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THE RELATIONS BETWEEN SOFT L-QUASI UNIFORMITIES AND SOFT L-NEIGHBORHOOD SYSTEMS

Jin Won Park[1], Yong Chan Kim[2 §]

[1]Department of Mathematics Education
Jeju, 63243, National University
Jeju, KOREA

[2]Department of Mathematics
Gangneung-Wonju University
Gangneung, Gangwondo, 25558, KOREA

Abstract: In this paper, we study the notions of soft L-neighborhood systems and soft L-quasi-uniformities in complete residuated lattices. We investigate the relations among soft L-topology, soft L-neighborhood systems and soft L-quasi-uniformities. We give their examples.

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1. Introduction

Recently, Molodtsov [14] introduced the soft set as a mathematical tool for dealing information as the uncertainty of data in engineering, physics, computer sciences and many other diverse field. Presently, the soft set theory is making progress rapidly [3,11-13,18,19]. Pawlak's rough set [15,16] can be viewed as a special case of soft rough sets [3]. The topological structures of soft sets have been developed by many researchers [7-9,18,19].

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§Correspondence author

On the other hand, Hájek [4] introduced a complete residuated lattice which is an algebraic structure for many valued logic. It is an important mathematical tool for algebraic structure [1,4,5,6-9,17]. Kim [7-9] introduced a fuzzy soft $F : A \to L^U$ as an extension as the soft $F : A \to P(U)$ where L is a complete residuated lattice. Kim [7-9] introduced the soft topological structures, L-fuzzy quasi-uniformities and soft L-fuzzy topogenous orders in complete residuated lattices.

In this paper, we study the notions of soft L-neighborhood systems and soft L-quasi-uniformities in complete residuated lattices. We investigate the relations among soft L-topology, soft L-neighborhood systems and soft L-quasi-uniformities. We give their examples.

2. Preliminaries

Definition 2.1. [1,4,5,17] An algebra $(L, \wedge, \vee, \odot, \to, 0, 1)$ is called a complete residuated lattice if it satisfies the following conditions:

(C1) $L = (L, \leq, \vee, \wedge, 1, 0)$ is a complete lattice with the greatest element 1 and the least element 0;

(C2) $(L, \odot, 1)$ is a commutative monoid;

(C3) $x \odot y \leq z$ iff $x \leq y \to z$ for $x, y, z \in L$.

In this paper, we assume that $(L, \leq, \odot, \to)$ is a complete residuated lattice and we denote $L_0 = L - \{0\}$.

Lemma 2.2. [1,4,5,17] For each $x, y, z, x_i, y_i, w \in L$, we have the following properties.

(1) $1 \to x = x$, $0 \odot x = 0$,
(2) If $y \leq z$, then $x \odot y \leq x \odot z$, $x \to y \leq x \to z$ and $z \to x \leq y \to x$,
(3) $x \odot y \leq x \wedge y \leq x \vee y \leq x \oplus y$,
(4) $x \odot (\bigvee_i y_i) = \bigvee_i (x \odot y_i)$,
(5) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$,
(6) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$,
(7) $x \to (\bigvee_i y_i) \geq \bigvee_i (x \to y_i)$,
(8) $(\bigwedge_i x_i) \to y \geq \bigvee_i (x_i \to y)$,
(9) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$,
(10) $x \odot (x \to y) \leq y$ and $x \to y \leq (y \to z) \to (x \to z)$,
(11) $(x \to y) \odot (z \to w) \leq (x \odot z) \to (y \odot w)$,
(12) $x \to y \leq (x \odot z) \to (y \odot z)$ and $(x \to y) \odot (y \to z) \leq x \to z$.

Definition 2.3. [7-9] Let X be an initial universe of objects and E the set of parameters (attributes) in X. A pair (F, A) is called a *fuzzy soft set* over X, where $A \subset E$ and $F : A \to L^X$ is a mapping. We denote $S(X, A)$ as the family of all fuzzy soft sets under the parameter A.

Definition 2.4.[7-9] Let (F, A) and (G, A) be two fuzzy soft sets over a common universe X.

(1) (F, A) is a fuzzy soft subset of (G, A), denoted by $(F, A) \le (G, A)$ if $F(a) \le G(a)$, for each $a \in A$.

(2) $(F, A) \wedge (G, A) = (F \wedge G, A)$ if $(F \wedge G)(a) = F(a) \wedge G(a)$ for each $a \in A$.

(3) $(F, A) \vee (G, A) = (F \vee G, A)$ if $(F \vee G)(a) = F(a) \vee G(a)$ for each $a \in A$.

(4) $(F, A) \odot (G, A) = (F \odot G, A)$ if $(F \odot G)(a) = F(a) \odot G(a)$ for each $a \in A$.

(6) $\alpha \odot (F, A) = (\alpha \odot F, A)$ for each $\alpha \in L$.

Definition 2.5. [7-9] A map $\tau \subset S(X, A)$ is called a soft L topology on X if it satisfies the following conditions.

(ST1) $(0_X, A), (1_X, A) \in \tau$, where $0_X(a)(x) = 0, 1_X(a)(x) = 1$ for all $a \in A, x \in X$,

(ST2) If $(F, A), (G, A) \in \tau$, then $(F, A) \odot (G, A)) \in \tau$,

(ST3) If $(F_i, A) \in \tau$ for each $i \in I$, $\bigvee_{i \in I}(F_i, A) \in \tau$.

The triple (X, A, τ) is called a soft L-topological space.

A soft L-topology is called enriched if $\alpha \odot (F, A) \in \tau$ for each $(F, A) \in \tau$ and $\alpha \in L$.

Let (X, A, τ_1) and (X, A, τ_2) be soft L-fuzzy topological spaces. Then τ_1 is finer than τ_2 if $(F, A) \in \tau_1$, for all $(F, A) \in \tau_2$.

