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ON SEMI GENERALIZED STAR b-CLOSED SET IN TOPOLOGICAL SPACES

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Abstract: In this paper, we introduce a new class of sets called semi generalized star b-closed sets in topological spaces (briefly sg^*b-closed set). Also we discuss some of their properties and investigate the relations between the associated topology.

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1. Introduction

In 1970, Levine introduced the concept of generalized closed set and discussed the properties of sets, closed and open maps, compactness, normal and separation axioms. Later in 1996 Andrjivic gave a new type of generalized closed set in topological space called b closed sets. The investigation on generalization of closed set has lead to significant contribution to the theory of separation axiom, generalization of continuity and covering properties. A.A. Omari and M.S.M. Noorani made an analytical study and gave the concepts of generalized b closed sets in topological spaces.

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In this paper, a new class of closed set called semi generalized star b-closed set is introduced to prove that the class forms a topology. The notion of semi generalized star b-closed set and its different characterizations are given in this paper. Throughout this paper (X, τ) and (Y, σ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

Let $A \subseteq X$, the closure of A and interior of A will be denoted by $cl(A)$ and $int(A)$ respectively, union of all b-open sets X contained in A is called b-interior of A and it is denoted by $bint(A)$, the intersection of all b-closed sets of X containing A is called b-closure of A and it is denoted by $bcl(A)$.

2. Preliminaries

Definition 2.1. Let A subset A of a topological space (X, τ), is called

1) a pre-open set [16] if $A \subseteq int(cl(A))$.

2) a semi-open set [13] if $A \subseteq cl(int(A))$.

3) a α -open set [17] if $A \subseteq int(cl(int(A)))$.

4) a α generalized closed set (briefly αg- closed) [14] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

5) a generalized $*$ closed set (briefly $g^{\square}$-closed)[20] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$ open in X.

6) a generalized b- closed set (briefly gb- closed) [2] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

7) a generalized semi-pre closed set (briefly gsp-closed) [9] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

8) a generalized pre- closed set (briefly gp- closed) [10] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

9) a generalized semi- closed set (briefly gs- closed) [9] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

10) a semi generalized closed set (briefly sg- closed) [6] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

11) a generalized pre regular closed set (briefly gpr-closed) [10] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

12) a semi generalized b- closed set (briefly sgb- closed) [11] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

13) a $\ddot{g}$-closed set [19] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg open in X.

3. Semi Generalized b Star-Closed Sets

In this section, we introduce semi generalized star b - closed set and investigate some of its properties.

Definition 3.1. A subset A of a topological space (X, τ), is called semi generalized star b - closed set (briefly $sg^{\star}b$-closed set) if $bcl(A) \subset U$ whenever $A \subset U$ and U is sg open in X.

Theorem 3.2. *Every closed set is $sg^{\star}b$ -closed.*

Proof. Let A be any closed set in X such that $A \subset U$, where U is sg open. Since $bcl(A) \subset cl(A) = A$. Therefore $bcl(A) \subset U$. Hence A is $sg^{\star}b$ -closed set in X. □

The converse of above theorem need not be true as seen from the following example.

Example 3.3. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{a, b\}\}$. The set $\{a, b\}$ is $sg^{\star}b$ - closed set but not a closed set.*

Theorem 3.4. *Every $\ddot{g}$ - closed set is $sg^{\star}b$ -closed set.*

Proof. Let A be any $\ddot{g}$ - closed set in X and U be any sg open set containing A. Then $bcl(A) \subset cl(A) \subset U$. Therefore $bcl(A) \subset U$. Hence A is $sg^{\star}b$ -closed set. □

The converse of above theorem need not be true as seen from the following example.

Example 3.5. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a, b\}\}$. The set $\{a, c\}$ is $sg^{\star}b$ - closed set but not a $\ddot{g}$ - closed set.*

Theorem 3.6. *Every semi closed set is $sg^{\star}b$ -closed set.*

Proof. Let A be any semi closed set in X and U be any sg open set containing A. Since A is semi closed set, $bcl(A) \subset scl(A) \subset U$. Therefore $bcl(A) \subset U$. Hence A is $sg^{\square}b$ closed set. $\square$

The converse of above theorem need not be true as seen from the following example.

Example 3.7. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a, b\}\}$. The set $\{b, c\}$ is $sg^{\square}b$ - closed set but not a semi closed set.*

Theorem 3.8. *Every α - closed set is $sg^{\square}b$ -closed set.*

Proof. Let A be any α - closed set in X and U be any sg open set containing A. Since A is α - closed, $bcl(A) \subset \alpha cl(A) \subset U$. Therefore $bcl(A) \subset U$. Hence A is $sg^{\square}b$-closed set. $\square$

The converse of above theorem need not be true as seen from the following example.

