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# BEING SEMI-$T_D$ IS SEMI-TOPOLOGICAL

Bhamini M.P. Nayar
Department of Mathematics
Morgan State University
1700 E. Cold Spring Lane, Baltimore, MD 21251, USA

**Abstract:** A property preserved under a semi-homeomorphism is called a semi-topological property. In the present article we prove that being semi-$T_D$ is a semi-topological property, where a space $X$ is called semi-$T_D$ if the derived set $d\{x\}$ of each $x$ in $X$ is semi-closed.

**AMS Subject Classification:** 54A05
**Key Words:** semi-$T_D$, semi-topological, semi-homeomorphism, semi-open sets

## 1. Introduction

In [9] we have proved the following Theorem A and used it to identify topological properties which are semi-topological.

**Theorem A.** *A topological property $P$ is semi-topological if and only if the statement '$(X, \mathcal{T})$ has $P$ if and only if $(X, F(\mathcal{T}))$ has $P$' is true where $F(\mathcal{T})$ is the finest topology on $X$ having the same family of semi-open sets as $(X, \mathcal{T})$.*

A set $A \subset X$ is *semi-open* (see [8]) if there is an open set $U \subset X$ such that $U \subset A \subset clU$, where $clU$ is the closure of $U$. A set is *semi-closed* , if its complement is semi-open. The *semi-closure* of a set $A$, denoted as $sclA$, is the intersection of all semi-closed sets containing $A$. A space $(X, \mathcal{T})$ is *semi-$T_D$* if the derived set $d\{x\} = \text{cl}\{x\}$ - $\{x\}$ is semi-closed for each $x \in X$. When more than one topology is under consideration, we shall denote $cl_{\mathcal{T}}\{x\}$, $scl_{\mathcal{T}}\{x\}$ , and $d_{\mathcal{T}}\{x\}$ to denote the closure, semi-closure and derived set, respectively, of $\{x\}$ in the topology $\mathcal{T}$.

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It is possible for more than one topology on a set $X$ to have the same family of semi-open sets. In [3], Crossley and Hildebrand characterized the finest topology $F(\mathcal{T})$, with the same class of semi-open sets as that of $(X, \mathcal{T})$, as $F(\mathcal{T}) = \{U - N : U \in \mathcal{T}$ and $N$ is a nowhere dense subset of$X\}$. Since every homeomorphism is a semi-homeomorphism, a semi-topological property is a topological property. But a topological property might fail to be semi-topological. Theorem A above enables one to identify topological properties which are semi-topological.

In [9] several examples are provided to demonstrate this property on several topological concepts. The space $(X, \mathcal{T})$ of Example 3.9 in [9] is in fact semi-$T_D$. Therefore it does not show that being semi-$T_D$ is not semi-topological.

In [6], Jankovic and Reilly studied $T_D$ type properties. A space $X$ is $T_D$ if the derived set $d\{x\}$ of each $x \in X$ is closed. In Proposition 6 of [6], they proved that being semi-$T_D$ is equivalent to the condition that $\{x\}$ is either open or nowhere dense for each $x \in X$ and is also equivalent to the condition that $scl\{x\} - \{x\}$ is semi-closed for each $x \in X$. We shall use Proposition 6 of [6] and Theorem A to prove that beig semi-$T_D$ is semi-topological.

**Theorem 1.** *A space $(X, \mathcal{T})$ is semi-$T_D$ if and only if $(X, F(\mathcal{T}))$ is semi-$T_D$.*

*Proof.* Suppose that $(X, \mathcal{T})$ is semi-$T_D$. We shall prove that $(X, F(\mathcal{T}))$ is semi-$T_D$. Let $x \in X$. Then $d_{\mathcal{T}}\{x\}$ is $\mathcal{T}$-semi-closed. Suppose $y \notin d_{F(\mathcal{T})}\{x\}$. If $y \neq x$, then there is a $U \in F(\mathcal{T})$ such that $y \in U$ and $x \notin U$. Since $U \in F(\mathcal{T})$, there is a $G \in \mathcal{T}$ and a nowhere dense subset $N$ of $(X, \mathcal{T})$ such that $U = G - N$. Now $U \cap cl_{F(\mathcal{T})}(\{x\}) = \emptyset$ and $\mathrm{scl}d_{F(\mathcal{T})}\{x\} \subset cl_{F(\mathcal{T})}(\{x\})$. Hence $y \notin \mathrm{scl}d_{F(\mathcal{T})}\{x\}$. If $y = x$, then $y \in (X - d_{\mathcal{T}}\{x\})$, which is $\mathcal{T}$ - semi-open and hence $F(\mathcal{T})$-semi-open and $(X - d_{\mathcal{T}}\{x\}) \cap d_{\mathcal{T}}\{x\} = \phi$. Thus, if $(X, \mathcal{T})$ is semi-$T_D$, then $(X, F(\mathcal{T}))$ is semi-$T_D$.

Suppose that $(X, F(\mathcal{T}))$ is semi-$T_D$ and $y \notin d_{\mathcal{T}}\{x\}$, for $x \in X$. Then $y \notin d_{F(\mathcal{T})}\{x\}$. Therefore, there is an $F(\mathcal{T})$-semi-open set, and hence a $\mathcal{T}$-semi-open set $U$ such that $y \in U$ and $U \cap d_{F(\mathcal{T})}\{x\} = \phi$. Hence, in view of Proposition 6 of [6], $U \cap (scl_{\mathcal{T}}\{x\} - \{x\}) = \phi$ and hence $U \cap d_{\mathcal{T}}\{x\} = \phi$. Thus $(X, \mathcal{T})$ is semi-$T_D$. □

**Theorem 2.** *Being semi-$T_D$ is semi-topological.*

*Proof.* Follows from Theorem A and Theorem 1. □

**Remark**. In recent studies of compactness of a space using special subsets, several long standing problems have been sloved [4],[5],[7]. The nature of the boundary of a set was very significant in thoses studies. The properties of the boundary of a single point set, using different closure operators, can be effectively used in investigating significant classes of spaces, especially the classes of spcaes focussing on localized concepts.

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