Definition 2.6. [7-9] A map $N : X \to (L^A)^{S(X,A)}$ is called a soft L-neighborhood system on X if $N = \{N_x = N(x) \mid x \in X\}$ satisfies the following conditions

(SN1) $N_x((1_X, A)) = (1_X, A)(x) = 1_A$ and $N_x((1_X, A)) = (0_X, A)(x) = 0_A$,

(SN2) $N_x((F, A) \odot (G, A)) \ge N_x((F, A)) \odot N_x((G, A))$ for each $(F, A), (G, A) \in S(X, A)$,

(SN3) If $(F, A) \le (G, A)$, then $N_x((F, A)) \le N_x((G, A))$,

(SN4) $N_x((F, A)) \le (F, A)(x)$ for all $(F, A) \in S(X, A)$ where $(F, A)(x) = F(-)(x)$.

(SN5) $N_x((F, A)) \le \bigvee\{N_x(((G, A))) \mid ((G, A))(y) \le N_y(((F, A))), \ \forall\ y \in X\}$.

A soft L-neighborhood system is called stratified if

(S) $N_x(\alpha \odot (F, A)) \ge \alpha \odot N_x((F, A))$ for all $(F, A) \in S(X, A)$ and $\alpha \in L$.

The triple (X, A, N) is called a soft L-neighborhood space.

Theorem 2.7. [8] Let $(X, A, \mathbf{U})$ be a soft L-quasi uniform space. Define two maps $rN^{\mathbf{U}}, lN^{\mathbf{U}} : X \to L^{L^X}$ by, $\forall$ $(F, A) \in S(X, A),$ $x \in X$,

$$rN_x^{\mathbf{U}}((F, A))(a) = \bigvee_{(U,A)\in\mathbf{U}} (\bigwedge_{y\in X} (U(a)(y, x) \to F(a)(y))),$$

$$lN_x^{\mathbf{U}}((F, A))(a) = \bigvee_{(U,A)\in\mathbf{U}} (\bigwedge_{y\in X} (U(a)(x, y) \to F(a)(y))).$$

Then we have the following properties.

(1) $(X, A, rN^{\mathbf{U}})$ is a stratified soft L-neighborhood space.

(2) $(X, A, lN^{\mathbf{U}})$ is a stratified soft L-neighborhood space.

Definition 2.8. [7-9] A subset $\mathbf{U} \subset S(X \times X, A)$ is called a soft L-quasi-uniformity on X iff it satisfies the properties.

(SU1) $(1_{X\times X}, A) \in \mathbf{U}$.

(SU2) If $(V, A) \leq (U, A)$ and $(V, A) \in \mathbf{U}$, then $(U, A) \in \mathbf{U}$.

(SU3) For every $(U, A), (V, A) \in \mathbf{U}$, $(U, A) \odot (V, A) \in \mathbf{U}$.

(SU4) If $(U, A) \in \mathbf{U}$ then $(1_\triangle, A) \leq (U, A)$ where

$$1_\triangle(a)(x, y) = \begin{cases} 1, & \text{if } x = y \\ \bot, & \text{if } x \neq y, \end{cases}$$

(SU5) For every $(U, A) \in \mathbf{U}$, there exists $(V, A) \in \mathbf{U}$ such that $(V, A) \circ (V, A) \leq (U, A)$ where

$$\begin{aligned} &((V, A) \circ (V, A))(a)(x, y) = (V(a) \circ V(a))(x, y) \\ &= \bigvee\nolimits_{z\in X} (V(a)(z, x) \odot V(a)(x, y)), \quad \forall\, x, y \in X, a \in A. \end{aligned}$$

The triple $(X, A, \mathbf{U})$ is called a soft L-quasi-uniform space.

A soft L-quasi-uniformity $\mathbf{U}$ is called stratified if $\alpha \odot (U, A) \in \mathbf{U}$ for all $\alpha \in L$ and $(U, A) \in \mathbf{U}$.

A soft L-quasi-uniformity $\mathbf{U}$ on X is said to be a soft L-uniformity if

(U) if $(U, A) \in \mathbf{U}$, then $(U^{-1}, A) \in \mathbf{U}$ where $U^{-1}(a)(x, y) = U(a)(y, x)$.

Theorem 2.9. [8] Let $(X, A, \mathbf{U})$ be a soft L-quasi-uniform space, $(X, A, rN^{\mathbf{U}})$ and $(X, A, lN^{\mathbf{U}})$ soft L-neighborhood spaces. Define $\tau^r_{\mathbf{U}}, \tau^l_{\mathbf{U}} \subset S(X, A)$ as follows

$$\tau^r_{\mathbf{U}} = \{(F, A) \in S(X, A) \mid F(a)(x) = rN_x^{\mathbf{U}}((F, A))(a), \forall x \in X\},$$

$$\tau^l_{\mathbf{U}} = \{(F,A) \in S(X,A) \mid (F,A)(x) = lN^{\mathbf{U}}_x((F,A))(a), \forall x \in X\}.$$

Then,
(1) $\tau^r_{\mathbf{U}}$ is an enriched soft L-topology on X.
(2) $\tau^l_{\mathbf{U}}$ is an enriched soft L-topology on X.
(3) $rN^{\mathbf{U}} = N^{\tau^r_{\mathbf{U}}}$.
(4) $lN^{\mathbf{U}} = N^{\tau^l_{\mathbf{U}}}$.

Lemma 2.10. [8,9] For every $(F,A),(G,A) \in S(X,A)$, we define (U_F,A), $(U_F^{-1},A) \in S(X \times X, A)$ by

$$U_F(a)(x,y) = F(a)(x) \to F(a)(y).$$

then we have the following statements
(1) $(1_{X\times X},A) = (U_{0_X},A) = (U_{1_X},A)$,
(2) $(1_\triangle,A) \le (U_F,A)$,
(3) $(U_F,A) \circ (U_F,A) = (U_F,A)$,
(4) $(U_F,A) \odot (U_G,A) \le (U_{F\odot G},A)$.

Theorem 2.11. [9] Let (X,A,τ) be a soft L-topological space. Define a function $\mathbf{U}_\tau : S(X\times X,A) \to L$ by

$$\mathbf{U}_\tau = \{(U,A) \in S(X \times X,A) \mid \odot_{i=1}^n (U_{G_i},A) \le (U,A), (G_i,A) \in \tau\}$$

where the first $\bigvee$ is taken over every finite family $\{U_{(G_i,A)} \mid i = 1,...,n\}$. Then
(1) $\mathbf{U}_\tau$ is a soft L-quasi-uniformity on X.
(2) $\tau \subset \tau^l_{\mathbf{U}_\tau}$.

