$.

Putting $x(t) = z(t)$ and $y(t) = w(t)$ in $(C_2)$, we have

$$\begin{aligned}
&d(z(t), T(t, w(t))) \\
&= d(S(t, z(t)), T(t, w(t))) \\
&\leq [\max\{d(A(t, z(t)), B(t, w(t))), d(A(t, z(t)), S(t, z(t))), \\
&\qquad d(B(t, w(t)), T(t, w(t))), d(S(t, z(t)), B(t, w(t))), \\
&\qquad d(A(t, z(t)), T(t, w(t)))\}]^{\lambda} \\
&= [\max\{1, 1, d(z(t), T(t, w(t))), 1, d(S(t, z(t)), T(t, w(t)))\}]^{\lambda} \\
&= d^{\lambda}(z(t), T(t, w(t))).
\end{aligned}$$

This implies that $d(z(t), T(t, w(t)) = 1$ and hence $T(t, w(t)) = z(t)$. Since $B$ and $T$ are compatible of type $(B)$ and $B(t, w(t)) = T(t, w(t)) = z(t)$, by Proposition 2.11, we have

$$B(t, z(t)) = B(t, T(t, w(t)) = T(t, B(t, w(t)) = T(t, z(t)).$$

Also, putting $x(t) = z(t)$ and $y(t) = z(t)$ in $(C_2)$, we have

$$\begin{aligned}
&d(z(t), B(t, z(t))) \\
&= d(S(t, z(t)), T(t, z(t))) \\
&\leq [\max\{d(A(t, z(t)), B(t, z(t))), d(A(t, z(t)), S(t, z(t))), \\
&\qquad d(B(t, z(t)), T(t, z(t))), d(S(t, z(t)), B(t, z(t))), \\
&\qquad d(A(t, z(t)), T(t, z(t)))\}]^{\lambda} \\
&= [\max\{d(z(t), B(t, z(t))), 1, 1, d(z(t), B(t, z(t))), \\
&\qquad d(z(t), B(t, z(t)))\}]^{\lambda}.
\end{aligned}$$

This implies that $z(t) = B(t, z(t)) = T(t, z(t))$. Hence $A(t, z(t)) = S(t, z(t)) = B(t, z(t)) = T(t, z(t)) = z(t)$ Therefore $z(t)$ is a common random fixed point of $A, B, S$ and $T$.

Similarly, we can prove theorem when $B$ is jointly continuous.

Uniqueness follows easily from $(C_2)$. Therefore, $A, B, S$ and $T$ have a unique common fixed point. This completes the proof. □

Next, we give the random fixed point theorem of compatible mappings of type $(C)$ for random mappings.

**Theorem 3.3.** *Let $(X, d)$ be a complete multiplicative metric space and $A, B, S$ and $T : \mathbb{R} \times X \to X$ be mappings satisfying the conditions $(C_1)$-$(C_3)$.*

*Assume that the pairs $A, S$ and $B, T$ are compatible of type $(C)$. Then $A, B, S$ and $T$ have a unique common random fixed point.*

*Proof.* From the proof of Theorem 3.2, $\{y_n(t)\}$ is a multiplicative Cauchy sequence and since $X$ is complete so $\{y_n(t)\}$ converges to a point $z(t) \in X$ as $n \to \infty$. Also subsequences of $\{y_n(t)\}$ also converges to a point $z(t) \in X$, that is

$$B(t, g_{2n+1}(t)) \to z(t), \quad S(t, g_{2n}(t)) \to z(t),$$
$$A(t, g_{2n}(t)) \to z(t), \quad T(t, g_{2n+1}(t)) \to z(t), \quad t \in \mathbb{R}$$

as $n \to \infty$.

Suppose that $S$ is jointly continuous. Then we have

$$S(t, S(t, g_{2n}(t))) \to S(t, z(t)), \quad S(t, A(t, g_{2n}(t))) \to S(t, z(t))$$

as $n \to \infty$. Since $A$ and $S$ are compatible of type $(C)$, it follows from Remark 2.13 that

$$A(t, A(t, g_{2n}(t))) \to S(t, z(t)) \quad \text{as } n \to \infty.$$

Putting $x(t) = A(t, g_{2n}(t))$ and $y(t) = g_{2n+1}(t)$ in $(C_2)$), we have

$$\begin{aligned}
&d(S(t, A(t, g_{2n}(t))), T(t, g_{2n+1}(t))) \\
&\le [\max\{d(A(t, A(t, g_{2n}(t))), B(t, g_{2n+1}(t))), \\
&\quad d(A(t, A(t, g_{2n}(t))), S(t, A(t, g_{2n}(t)))), \\
&\quad d(B(t, g_{2n+1}(t)), T(t, g_{2n+1}(t))), d(S(t, A(t, g_{2n}(t))), B(t, g_{2n+1}(t))), \\
&\quad d(A(t, A(t, g_{2n}(t))), T(t, g_{2n+1}(t)))\}]^{\lambda}.
\end{aligned}$$