Example 3.9. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{a, b\}\}$. The set $\{a, b\}$ is $sg^{\square}b$ - closed set but not a α -closed set.*

Theorem 3.10. *Every pre - closed set is $sg^{\square}b$-closed set.*

Proof. Let A be any pre -closed set in X and U be any sg open set containing A. Since every A pre close set, $bcl(A) \subset pcl(A) \subset U$. Therefore $bcl(A) \subset U$. Hence A is $sg^{\square}b$-closed set. $\square$

The converse of above theorem need not be true as seen from the following example.

Example 3.11. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. The set $\{a\}$ is $sg^{\square}b$-closed set but not a pre - closed set.*

Theorem 3.12. *Every αg-closed set is $sg^{\square}b$-closed set.*

Proof. Let A be αg -closed set in X and U be any open set containing A. Since every open set is sg-open sets, we have $bcl(A) \subset \alpha cl(A) \subset U$. Therefore $bcl(A) \subset U$. Hence A is $sg^{\square}b$- closed set. $\square$

The converse of above theorem need not be true as seen from the following example.

Example 3.13. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a, b\}\}$. The set $\{b\}$ is $sg^{\square}b$- closed set but not a αg- closed set.*

Theorem 3.14. *Every $sg^{\square}b$- closed set is gsp- closed set.*

Proof. Let A be any $sg^{\star}b$-closed set such that U be any open set containing A. Since every open set is sg-open, we have $bcl(A) \subset spcl(A) \subset U$. Therefore $bcl(A) \subset U$. Hence A is gsp-closed set. □

The converse of above theorem need not be true as seen from the following example.

Example 3.15. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{a, c\}\}$. The set $\{a, b\}$ is gsp- closed set but not a $sg^{\star}b$- closed set.*

Theorem 3.16. *Every $sg^{\star}b$- closed set is gb- closed set.*

Proof. Let A be any $sg^{\star}b$-closed set in X such that U be any open set containing A. Since every open set is sg open, we have $bcl(A)$. Hence A is gb-closed set. □

The converse of above theorem need not be true as seen from the following example.

Example 3.17. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{a, b\}\}$. The set $\{a, c\}$ is gb- closed set but not a $sg^{\star}b$- closed set.*

Theorem 3.18. *Every sg- closed set is $sg^{\star}b$- closed set.*

Proof. Let A be any sg-closed set in X such that U be any semi open set containing A. Since every semi open set is sg open, we have $bcl(A) \subset scl(A) \subset U$. Therefore $bcl(A) \subset U$. Hence A is $sg^{\star}b$-closed set. □

The converse of above theorem need not be true as seen from the following example.

Example 3.19. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{a, b\}\}$. The set $\{a, b\}$ is $sg^{\star}b$- closed set but not a sg- closed set.*

4. Characteristics of sg^*b-Closed Sets

Theorem 4.1. *If a set A is $sg^{\star}b$-closed set then $bcl(A) - A$ contains no non empty sg closed set.*

Proof. Let F be a sg closed set in X such that $F \subseteq bcl(A) - A$. Then $A \subseteq X - F$. Since A is $sg^{\star}b$-closed set and $X - F$ is sg open then $bcl(A) \subseteq X - F$. (i.e.) $F \subseteq X - bcl(A)$. So F $\subseteq (X - bcl(A)) \cap (bcl(A) - A)$. Therefore $F = \phi$. □

Theorem 4.2. *If A is $sg^{\star}b$-closed set in X and $A \subseteq B \subseteq bcl(A)$. Then B is $sg^{\star}b$- closed set in X.*

Proof. Since $B \subseteq bcl(A)$, we have $bcl(B) \subseteq bcl(A)$ then $bcl(B) - B \subseteq bcl(A) - A$. By Theorem 4.1, $bcl(A) - A$ contains no non empty sg closed set. Hence $bcl(B) - B$ contains no non empty sg closed set. Therefore B is $sg^{\star}b$-closed set in X. □

Theorem 4.3. *If $A \subseteq Y \subseteq X$ and suppose that A is $sg^{\star}b$ closed set in X then A is $sg^{\star}b$- closed set relative to Y.*

Proof. Given that $A \subseteq Y \subseteq X$ and A is $sg^{\star}b$-closed set in X. To prove that A is $sg^{\star}b$- closed set relative to Y. Let us assume that $A \subseteq Y \cap U$, where U is sg- open in X. Since A is $sg^{\star}b$- closed set, $A \subseteq U$ implies $bcl(A) \subseteq U$. It follows that $Y \cap bcl(A) \subseteq Y \cap U$. That is A is $sg^{\star}b$- closed set relative to Y. □

Theorem 4.4. *If A is both sg open and $sg^{\star}b$-closed set in X, then A is b closed set.*