3. Soft L-Uniformities and Soft L-Neighborhood Systems

Theorem 3.1. Let (X,A,N) be a soft L-neighborhood space. Define a function $\mathbf{U}_N : S(X\times X,A) \to L$ by

$$\mathbf{U}_N = \{(U,A) \in S(X\times X,A) \mid \odot_{i=1}^n (U_{N_-((G_i,A))},A) \le (U,A), (G_i,A) \in S(X,A)\}$$

where the first $\bigvee$ is taken over every finite family $\{U_{(G_i,A)} \mid i = 1,...,n\}$. Then
(1) $\mathbf{U}_N$ is a soft L-quasi-uniformity on X.
(2) $lN^{\mathbf{U}_N}_x \ge N_x$.

Proof (1) (SU1) Since $N_-((1_X,A)) = (1_X,A)$, there exists $(U_{N_-((1_X,A))},A) \in S(X\times X,A)$ such that $(U_{1_X},A) \in \mathbf{U}_N$.

(SU2) It is trivial.

(SU3) For $(U,A),(V,A) \in \mathbf{U}_N$, there exist two finite families $\{N_-((F_i,A)) \mid \odot_{i=1}^m (U_{N_-((F_i,A))},A) \leq (U,A)\}$ and $\{N_-((G_j,A)) \mid \odot_{j=1}^n (U_{N_-((G_j,A))},A) \leq (V,A)\}$. Then $(U,A)\odot(V,A) \geq (\odot_{i=1}^m (U_{N_-((F_i,A))},A))\odot(\odot_{j=1}^n (U_{N_-((G_j,A))},A))$. So, $(U,A)\odot(V,A) \in \mathbf{U}_N$.

(SU4) Let $(U,A) \in \mathbf{U}_N$. Then there exists a finite family $\{N_-((F_i,A)) \mid \odot_{i=1}^m (U_{N_-((F_i,A))},A) \leq (U,A)\}$. Since $(1_\triangle,A) \leq (U_{N_-((F_i,A))},A)$ from Lemma 2.10(2),

$$(1_\triangle,A) \leq \odot_{i=1}^m (U_{N_-((F_i,A))},A) \leq (U,A).$$

(SU5) Let $(U,A) \in \mathbf{U}_N$. Then there exists a finite family $\{N_-((F_i,A)) \mid \odot_{i=1}^m (U_{N_-((F_i,A))},A) \leq (U,A)\}$. Since

$$(U_{N_-((F_i,A))},A) \circ (U_{N_-((F_i,A))},A) = (U_{N_-((F_i,A))},A)$$

for each $i \in \{1,...,m\}$ from Lemma 2.10(3), we have $(\odot_{i=1}^m (U_{N_-((F_i,A))},A) \circ (\odot_{i=1}^m (U_{N_-((F_i,A))},A)) \leq \odot_{i=1}^m (U_{N_-((F_i,A))},A)$ from

$$\begin{aligned}
&\bigvee_{y\in X}((\odot_{i=1}^m U_{N_-((F_i,A))}(a)(x,y)) \odot (\odot_{i=1}^m U_{N_-((F_i,A))}(a)(y,z))) \\
&= \bigvee_{y\in X}((\odot_{i=1}^m (N_x((F_i,A))(a) \to N_y((F_i,A))(a)) \\
&\odot(\odot_{i=1}^m (N_y((F_i,A))(a) \to N_z((F_i,A))(a)))) \\
&= \bigvee_{y\in X}((\odot_{i=1}^m (N_x((F_i,A))(a) \to N_y((F_i,A))(a)) \\
&\odot(N_y((F_i,A))(a) \to N_z((F_i,A))(a)))) \\
&\leq \odot_{i=1}^m (N_x((F_i,A))(a) \to N_z((F_i,A))(a)).
\end{aligned}$$

Put $(V,A) = \odot_{i=1}^m (U_{N_-((F_i,A))},A)$. Then $(V,A) \in \mathbf{U}_N$ with $(V,A)\circ(V,A) \leq (U,A)$. Hence $\mathbf{U}_N$ is a soft L-quasi-uniformity on X.

(2)

$$\begin{aligned}
&lN_x^{\mathbf{U}_N}((F,A))(a) \geq lN_x^{\mathbf{U}_N}(N_-((F,A)))(a) \\
&\geq \bigwedge_{y\in X}((N_x((F,A))(a) \to N_y((F,A))(a)) \to N_y((F,A))(a)) \geq N_x((F,A))(a).
\end{aligned}$$

Theorem 3.2. Let N^1 and N^2 be soft L-neighborhood systems on X satisfying $N_x^1((G_1,A)) \odot N_x^2((G_2,A)) = 0_A$ for all $(G_1,A)\odot(G_2,A) = (0_X,A)$. Then

(1) $N^1 \oplus N^2$ is a soft L-neighborhood system defined for all $(G,A) \in S(X,A)$ by

$$(N^1\oplus N^2)((G,A)) = \bigvee\{N^1((G_1,A))\odot N^2((G_2,A)) \mid (G_1,A)\odot(G_2,A) \leq (G,A)\}.$$

(2) $\tau_{N^1\oplus N^2} = \tau_{N^1} \oplus \tau_{N^2}$.

Proof. (1) (SN1)

$$(N^1 \oplus N^2)((G,A))((1_X,A)) \geq \{N^1((1_X,A)) \odot N^2((1_X,A)) \mid (1_X,A) \odot (1_X,A) = (1_X,A)\} = (1_X,A).$$

From the Theorem's condition, we have

$$(N^1 \oplus N^2)((0_X,A)) \geq \{N^1((F,A)) \odot N^2((G,A)) \mid (F,A) \odot (G,A) = (1_X,A)\} = (0_X,A).$$