Letting $n \to \infty$, we have

$$\begin{aligned}
&d(S(t, z(t)), z(t)) \\
&\le [\max\{d(S(t, z(t)), z(t)), 1, 1, d(S(t, z(t)), z(t)), d(S(t, z(t)), z(t))\}]^{\lambda} \\
&= d^{\lambda}(S(t, z(t)), z(t)).
\end{aligned}$$

This implies that $d(S(t, z(t)), z(t)) = 1$ and hence $S(t, z(t)) = z(t)$. Since $z(t) = S(t, z(t)) \in S(t, X) \subset B(t, X)$, there exists $u(t) \in X$ such that $z(t) = B(t, u(t))$, $t \in \mathbb{R}$.

Putting $x(t) = A(t, g_{2n}(t))$ and $y(t) = u(t)$ in $(C_2)$, we have

$$\begin{aligned}
&d(S(t, A(t, g_{2n}(t))), T(t, u(t))) \\
&\le [\max\{d(A(t, A(t, g_{2n}(t))), B(t, u(t))), \\
&\quad d(A(t, A(t, g_{2n}(t))), S(t, A(t, g_{2n}(t)))), \\
&\quad d(B(t, u(t)), T(t, u(t))), d(S(t, A(t, g_{2n}(t))), B(t, u(t))), \\
&\quad d(A(t, A(t, g_{2n}(t))), T(t, u(t)))\}]^{\lambda}.
\end{aligned}$$

Letting $n \to \infty$, we obtain

$$\begin{aligned} & d(S(t, z(t)), T(t, u(t))) \\ & \leq [\max\{d(S(t, z(t)), z(t)), 1, d(z(t), T(t, u(t))), \\ & \quad d(S(t, z(t)), z(t)), d(S(t, z(t)), T(t, u(t)))\}]^{\lambda} \\ & = d^{\lambda}(z(t), T(t, u(t)))^{\lambda}. \end{aligned}$$

This implies that $d(z(t), T(t, u(t)) = 1$ and hence $T(t, u(t)) = z(t)$. Since $B$ and $T$ are compatible of type $(C)$ and $B(t, u(t)) = T(t, u(t)) = z(t)$, by Remark 2.13, we have

$$B(t, z(t)) = B(t, T(t, u(t)) = T(t, B(t, u(t)) = T(t, z(t)).$$

Putting $x(t) = g_{2n}(t)$ and $y(t) = z(t)$ in $(C_2)$, we have

$$\begin{aligned} & d(S(t, g_{2n}(t)), T(t, z(t))) \\ & \leq [\max\{d(A(t, g_{2n}(t)), B(t, z(t))), d(A(t, g_{2n}(t)), S(t, g_{2n}(t))), \\ & \quad d(B(t, z(t)), T(t, z(t))), d(S(t, g_{2n}(t)), B(t, z(t))), \\ & \quad d(A(t, g_{2n}(t)), T(t, z(t)))\}]^{\lambda}. \end{aligned}$$

Letting $n \to \infty$, we obtain

$$\begin{aligned} & d(z(t), T(t, z(t)) \\ & \leq [\max\{d(z(t), T(t, z(t)), 1, 1, d(z(t), T(t, z(t)), d(z(t), T(t, z(t)))\}]^{\lambda} \\ & = d^{\lambda}(z(t), T(t, z(t))). \end{aligned}$$

This implies that $d(z(t), T(t, z(t)) = 1$ and hence $T(t, z(t)) = z(t)$. Since $T(t, X) \subset A(t, X)$, there exists a point $v(t) \in X$ such that $z(t) = T(t, z(t)) = A(t, v(t))$.

