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L-FUZZY (K,E)-SOFT PRE-PROXIMITIES INDUCED BY L-FUZZY (K,E)-SOFT QUASI-UNIFORM SPACES

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Abstract: In a strictly two-sided commutative quantale having an order reversing involution, we introduce the notion of L- fuzzy (K,E)-soft pre-proximity spaces and L-fuzzy (K,E)-soft pre-uniform spaces. We investigate their properties. In particular, an L-fuzzy (K,E)-soft pre-uniformity induces an L–fuzzy (K,E)-soft pre-proximity. We give their examples.

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1. Introduction

In 1999 Molodtsov [14] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. In [15], Molodtsov applied successfully in directions such as, smoothness of func-

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tions, game theory, operations research, Riemann-integration, Perron integration, probability and theory of measurement.

Maji et al. [12,13] gave the first practical application of soft sets in decision making problems. In 2003, Maji et al. [13] defined and studied several basic notions of soft set theory. Many researchers have contributed towards the algebraic structure of soft set theory [1-3,7]. In 2011, Shabir and Naz [19] initiated the study of soft topological spaces. They defined soft topology on the collection of soft sets over X and established their several properties. Aygünoglu et.al [4] introduced the concept of soft topology in the sense of Šostak [10]. Cetkin et.al [5] studied soft proximities and discuss their properties.

Hájek [8] introduced a complete residuated lattice which is an algebraic structure for many valued logic and decision rules in complete residuated lattices. Höhle [9,10] introduced L-fuzzy topologies with algebraic structure L(cqm, quantales, MV-algebra). It has developed in many directions [17-19].

The notion of an L-fuzzy pre-proximity spaces, L is a strictly two sided commutative quantale lattice having a strong negation was introduced by Kim et al. [11]. Lather, Ramadan et al. [17,18] define the the concept of L- fuzzy soft topogenous orders, L- fuzzy soft uniform spaces, L- fuzzy soft topological spaces in strictly two sided commutative quantales and investigated the relation between them.

The purpose of this paper is to introduce the notion of L-fuzzy (K, E)-soft pre-proximity spaces and L-fuzzy (K, E)-soft pre-uniform spaces. We investigate their properties. In particular, an L-fuzzy (K, E)-soft pre-uniformity induces an L–fuzzy (K, E)-soft pre-proximity. We give their examples.

2. Preliminaries

Let $L = (L, \leq, \vee, \wedge, 0, 1)$ be a completely distributive lattice with the least element 0 and the greatest element 1 in L.

Definition 2.1. [9-11] A complete lattice $(L, \leq, \odot)$ is called a strictly two-sided commutative quantale (stsc-quantale, for short) iff it satisfies the following properties.

(L1) $(L, \odot)$ is a commutative semigroup,

(L2) $x = x \odot 1$, for each $x \in L$ and 1 is the universal upper bound,

(L3) $\odot$ is distributive over arbitrary joins, i.e. $(\bigvee_i x_i) \odot y = \bigvee_i (x_i \odot y)$.

There exists a further binary operation $\rightarrow$ (called the implication operator or residuated) satisfying the following condition

$$x \rightarrow y = \bigvee \{z \in L | x \odot z \leq y\}.$$

Then it satisfies Galois correspondence; i.e, $(x \odot z) \leq y$ iff $z \leq (x \to y)$.

In this paper, we always assume that $(L, \leq, \odot, \to, \oplus, ^*)$ is a stsc-quantales with an order reversing involution * which is defined by

$$x \oplus y = (x^* \odot y^*)^*, \quad x^* = x \to 0$$

unless otherwise specified.

Remark 2.2. Every completely distributive lattice $(L, \leq, \wedge, \vee, ^*)$ with order reversing involution * is a stsc-quantale $(L, \leq, \odot, \oplus, ^*)$ with a strong negation * where $\odot = \wedge$ and $\oplus = \vee$.

Lemma 2.3. [9-11] For each $x, y, z, x_i, y_i, w \in L$, we have the following properties.

(1) $1 \to x = x$, $0 \odot x = 0$,

(2) If $y \leq z$, then $x \odot y \leq x \odot z$, $x \oplus y \leq x \oplus z$, $x \to y \leq x \to z$ and $z \to x \leq y \to x$,

(3) $x \leq y$ iff $x \to y = 1$.

(4) $(\bigwedge_i y_i)^* = \bigvee_i y_i^*$, $(\bigvee_i y_i)^* = \bigwedge_i y_i^*$,

(5) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$,

(6) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$,

(7) $x \to (\bigvee_i y_i) \geq \bigvee_i (x \to y_i)$,

(8) $(\bigwedge_i x_i) \to y \geq \bigvee_i (x_i \to y)$,

(9) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$,

(10) $x \odot y = (x \to y^*)^*$ and $x \oplus y = x^* \to y$,

(11) $(x \to y) \odot (z \to w) \leq (x \odot z) \to (y \odot w)$,

(12) $x \to y \leq (x \odot z) \to (y \odot z)$ and $(x \to y) \odot (y \to z) \leq x \to z$,

(13) $(x \to y) \odot (z \to w) \leq (x \oplus z) \to (y \oplus w)$.