(SN2) For $(F_1,A),(G_1,A) \in L^X$ and $(F_2,A),(G_2,A) \in L^Y$, we can prove that

$$\begin{aligned}
&(N^1 \oplus N^2)((F,A)) \odot (N^1 \oplus N^2)((G,A))\\
&= \bigvee\{N^1((F_1,A)) \odot N^2((F_2,A)) \mid (F_1,A) \odot (F_2,A) \leq (F,A)\}\\
&\odot \bigvee\{N^1((G_1,A)) \odot N^2((G_2,A)) \mid (G_1,A) \odot (G_2,A) \leq (G,A)\}\\
&\leq \bigvee\{N^1((F_1,A)) \odot N^1((G_1,A)) \odot N^2((F_2,A)) \odot N^2((G_2,A))\\
&\quad \mid ((F_1,A) \odot (F_2,A)) \odot ((G_1,A) \odot (G_2,A)) \leq (F,A) \odot (G,A)\}\\
&\leq \bigvee\{N^1((F_1,A) \odot (G_1,A)) \odot N^2((F_2,A) \odot (G_2,A))\\
&\quad \mid ((F_1,A) \odot (G_1,A)) \odot ((F_2,A) \odot (G_2,A)) \leq (F,A) \odot (G,A)\}\\
&\leq \bigvee\{N^1((H,A)) \odot N^2((K,A)) \mid (H,A) \odot (K,A) \leq (F,A) \odot (G,A)\}\\
&= (N^1 \oplus N^2)((F,A) \odot (G,A)).
\end{aligned}$$

(SN3) and (SN4) are clearly true.

(SN5) Let $(F,A) \in L^{X\times Y}$, $(F_1,A) \in L^X$ and $(F_2,A) \in L^Y$, then we have

$$\begin{aligned}
&(N^1 \oplus N^2)_x((F,A)) = \bigvee\{N^1_x((F_1,A)) \odot N^2_x((F_2,A)) \mid (F_1,A) \odot (F_2,A) \leq (F,A)\}\\
&\leq \bigvee\{\bigvee\{N^1_x((G_1,A)) \mid (G_1,A)(y) \leq N^1_y((F_1,A))\}\\
&\odot \bigvee\{N^2_x((G_2,A)) \mid (G_2,A)(y) \leq N^2_y((F_2,A))\} \mid (F_1,A) \odot (F_2,A) \leq (F,A)\}\\
&\leq \bigvee\{\bigvee\{N^1_x((G_1,A)) \odot N^2_x((G_2,A)) \mid (G_1,A)(y) \odot (G_2,A)(y) \leq\\
&N^1_y((F_1,A)) \odot N^2_y((F_2,A))\} \mid (F_1,A) \odot (F_2,A) \leq (F,A)\}\\
&= \bigvee\{\bigvee\{N^1_x((G_1,A)) \odot N^2_x((G_2,A)) \mid (G_1,A)(y) \odot (G_2,A)(y) \leq\\
&N^1_y((F_1,A)) \odot N^2_y((F_2,A))\} \mid (F_1,A) \odot (F_2,A) \leq (F,A)\}\\
&\leq \bigvee\{(N^1 \oplus N^2)_x((G_1,A) \odot (G_2,A)) \mid ((G_1,A) \odot (G_2,A))(y) \leq\\
&\bigvee\{(N^1 \oplus N^2)_y((F_1,A) \odot (F_2,A))\}\}\\
&\leq \bigvee\{(N^1 \oplus N^2)_x((G,A)) \mid (G,A)(y) \leq (N^1 \oplus N^2)_y((F,A))\}.
\end{aligned}$$

(2) Let $(F,A) \in \tau_{N^1} \oplus \tau_{N^2}$. Then $(F,A) = (F_1,A) \odot (F_2,A)$ for $(F_i,A) \in \tau_{N^i}, i = 1,2$, that is, $(F_i,A)(x) = N^i_x((F_i,A))$. Thus

$$(F_1,A)(x) \odot (F_2,A)(x) \leq (N^1 \oplus N^2)_x((F,A)) \leq (F_1,A)(x) \odot (F_2,A)(x).$$

So, $(F,A) \in \tau_{N^1 \oplus N^2}$. Hence $\tau_{N^1} \oplus \tau_{N^2} \subset \tau_{N^1 \oplus N^2}$.

Let $(G,A) \in \tau_{N^1 \oplus N^2}$. Then $(G,A) = \bigvee\{N^1((G_1,A)) \odot N^2((G_2,A)) \mid (G_1,A) \odot (G_2,A) \le (G,A)\}$. Since $N^i_x((G_i,A)) \le N^i_x(N^i((G_i,A))) \le N^i_x((G_i,A))$ for $i = 1,2$, $N^i((G_i,A)) \in \tau_{N^i}$. So, $(G,A) \in \tau_{N^1} \oplus \tau_{N^2}$. $\tau_{N^1 \oplus N^2} \subset \tau_{N^1} \oplus \tau_{N^2}$.

Theorem 3.3. Let $\mathbf{U_1}$ and $\mathbf{U_2}$ be soft L-quasi-uniformities on X. We define, $(U_1,A) \in \mathbf{U_1}, (U_2,A) \in \mathbf{U_2}$,

$$\mathbf{U_1} \oplus \mathbf{U_2} = \{(U,A) \in S(X \times X, A) \mid (U_1,A) \odot (U_2,A) \le (U,A)\}.$$

Then we have the following properties.

(1) $\mathbf{U_1} \oplus \mathbf{U_2}$ is the coarsest L-quasi-uniformity on X which is finer than $\mathbf{U_1}$ and $\mathbf{U_2}$.

(2) $rN^{\mathbf{U_1}} \oplus rN^{\mathbf{U_2}} = rN^{\mathbf{U_1} \oplus \mathbf{U_2}}$ and $lN^{\mathbf{U_1}} \oplus lN^{\mathbf{U_2}} = lN^{\mathbf{U_1} \oplus \mathbf{U_2}}$.

(3) $\tau^r_{\mathbf{U_1}} \oplus \tau^r_{\mathbf{U_2}} = \tau^r_{\mathbf{U_1} \oplus \mathbf{U_2}}$ where

$$\tau^r_{\mathbf{U_1}} \oplus \tau^r_{\mathbf{U_2}} = \{(F,A) = (F_1,A) \odot (F_2,A) \mid (F_i,A) \in \tau^r_{\mathbf{U}_i}, i = 1,2\}.$$

(4) $\tau^l_{\mathbf{U_1}} \oplus \tau^l_{\mathbf{U_2}} = \tau^l_{\mathbf{U_1} \oplus \mathbf{U_2}}$ where

$$\tau^l_{\mathbf{U_1}} \oplus \tau^l_{\mathbf{U_2}} = \{(F,A) = (F_1,A) \odot (F_2,A) \mid (F_i,A) \in \tau^l_{\mathbf{U}_i}, i = 1,2\}.$$

(5) If (X,A,N^1) and (X,A,N^2) are soft L-neighborhood spaces, then

$$\mathbf{U}_{N^1 \oplus N^2} \subset \mathbf{U}_{N^1} \oplus \mathbf{U}_{N^2}.$$

Proof. (1) It follows from Theorem 3.1 in [9].