Putting $x(t) = v(t)$ and $y(t) = z(t)$ in $(C_2)$, we have

$$\begin{aligned} & d(S(t, v(t)), z(t)) \\ & = d(S(t, v(t)), T(t, z(t))) \\ & \leq [\max\{d(A(t, v(t)), B(t, z(t))), d(A(t, v(t)), S(t, v(t))), \\ & \quad d(B(t, z(t)), T(t, z(t))), d(S(t, v(t)), B(t, z(t))), \\ & \quad d(A(t, v(t)), T(t, z(t)))\}]^{\lambda} \\ & = [\max\{d(z(t), z(t)), d(z(t), S(t, v(t))), 1, \\ & \quad d(S(t, v(t)), z(t)), d(z(t), z(t))\}]^{\lambda}. \end{aligned}$$

This implies that $S(t, v(t)) = z(t)$. Since $A$ and $S$ are compatible $(C)$ and $S(t, v(t)) = A(t, v(t)) = z(t)$, it follows from Remark 2.13 that

$$A(t, z(t)) = A(t, S(t, v(t)) = S(t, A(t, v(t)) = S(t, z(t)).$$

Hence

$$A(t, z(t)) = S(t, z(t)) = B(t, z(t)) = T(t, z(t)) = z(t).$$

Therefore $z(t)$ is a common random fixed point of $A, B, S$ and $T$.

Similarly, we can complete the proof when $A$ or $B$ or $T$ is jointly continuous.

The uniqueness follows easily. Therefore $A, B, S$ and $T$ have a unique common random fixed point. This completes the proof. □

Finally, we give the random fixed point theorem of compatible mappings of type $(P)$ for random mappings.

**Theorem 3.4.** *Let $(X, d)$ be a complete multiplicative metric space and $A, B, S$ and $T : \mathbb{R} \times X \to X$ be mappings satisfying the conditions $(C_1)$-$(C_3)$.*

*Assume that the pairs $A, S$ and $B, T$ are compatible of type $(P)$. Then $A, B, S$ and $T$ have a unique common random fixed point.*

*Proof.* From the proof of Theorem 3.2, $\{y_n(t)\}$ is a multiplicative Cauchy sequence and since $X$ is complete so $\{y_n(t)\}$ converges to a point $z(t) \in X$ as $n \to \infty$. Also subsequences of $\{y_n(t)\}$ also converges to a point $z(t) \in X$, that is,

$$\begin{aligned} &B(t, g_{2n+1}(t)) \to z(t), \quad S(t, g_{2n}(t)) \to z(t), \\ &A(t, g_{2n}(t)) \to z(t), \quad T(t, g_{2n+1}(t)) \to z(t), \quad t \in \mathbb{R} \end{aligned}$$

as $n \to \infty$.

Now suppose that $S$ is jointly continuous. Then we have

$$S(t, S(t, g_{2n}(t))) \to S(t, z(t)), \quad S(t, A(t, g_{2n}(t))) \to S(t, z(t))$$

as $n \to \infty$. Since $A$ and $S$ are compatible of type $(P)$, it follows from Remark 2.14 that

$$A(t, A(t, g_{2n}(t))) \to S(t, z(t)) \quad \text{as } n \to \infty.$$

Putting $x(t) = A(t, g_{2n}(t))$ and $y(t) = g_{2n+1}(t)$ in $(C_2)$, we have

$$\begin{aligned} &d(S(t, A(t, g_{2n}(t))), T(t, g_{2n+1}(t))) \\ &\leq [\max\{d(A(t, A(t, g_{2n}(t))), B(t, g_{2n+1}(t))), \\ &\quad d(A(t, A(t, g_{2n}(t))), S(t, A(t, g_{2n}(t)))), \\ &\quad d(B(t, g_{2n+1}(t)), T(t, g_{2n+1}(t))), d(S(t, A(t, g_{2n}(t))), B(t, g_{2n+1}(t))), \\ &\quad d(A(t, A(t, g_{2n}(t))), T(t, g_{2n+1}(t)))\}]^{\lambda}. \end{aligned}$$

Letting $n \to \infty$, we have

$$\begin{aligned}
&d(S(t,z(t)),z(t))\\
&\leq [\max\{d(S(t,z(t)),z(t)),d(S(t,z(t)),S(t,z(t))),d(z(t),z(t)),\\
&\qquad d(S(t,z(t)),z(t)),d(S(t,z(t)),z(t))\}]^{\lambda}\\
&= d^{\lambda}(S(t,z(t)),z(t)).
\end{aligned}$$

This implies that $d(S(t,z(t)),z(t)) = 1$ and hence $S(t,z(t)) = z(t)$. Since $z(t) = S(t,z(t)) \in S(t,X) \subset B(t,X)$, there exists $u(t) \in X$ such that $z(t) = B(t,u(t))$, $t \in \mathbb{R}$.