Proof. Since A is sg open and $sg^{\star}b$ closed in X, $bcl(A) \subseteq A$. But $A \subseteq bcl(A)$. Therefore $A = bcl(A)$. Hence A is b closed set. □

Theorem 4.5. *For $xinX$, then the set $X - \{x\}$ is a $sg^{\star}b$-closed set or sg-open.*

Proof. Suppose that $X - \{x\}$ is not sg open, then X is the only sg open set containing $X - \{x\}$. (i.e.) $bcl(X - \{x\}) \subseteq X$. Then $X - \{x\}$ is $sg^{\star}b$-closed in X. □

Note 4.6. $g^{\star}$-closed set and $sg^{\star}b$-closed set are independent to each other as seen from the following examples

Example 4.7. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. The set $\{b\}$ is $sg^{\star}b$-closed set but not a $g^{\star}$-closed set.*

Example 4.8. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{c\}, \{b, c\}\}$. The set $\{a, c\}$ is $g^{\star}$-closed set but not a $sg^{\star}b$-closed set.*

Note 4.9. gp-closed set and $sg^{\star}b$ closed set are independent to each other as seen from the following examples.

Example 4.10. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{c\}, \{a, c\}\}$. The set $\{b, c\}$ is gp -closed set but not a $sg^{\star}b$-closed set.*

Example 4.11. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{a, b\}\}$. The set $\{a, b\}$ is $sg^{\square}b$-closed set but not a gp-closed set.*

Note 4.12. $sg^{\square}b$ closed set and *gpr* closed set are independent to each other as seen from the following examples.

Example 4.13. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}\{c\}, \{a, c\}\}$. The set $\{a\}$ is $sg^{\square}b$-closed set but not a gpr-closed set.*

Example 4.14. *Let $X = \{a, b, c\}$ with $\tau = \{X, \phi, \{c\}, \{a, c\}\}$. The set $\{b, c\}$ is spr-closed set but not a $sg^{\square}b$-closed set.*

5. Semi Generalized Star b-Open Sets and Semi Generalized Star b-Neighbourhoods

In this section, we introduce semi generalized star b-open sets (briefly $sg^{\square}b$-open) and semi generalized star b-neighbourhoods (briefly $sg^{\square}b$-neighbourhood) in topological spaces by using the notions of $sg^{\square}b$-open sets and study some of their properties.

Definition 5.1. A subset A of a topological space (X, τ), is called semi generalized star b- open set (briefly $sg^{\square}b$-open set) if A^c is $sg^{\square}b$-closed in X. We denote the family of all $sg^{\square}b$-open sets in X by $sg^{\square}b$-$O(X)$.

Theorem 5.2. *If A and B are $sg^{\square}b$-open sets in a space X. Then $A \cap B$ is also $sg^{\square}b$-open set in X.*

Proof. If A and B are $sg^{\square}b$-open sets in a space X. Then A^c and B^c are $sg^{\square}b$-closed sets in a space X. By Theorem 4.6 $A^c \cup B^c$ is also $sg^{\square}b$-closed set in X. (i.e.) $A^c \cup B^c = (A \cap B)^c$ is a $sg^{\square}b$-closed set in X. Therefore $A \cap B$ $sg^{\square}b$-open set in X. $\square$

Theorem 5.3. *If $int(B) \subseteq B \subseteq A$ and if A is $sg^{\square}b$-open in X, then B is $sg^{\square}b$-open in X.*

Proof. Suppose that $int(B) \subseteq B \subseteq A$ and A is $sg^{\square}b$-open in X then $A^c \subseteq B^c \subseteq cl(A^c)$. Since A^c is $sg^{\square}b$-closed in X, by Theorem B is $sg^{\square}b$-open in X. $\square$

Definition 5.4. Let x be a point in a topological space X and let $x \in X$. A subset N of X is said to be a $sg^{\square}b$-neighbourhood of x iff there exists a $sg^{\square}b$-open set G such that $x \in G \subset N$.

Definition 5.5. A subset N of Space X is called a $sg^{\square}b$-neighbourhood of $A \subset X$ iff there exists a $sg^{\square}b$- open set G such that $A \subset G \subset N$.

Theorem 5.6. *Every neighbourhood N of $x \in X$ is a $sg^{\square}b$-neighbourhood of x.*

Proof. Let N be a neighbourhood of point $x \in X$. To prove that N is a $sg^{\square}b$-neighbourhood of x. By Definition of neighbourhood, there exists an open set G such that $x \in G \subset N$. Hence N is a $sg^{\square}b$-neighbourhood of x. $\square$

Remark 5.7. In general, a $sg^{\square}b$-neighbourhood of $x \in X$ need not be a neighbourhood of x in X as seen from the following example.