(2)

$$
\begin{aligned}
&(rN^{\mathbf{U_1}}_x \oplus rN^{\mathbf{U_2}}_x)((H,A))(a)\\
&= \bigvee_{(F,A)\odot(G,A)\le(H,A)} rN^{\mathbf{U_1}}_x((F,A))(a) \odot rN^{\mathbf{U_2}}_x((G,A))(a)\\
&= \bigvee_{(F,A)\odot(G,A)\le(H,A)}\{\{\bigvee_{(U_1,A)\in\mathbf{U_1}} \bigwedge_{y\in X}(U_1(a)(y,x) \to F(a)(y))\}\\
&\odot\{\bigvee_{(U_2,A)\in\mathbf{U_2}} \bigwedge_{z\in X}(U_2(a)(z,x) \to G(a)(z))\}\}\\
&\le \bigvee_{(F,A)\odot(G,A)\le(H,A)}\{\bigvee_{(U_1,A)\in\mathbf{U_1},(U_2,A)\in\mathbf{U_2}} \bigwedge_{y\in X}\{(U_1(a)(y,x) \to F(a)(y))\\
&\odot(U_2(a)(y,x) \to G(a)(y))\}\} \quad \text{(by Lemma 2.2(11))}\\
&\le \bigvee_{(F,A)\odot(G,A)\le(H,A)}\{\bigvee_{(U_1\odot U_2,A)\in\mathbf{U_1}\oplus\mathbf{U_2}} \bigwedge_{y\in X}\{((U_1(a) \odot U_2(a))(y,x)\\
&\to (F(a) \odot G(a))(y))\}\}\\
&\le \bigvee_{(F,A)\odot(G,A)\le(H,A)}\{\bigvee_{(U,A)\in\mathbf{U_1}\oplus\mathbf{U_2}} \bigwedge_{y\in X}\{(U(a)(y,x) \to H(a)(y))\}\}\\
&= rN^{\mathbf{U_1}\oplus\mathbf{U_2}}_x((H,A))(a).
\end{aligned}
$$

Suppose there exist $x \in X, (H, A) \in L^X$ such that

$$rN_x^{\mathbf{U_1} \oplus \mathbf{U_2}}((H, A))(a) \not\leq (rN_x^{\mathbf{U_1}} \oplus rN_x^{\mathbf{U_2}})((H, A))(a).$$

Then there exist $(U, A) \in \mathbf{U_1} \oplus \mathbf{U_2}$ such that

$$\bigwedge_{y \in X} (U(a)(y, x) \to H(a)(y)) \not\leq (rN_x^{\mathbf{U_1}} \oplus rN_x^{\mathbf{U_2}})((H, A))(a).$$

Then there exist $(U_i, A) \in \mathbf{U_i}$ with $(U_1, A) \odot (U_2, A) \leq (U, A)$ such that

$$\bigwedge_{y \in X} ((U_1(a) \odot U_2(a))(y, x) \to H(a)(y)) \not\leq (rN_x^{\mathbf{U_1}} \oplus rN_x^{\mathbf{U_2}})((H, A))(a).$$

$$\begin{aligned} &\textstyle\bigwedge_{y \in X} ((U_1(a) \odot U_2(a))(y, x) \to H(a)(y)) \\ &\textstyle\bigwedge_{y \in X} (U_1(a)(y, x) \to (U_2(a)(y, x) \to H(a)(y))) \\ &\not\leq (rN_x^{\mathbf{U_1}} \oplus rN_x^{\mathbf{U_2}})((H, A))(a). \end{aligned}$$

Since $(U_2(a)(y, x) \to H(a)(y)) \odot U_2(a)(y, x) \leq H(a)(y)$, then $(U_2, A)(-, x) \to (H, A)) \odot (U_2, A)(-, x) \leq (H, A)$. We have

$$\begin{aligned} &rN_x^{\mathbf{U_1} \oplus \mathbf{U_2}}((H, A))(a) \\ &\geq rN_x^{\mathbf{U_1}}((U_2, A)(-, x) \to (H, A))(a) \odot rN_x^{\mathbf{U_1}}((U_2, A)(-, x))(a) \\ &\geq \textstyle\bigwedge_{y \in X} ((U_1(a)(y, x) \to (U_2(a)(y, x) \to H(a)(y)) \odot \bigwedge_{y \in X} (U_2(a)(y, x) \\ &\qquad \to U_2(y, x)) \\ &\geq \textstyle\bigwedge_{y \in X} ((U_1(a)(y, x) \to (U_2(a)(y, x) \to H(a)(y)) \end{aligned}$$

It is a contradiction. Hence $rN_x^{\mathbf{U_1} \oplus \mathbf{U_2}} \leq rN_x^{\mathbf{U_1}} \oplus rN_x^{\mathbf{U_2}}$. Thus, $rN_x^{\mathbf{U_1} \oplus \mathbf{U_2}} = rN_x^{\mathbf{U_1}} \oplus rN_x^{\mathbf{U_2}}$.

Similarly, $lN_x^{\mathbf{U_1} \oplus \mathbf{U_2}} = lN_x^{\mathbf{U_1}} \oplus lN_x^{\mathbf{U_2}}$.