Putting $x(t) = g_{2n}(t)$ and $y(t) = u(t)$ in $(C_2)$, we have

$$\begin{aligned}
&d(S(t,g_{2n}(t)),T(t,u(t)))\\
&\leq [\max\{d(A(t,g_{2n}(t)),B(t,u(t))),d(A(t,g_{2n}(t)),S(t,g_{2n}(t))),\\
&\qquad d(B(t,u(t)),T(t,u(t))),d(S(t,g_{2n}(t)),B(t,u(t))),\\
&\qquad d(A(t,g_{2n}(t)),T(t,u(t)))\}]^{\lambda}.
\end{aligned}$$

Letting $n \to \infty$, we obtain

$$\begin{aligned}
&d(z(t),T(t,u(t)))\\
&\leq [\max\{d(z(t),z(t)),d(z(t),z(t)),d(z(t),T(t,u(t))),\\
&\qquad d(z(t),z(t)),d(z(t),T(t,u(t)))\}]^{\lambda}\\
&= d^{\lambda}(z(t),T(t,u(t))).
\end{aligned}$$

This implies that $d(z(t),T(t,u(t)) = 1$ and hence $T(t,u(t)) = z(t)$. Therefore, $B(t,u(t)) = T(t,u(t)) = z(t)$.

Since $B$ and $T$ are compatible of type $(P)$ and $B(t,u(t)) = T(t,u(t)) = z(t)$, by Remark 2.13, we have

$$B(t,z(t)) = B(t,T(t,u(t)) = T(t,B(t,u(t)) = T(t,z(t)).$$

Putting $x(t) = g_{2n}(t)$ and $y(t) = z(t)$ in $(C_2)$, we have

$$\begin{aligned}
&d(S(t,g_{2n}(t)),T(t,z(t)))\\
&\leq [\max\{d(A(t,g_{2n}(t)),B(t,z(t))),d(A(t,g_{2n}(t)),S(t,g_{2n}(t))),\\
&\qquad d(B(t,z(t)),T(t,z(t))),d(S(t,g_{2n}(t)),B(t,z(t))),\\
&\qquad d(A(t,g_{2n}(t)),T(t,z(t)))\}]^{\lambda}.
\end{aligned}$$

Letting $n \to \infty$, we obtain

$$\begin{aligned}
&d(S(t, g_{2n}(t)), T(t, z(t))) \\
&\leq [\max\{d(z(t), T(t, z(t)), 1, 1, d(z(t), T(t, z(t)), d(z(t), T(t, z(t)))\}]^{\lambda} \\
&= d^{\lambda}(z(t), T(t, z(t))).
\end{aligned}$$

This implies that $d(z(t), T(t, z(t)) = 1$ and hence $T(t, z(t)) = z(t)$. Therefore, $B(t, z(t)) = T(t, z(t)) = z(t)$. Since $T(t, X) \subset A(t, X)$, there exists a point $v(t) \in X$ such that $z(t) = T(t, z(t)) = A(t, v(t))$.

Putting $x(t) = v(t)$ and $y(t) = z(t)$ in $(C_2)$, we have

$$\begin{aligned}
&d(S(t, v(t)), z(t)) \\
&= d(S(t, v(t)), T(t, z(t))) \\
&\leq [\max\{d(A(t, v(t)), B(t, z(t))), d(A(t, v(t)), S(t, v(t))), \\
&\qquad d(B(t, T(t, z(t)), T(t, z(t))), d(S(t, v(t)), B(t, z(t))), \\
&\qquad d(A(t, v(t)), T(t, z(t)))\}]^{\lambda} \\
&= [\max\{d(z(t), z(t)), d(z(t), S(t, v(t))), 1, \\
&\qquad d(S(t, v(t)), z(t)), d(z(t), z(t))\}]^{\lambda}.
\end{aligned}$$

This implies that $S(t, v(t)) = z(t)$. Therefore $A(t, v(t)) = S(t, v(t)) = z(t)$. Since $A$ and $S$ are compatible $(P)$ and $S(t, v(t)) = A(t, v(t)) = z(t)$, it follows from Remark 2.13 that

$$A(t, z(t)) = A(t, S(t, v(t))) = S(t, A(t, v(t))) = S(t, z(t)).$$

Hence

$$A(t, z(t)) = S(t, z(t)) = B(t, z(t)) = T(t, z(t)) = z(t).$$

Therefore $z(t)$ is a common random fixed point of $A, B, S$ and $T$.

Similarly, we can complete the proof when $A$ or $B$ or $T$ is jointly continuous.

The uniqueness follows easily. Therefore $A, B, S$ and $T$ have a unique common random fixed point. This completes the proof. □

**Remark 3.5.** If we take $\mathbb{R}$ to be a singleton set, then the results can reduce into the result of Kang et al. [2].

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