Example 5.8. *Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{c\}, \{a, c\}\}$. Then $sg^{\square}b$-$O(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$. The set $\{b, c\}$ is $sg^{\square}b$-neighbourhood of point c, since the $sg^{\square}b$-open sets $\{c\}$ is such that $c \in \{c\} \subset \{b, c\}$. However, the set $\{b, c\}$ is not a neighbourhood of the point c, since no open set G exists such that $c \in G \subset \{b, c\}$.*

Remark 5.9. The $sg^{\square}b$-neighbourhood N of $x \in X$ need not be a $sg^{\square}b$-open in X.

Theorem 5.10. *If a subset N of a space X is $sg^{\square}b$-open, then N is $sg^{\square}b$-neighbourhood of each of its points.*

Proof. Suppose N is $sg^{\square}b$-open. Let $x \in N$. We claim that N is $sg^{\square}b$-neighbourhood of x. For N is a $sg^{\square}b$-open set such that $x \in N \subset N$. Since x is an arbitrary point of N, it follows that N is a $sg^{\square}b$-neighbourhood of each of its points. $\square$

Theorem 5.11. *Let X be a topological space. If F is $sg^{\square}b$-closed subset of X and $x \in F^c$. Prove that there exists a $sg^{\square}b$-neighbourhood N of x such that $N \cap F = \phi$.*

Proof. Let F be $sg^{\square}b$-closed subset of X and $x \in F^c$. Then F^c is $sg^{\square}b$-open set of X. So by Theorem 5.10 F^c contains a $sg^{\square}b$-neighbourhood of each of its points. Hence there exists a $sg^{\square}b$-neighbourhood N of x such that $N \subset F^c$. (i.e.) $N \cap F = \phi$. $\square$

Definition 5.12. Let x be a point in a topological space X. The set of all $sg^{\square}b$-neighbourhood of x is called the $sg^{\square}b$-neighbourhood system at x, and is denoted by $sg^{\square}b$-$N(x)$.

Theorem 5.13. *Let a $sg^{\square}b$-neighbourhood N of X be a topological space and each $x \in X$, Let $sg^{\square}b$- $N(X,\tau)$ be the collection of all $sg^{\square}b$-neighbourhood of x. Then we have the following results.*

(i) $x \in, sg^{\square}b - N(x) \neq \phi$.

(ii) $N \in sg^{\square}b - N(x) \Rightarrow x \in N$.

(iii) $N \in sg^{\square}b - N(x), M \supset N \Rightarrow M \in sg^{\square}b - N(x)$.

(iv) $N \in sg^{\square}b - N(x), M \in sg^{\square}b - N(x) \Rightarrow N \cap M \in sg^{\square}b - N(x)$, *if finite intersection of $sg^{\square}b$ open set is $sg^{\square}b$ open.*

(v) $N \in sg^{\square}b - N(x) \Rightarrow$ *there exists* $M \in sg^{\square}b - N(x)$ *such that* $M \subset N$ *and* $M \in sg^{\square}b - N(y)$ *for every* $y \in M$.

Proof. 1. Since X is $sg^{\square}b$-open set, it is a $sg^{\square}b$-neighbourhood of every $x \in X$. Hence there exists at least one $sg^{\square}b$-neighbourhood (namely-X) for each $x \in X$. Therefore $sg^{\square}b - N(x) \neq \phi$ for every $x \in X$.

2. If $N \in sg^{\square}b - N(x)$, then N is $sg^{\square}b$-neighbourhood of x. By Definition of $sg^{\square}b$-neighbourhood, $x \in N$.

3. Let $N \in sg^{\square}b - N(x)$ and $M \supset N$. Then there is a $sg^{\square}b$-open set G such that $x \in G \subset N$. Since $N \subset M$, $x \in G \subset M$ and so M is $sg^{\square}b$-neighbourhood of x. Hence $M \in sg^{\square}b - N(x)$.

4. Let $N \in sg^{\square}b - N(x)$, $M \in sg^{\square}b - N(x)$. Then by Definition of $sg^{\square}b$-neighbourhood, there exists $sg^{\square}b$- open sets G_1 and G_2 such that $x \in G_1 \subset N$ and $x \in G_2 \subset M$. Hence

$$x \in G_1 \cap G_2 \subset N \cap M \tag{1}$$

Since $G_1 \cap G_2$ is a $sg^{\square}b$-open set, it follows from (1) that $N \cap M$ is a $sg^{\square}b$-neighbourhood of x.
Hence $N \cap M \in sg^{\square}b - N(x)$.

5. Let $N \in sg^{\square}b - N(x)$, Then there is a $sg^{\square}b$-open set M such that $x \in M \subset N$. Since M is $sg^{\square}b$-open set, it is $sg^{\square}b$-neighbourhood of each of its points.
Therefore $M \in sg^{\square}b - N(y)$ for every $y \in M$.

□

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