(3) Let $(F, A) \in \tau^r_{\mathbf{U_1}} \oplus \tau^r_{\mathbf{U_2}}$ such that $(F, A) = \bigvee_{j \in J} (F_j, A)$. Then there exist $(F_{ji}, A) \in \tau^r_{U_i}, i = 1, 2$ such that

$$\begin{aligned} &(F_j, A)(x) = (F_{j1}, A)(x) \odot (F_{j2}, A)(x) = rN_x^{\mathbf{U_1}}((F_{j1}, A) \odot rN_x^{\mathbf{U_2}}((F_{j2}, A)) \\ &\leq rN_x^{\mathbf{U_1}} \oplus rN_x^{\mathbf{U_2}}((F_j, A)) = rN_x^{\mathbf{U_1} \oplus \mathbf{U_2}}((F_j, A)) \leq (F_j, A)(x). \end{aligned}$$

Then $(F_j, A)(x) = rN_x^{\mathbf{U_1} \oplus \mathbf{U_2}}((F_j, A))$ for each $j \in J$.

$$\begin{aligned} &(F, A)(x) = \textstyle\bigvee_{j \in J} (F_j, A)(x) = \bigvee_{j \in J} rN_x^{\mathbf{U_1} \oplus \mathbf{U_2}}((F_j, A)) \\ &\leq rN_x^{\mathbf{U_1} \oplus \mathbf{U_2}}(\textstyle\bigvee_{j \in J} ((F_j, A)) \leq \bigvee_{j \in J} (F_j, A)(x) = (F, A)(x). \end{aligned}$$

Hence $(F, A)(x) = rN_x^{\mathbf{U_1} \oplus \mathbf{U_2}}((F, A))$. So, $(F, A) \in \tau^r_{\mathbf{U_1} \oplus \mathbf{U_2}}$.

Let $(H,A) \in \tau^r_{\mathbf{U_1}\oplus\mathbf{U_2}}$. Then $(H,A)(x) = rN_x^{\mathbf{U_1}\oplus\mathbf{U_2}}((H,A)) = (rN_x^{\mathbf{U_1}} \oplus rN_x^{\mathbf{U_2}})((H,A)) = \bigvee\{rN_x^{\mathbf{U_1}}((H_1,A)) \odot rN_x^{\mathbf{U_2}}((H_2,A)) \mid (H_1,A)\odot(H_2,A) \leq (H,A)\}$. Since $rN_x^{U_i}(rN_-^{U_i}((H_i,A))) = rN_x^{U_i}((H_i,A)), i=1,2$ from (SN5), then $rN_-^{U_i}((H_i,A)) \in \tau^r_{U_i}$. So, $(H,A) \in \tau^r_{\mathbf{U_1}} \oplus \tau^r_{\mathbf{U_2}}$.

(4) It is similarly proved as (3).

(5) Let $(U,A) \in \mathbf{U}_{N^1\oplus N^2}$. Then there exist $N^i_-(F_{j_i},A)$ such that

$$\odot_{j=1}^n (U_{N^1_-(F_{j_1},A)\odot N^2_-(F_{j_2},A)}, A) \leq (U,A).$$

Since $(U_{N^1_-(F_{j_1},A)},A)\odot(U_{N^2_-(F_{j_2},A)},A) \leq (U_{N^1_-(F_{j_1},A)\odot N^2_-(F_{j_2},A)},A)$ from Lemma 2.10(4), we have

$$\begin{aligned}&\odot_{j=1}^n (U_{N^1_-(F_{j_1},A)},A) \odot (\odot_{j=1}^n (U_{N^2_-(F_{j_2},A)},A))\\ &\qquad\leq \odot_{j=1}^n (U_{N^1_-(F_{j_1},A)\odot N^2_-(F_{j_2},A)},A) \leq (U,A).\end{aligned}$$

Since $\odot_{j=1}^n (U_{N^1_-(F_{j_1},A)},A) \in \mathbf{U}_{N^1},\ \odot_{j=1}^n (U_{N^2_-(F_{j_2},A)},A) \in \mathbf{U}_{N^2}$, we have $(U,A) \in \mathbf{U}_{N^1} \oplus \mathbf{U}_{N^2}$.

From Theorem 3.3, we can obtain the following corollary.

Corollary 3.4. Let $\mathbf{U}$ be a soft quasi-uniformities on X. We define

$\mathbf{U^{-1}} = \{(U,A) \in S(X\times X,A) \mid (U^{-1},A) \in \mathbf{U}\}$.
$\mathbf{U}\oplus\mathbf{U^{-1}} = \{(U,A) \in S(X\times X,A) \mid (U_1,A)\odot(U_2,A) \leq (U,A),$
$(U_1,A) \in \mathbf{U}, (U_2,A) \in \mathbf{U^{-1}}\}$.

Then we have the following properties.

(1) $\mathbf{U^{-1}}$ a soft quasi-uniformities on X.

(2) $\mathbf{U}\oplus\mathbf{U^{-1}}$ is the coarsest uniformity on X which is finer than $\mathbf{U}$ and $\mathbf{U^{-1}}$.

(3) $rN^{\mathbf{U}} = lN^{\mathbf{U^{-1}}}, lN^{\mathbf{U}} = rN^{\mathbf{U^{-1}}}$,

$$rN^{\mathbf{U}} \oplus rN^{\mathbf{U^{-1}}} = lN^{\mathbf{U}} \oplus lN^{\mathbf{U^{-1}}} = rN^{\mathbf{U}\oplus\mathbf{U^{-1}}} = lN^{\mathbf{U}\oplus\mathbf{U^{-1}}}.$$

(4) $\tau^r_{\mathbf{U}} = \tau^l_{\mathbf{U^{-1}}},\ \tau^l_{\mathbf{U}} = \tau^r_{\mathbf{U^{-1}}},\ \tau^r_{\mathbf{U}} \oplus \tau^r_{\mathbf{U^{-1}}} = \tau^r_{\mathbf{U}\oplus\mathbf{U^{-1}}} = \tau^l_{\mathbf{U}\oplus\mathbf{U^{-1}}}$ where

$\tau^r_{\mathbf{U}} \oplus \tau^r_{\mathbf{U^{-1}}} = \{(F,A) = (F_1,A)\odot(F_2,A) \mid (F_1,A) \in \tau^r_{\mathbf{U}}, (F_2,A) \in \tau^r_{\mathbf{U}^{-1}}\}$
$= \tau^l_{\mathbf{U}} \oplus \tau^l_{\mathbf{U^{-1}}}$.

Example 3.5. Let $X = \{h_i \mid i = \{1,...,4\}\}$ with h_i=house and $E_Y = \{e,b,w,c,i\}$ with e=expensive,b= beautiful, w=wooden, c= creative, i=in the green surroundings.

Let $(L=[0,1],\odot,\rightarrow)$ be a complete residuated lattice defined by

$$x\odot y=x\wedge y,\quad x\rightarrow y=\begin{cases}1, & \text{if } x\leq y,\\ y, & \text{otherwise.}\end{cases}$$

Let $X=\{x,y,z\}$ be a set and $W_i(e),W_i(b)\in S(X\times X,A)$ such that

$$W_1(e)=\begin{pmatrix}1 & 0.5 & 0.5\\ 0.7 & 1 & 0.8\\ 0.4 & 0.4 & 1\end{pmatrix},\ W_1(b)=\begin{pmatrix}1 & 0.6 & 0.7\\ 0.4 & 1 & 0.4\\ 0.5 & 0.6 & 1\end{pmatrix}$$

$$W_2(e)=\begin{pmatrix}1 & 0.4 & 0.3\\ 0.4 & 1 & 0.3\\ 0.6 & 0.5 & 1\end{pmatrix},\ W_2(b)=\begin{pmatrix}1 & 0.3 & 0.3\\ 0.6 & 1 & 0.7\\ 0.5 & 0.4 & 1\end{pmatrix}$$

$$(W_1\wedge W_2)(e)=\begin{pmatrix}1 & 0.4 & 0.3\\ 0.4 & 1 & 0.3\\ 0.4 & 0.4 & 1\end{pmatrix},\ (W_1\wedge W_2)(b)=\begin{pmatrix}1 & 0.3 & 0.3\\ 0.4 & 1 & 0.4\\ 0.5 & 0.4 & 1\end{pmatrix}$$

Define $\mathbf{U}_i=\{(U,A)\in S(X\times X,A)\mid (U,A)\geq (W_i,A)\}$ for $i=1,2$.

(1) Since $W_i(e)\circ W_i(e)=W_i(e)$ and $W_i(b)\circ W_i(b)=W_i(b)$, $\mathbf{U}_i$ is a soft L-quasi-uniformity on X.

(2) Since $rN_x^{\mathbf{U}}((F,A))(e)=\bigvee_{(U,A)\in\mathbf{U}}\bigwedge_{y\in X}(U(e)(y,x)\rightarrow F(e)(y))$, we have

$$\begin{aligned}
rN_x^{\mathbf{U}_1}((F,A))(e)&=(F(e)(x)\wedge(0.7\rightarrow F(e)(y))\wedge(0.4\rightarrow F(e)(z)),\\
rN_y^{\mathbf{U}_1}((F,A))(e)&=(0.5\rightarrow F(e)(x))\wedge F(e)(y)\wedge(0.4\rightarrow F(e)(z)),\\
rN_z^{\mathbf{U}_1}((F,A))(e)&=(0.5\rightarrow F(e)(x))\wedge(0.8\rightarrow F(e)(y))\wedge F(e)(z),
\end{aligned}$$

$$\begin{aligned}
rN_x^{\mathbf{U}_1}((F,A))(b)&=(F(b)(x)\wedge(0.4\rightarrow F(b)(y))\wedge(0.5\rightarrow F(b)(z)),\\
rN_y^{\mathbf{U}_1}((F,A))(b)&=(0.6\rightarrow F(b)(x))\wedge F(b)(y)\wedge(0.6\rightarrow F(b)(z)),\\
rN_z^{\mathbf{U}_1}((F,A))(b)&=(0.7\rightarrow F(b)(x))\wedge(0.4\rightarrow F(b)(y))\wedge F(b)(z),
\end{aligned}$$

$$\begin{aligned}
rN_x^{\mathbf{U}_2}((F,A))(e)&=(F(e)(x)\wedge(0.4\rightarrow F(e)(y))\wedge(0.6\rightarrow F(e)(z)),\\
rN_y^{\mathbf{U}_2}((F,A))(e)&=(0.4\rightarrow F(e)(x))\wedge F(e)(y)\wedge(0.5\rightarrow F(e)(z)),\\
rN_z^{\mathbf{U}_2}((F,A))(e)&=(0.3\rightarrow F(e)(x))\wedge(0.3\rightarrow F(e)(y))\wedge F(e)(z),
\end{aligned}$$

$$\begin{aligned}
rN_x^{\mathbf{U}_2}((F,A))(b)&=(F(b)(x)\wedge(0.6\rightarrow F(b)(y))\wedge(0.5\rightarrow F(b)(z)),\\
rN_y^{\mathbf{U}_2}((F,A))(b)&=(0.3\rightarrow F(b)(x))\wedge F(b)(y)\wedge(0.4\rightarrow F(b)(z)),\\
rN_z^{\mathbf{U}_2}((F,A))(b)&=(0.3\rightarrow F(b)(x))\wedge(0.7\rightarrow F(b)(y))\wedge F(b)(z).
\end{aligned}$$

(3) From Theorem 3.3(2), we obtain $rN^{\mathbf{U_1}} \oplus rN^{\mathbf{U_2}} = rN^{\mathbf{U_1} \oplus \mathbf{U_2}}$ as follows:

$$\begin{aligned}
rN_x^{\mathbf{U}_1 \oplus \mathbf{U}_2}((F,A))(e) &= (F(e)(x) \wedge (0.4 \to F(e)(y)) \wedge (0.4 \to F(e)(z)),\\
rN_y^{\mathbf{U}_1 \oplus \mathbf{U}_2}((F,A))(e) &= (0.4 \to F(e)(x)) \wedge F(e)(y) \wedge (0.4 \to F(e)(z)),\\
rN_z^{\mathbf{U}_1 \oplus \mathbf{U}_2}((F,A))(e) &= (0.3 \to F(e)(x)) \wedge (0.3 \to F(e)(y)) \wedge F(e)(z),
\end{aligned}$$

$$\begin{aligned}
rN_x^{\mathbf{U}_1 \oplus \mathbf{U}_2}((F,A))(b) &= (F(b)(x) \wedge (0.4 \to F(b)(y)) \wedge (0.5 \to F(b)(z)),\\
rN_y^{\mathbf{U}_1 \oplus \mathbf{U}_2}((F,A))(b) &= (0.3 \to F(b)(x)) \wedge F(b)(y) \wedge (0.4 \to F(b)(z)),\\
rN_z^{\mathbf{U}_1 \oplus \mathbf{U}_2}((F,A))(b) &= (0.3 \to F(b)(x)) \wedge (0.4 \to F(b)(y)) \wedge F(b)(z).
\end{aligned}$$

Similarly, we obtain $lN^{\mathbf{U_1}} \oplus lN^{\mathbf{U_2}} = lN^{\mathbf{U_1} \oplus \mathbf{U_2}}$ as follows:

$$\begin{aligned}
lN_x^{\mathbf{U}_1 \oplus \mathbf{U}_2}((F,A))(e) &= (F(e)(x) \wedge (0.4 \to F(e)(y)) \wedge (0.3 \to F(e)(z)),\\
lN_y^{\mathbf{U}_1 \oplus \mathbf{U}_2}((F,A))(e) &= (0.4 \to F(e)(x)) \wedge F(e)(y) \wedge (0.3 \to F(e)(z)),\\
lN_z^{\mathbf{U}_1 \oplus \mathbf{U}_2}((F,A))(e) &= (0.4 \to F(e)(x)) \wedge (0.4 \to F(e)(y)) \wedge F(e)(z),
\end{aligned}$$

$$\begin{aligned}
lN_x^{\mathbf{U}_1 \oplus \mathbf{U}_2}((F,A))(b) &= (F(b)(x) \wedge (0.3 \to F(b)(y)) \wedge (0.3 \to F(b)(z)),\\
lN_y^{\mathbf{U}_1 \oplus \mathbf{U}_2}((F,A))(b) &= (0.4 \to F(b)(x)) \wedge F(b)(y) \wedge (0.4 \to F(b)(z)),\\
lN_z^{\mathbf{U}_1 \oplus \mathbf{U}_2}((F,A))(b) &= (0.5 \to F(b)(x)) \wedge (0.4 \to F(b)(y)) \wedge F(b)(z).
\end{aligned}$$

(4) Since $\tau^r_{\mathbf{U}} = \{(F,A) \in L^X \mid (F,A)(x) = rN^{\mathbf{U}}_x((F,A)), \forall x \in X\}$, we have

$$(F,A) \in \tau^r_{\mathbf{U}_1} \text{ iff } \begin{cases} (F,A) = \alpha_X, \\ F(e)(x) \leq 0.7 \to F(e)(y), F(e)(x) \leq 0.4 \to F(e)(z), \\ F(b)(x) \leq 0.4 \to F(b)(y), F(b)(x) \leq 0.5 \to F(b)(z), \\ F(e)(y) \leq 0.5 \to F(e)(x), (F(e)(y) \leq 0.4 \to F(e)(z), \\ F(b)(y) \leq 0.6 \to F(b)(x), F(b)(y) \leq 0.6 \to F(b)(z), \\ F(e)(z) \leq 0.5 \to F(e)(x), F(e)(z) \leq 0.8 \to F(e)(y), \\ F(b)(z) \leq 0.7 \to F(b)(x), F(b)(z) \leq 0.4 \to F(b)(y). \end{cases}$$

$$(F,A) \in \tau^r_{\mathbf{U}_2} \text{ iff } \begin{cases} (F,A) = \alpha_X, \\ F(e)(x) \leq 0.4 \to F(e)(y), F(e)(x) \leq 0.6 \to F(e)(z), \\ F(b)(x) \leq 0.6 \to F(b)(y), F(b)(x) \leq 0.5 \to F(b)(z), \\ F(e)(y) \leq 0.4 \to F(e)(x), (F(e)(y) \leq 0.5 \to F(e)(z), \\ F(b)(y) \leq 0.3 \to F(b)(x), F(b)(y) \leq 0.4 \to F(b)(z), \\ F(e)(z) \leq 0.3 \to F(e)(x), F(e)(z) \leq 0.3 \to F(e)(y), \\ F(b)(z) \leq 0.3 \to F(b)(x), F(b)(z) \leq 0.7 \to F(b)(y). \end{cases}$$

(5) From Theorem 3.3(3), we obtain $\tau^r_{\mathbf{U_1}} \oplus \tau^r_{\mathbf{U_2}} = \tau^r_{\mathbf{U_1 \oplus U_2}}$ as follows:

$$(F,A) \in \tau^r_{\mathbf{U}_1 \oplus \mathbf{U}_2} \text{ iff } \begin{cases} (F,A) = \alpha_X, \\ F(e)(x) \le 0.4 \to F(e)(y), F(e)(x) \le 0.4 \to F(e)(z), \\ F(b)(x) \le 0.4 \to F(b)(y), F(b)(x) \le 0.5 \to F(b)(z), \\ F(e)(y) \le 0.4 \to F(e)(x), (F(e)(y) \le 0.4 \to F(e)(z), \\ F(b)(y) \le 0.3 \to F(b)(x), F(b)(y) \le 0.4 \to F(b)(z), \\ F(e)(z) \le 0.3 \to F(e)(x), F(e)(z) \le 0.3 \to F(e)(y), \\ F(b)(z) \le 0.3 \to F(b)(x), F(b)(z) \le 0.4 \to F(b)(y). \end{cases}$$

Similarly, we obtain $\tau^l_{\mathbf{U_1}} \oplus \tau^l_{\mathbf{U_2}} = \tau^l_{\mathbf{U_1 \oplus U_2}}$ as follows:

$$(F,A) \in \tau^l_{\mathbf{U}_1 \oplus \mathbf{U}_2} \text{ iff } \begin{cases} (F,A) = \alpha_X, \\ F(e)(x) \le 0.4 \to F(e)(y), F(e)(x) \le 0.3 \to F(e)(z), \\ F(b)(x) \le 0.3 \to F(b)(y), F(b)(x) \le 0.3 \to F(b)(z), \\ F(e)(y) \le 0.4 \to F(e)(x), (F(e)(y) \le 0.3 \to F(e)(z), \\ F(b)(y) \le 0.4 \to F(b)(x), F(b)(y) \le 0.4 \to F(b)(z), \\ F(e)(z) \le 0.4 \to F(e)(x), F(e)(z) \le 0.4 \to F(e)(y), \\ F(b)(z) \le 0.5 \to F(b)(x), F(b)(z) \le 0.5 \to F(b)(y). \end{cases}$$

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