(14) $x \to y = y^* \to x^*$.

(15) $(x \vee y) \odot (z \vee w) \leq (x \vee z) \vee (y \odot w) \leq (x \oplus z) \vee (y \odot w)$.

(16) $\bigvee_{i \in \Gamma} x_i \to \bigvee_{i \in \Gamma} y_i \geq \bigwedge_{i \in \Gamma} (x_i \to y_i)$ and $\bigwedge_{i \in \Gamma} x_i \to \bigwedge_{i \in \Gamma} y_i \geq \bigwedge_{i \in \Gamma} (x_i \to y_i)$,

(17) $(x \odot y) \odot (z \oplus w) \leq (x \odot z) \oplus (y \odot w)$.

Throughout this paper, X refers to an initial universe, E and K are the sets of all parameters for X, and L^X is the set of all L-fuzzy sets on X.

Definition 2.4. [4] A map f is called an L- fuzzy soft set on X, where f is a mapping from E into L^X, i.e., $f_e := f(e)$ is an L- fuzzy set on X, for each $e \in E$. The family of all L- fuzzy soft sets on X is denoted by $(L^X)^E$. Let f and g be two L- fuzzy soft sets on X.

(1) f is an L-fuzzy soft subset of g and we write $f \sqsubseteq g$ if $f_e \leq g_e$, for each $e \in E$. f and g are equal if $f \sqsubseteq g$ and $g \sqsubseteq f$.

(2) The intersection of f and g is an L- fuzzy soft set $h = f \sqcap g$, where $h_e = f_e \wedge g_e$, for each $e \in E$.

(3) The union of f and g is an L- fuzzy soft set $h = f \sqcup g$, where $h_e = f_e \vee g_e$, for each $e \in E$.

(4) An L- fuzzy soft set $h = f \odot g$ is defined as $h_e = f_e \odot g_e$, for each $e \in E$.

(5) An L- fuzzy soft set $h = f \oplus g$ is defined as $h_e = f_e \oplus g_e$, for each $e \in E$.

(6) The complement of an L- fuzzy soft sets on X is denoted by f^*, where $f^* : E \to L^X$ is a mapping given by $f^*_e = (f_e)^*$, for each $e \in E$.

(7) f is called a null L- fuzzy soft set and is denoted by 0_X , if $f_e(x) = 0$, for each $e \in E$, $x \in X$.

(8) f is called an absolute L- fuzzy soft set and is denoted by 1_X , if $f_e(x) = 1$, for each $e \in E$, $x \in X$ and $(1_x)_e(x) = 1$.

Definition 2.5. [4] Let $\varphi : X \to Y$ and $\psi : E \to K$ be two mappings, where E and K are parameters sets for the crisp sets X and Y, respectively. Then $\varphi_\psi : (X, E) \to (Y, K)$ is called a fuzzy soft mapping. Let f and g be two fuzzy soft sets over X and Y, respectively and let φ_ψ be a fuzzy soft mapping from (X, E) into (Y, K).

(1) The image of f under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi(f)$ is the fuzzy soft set on Y defined by

$$\varphi(f)_k(y) = \begin{cases} \bigvee_{\varphi(x)=y} \left(\bigvee_{\psi(e)=k} f_e(x) \right), & \text{if } x \in \varphi^{-1}(y) \\ 0, & \text{otherwise,} \end{cases}$$

$\forall k \in K, \forall y \in Y$.

(2) The pre-image of g under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi^{-1}(g)$ is the fuzzy soft set on X defined by

$$\varphi_\psi^{-1}(g)_e(x) = g_{\psi(e)}(\varphi(x)), \forall e \in E, \forall x \in X.$$

Definition 2.6. [16] An L- fuzzy (K, E)-soft pre-uniformity is a mapping $\mathcal{U} : K \to L^{(L^{X \times X})^E}$ which satisfies the following conditions .

(SU1) There exists $u \in (L^{X \times X})^E$ such that $\mathcal{U}_k(u) = 1$.

(SU2) If $v \sqsubseteq u$, then $\mathcal{U}_k(v) \leq \mathcal{U}_k(u)$.

(SU3) For every $u, v \in (L^{X \times X})^E, \mathcal{U}_k(u \odot v) \geq \mathcal{U}_k(u) \odot \mathcal{U}_k(v)$.

(SU4) If $\mathcal{U}_k(u) \neq 0$ then $\top_\triangle \sqsubseteq u$ where, for each $e \in E$,

$$(\top_\triangle)_e(x, y) = \begin{cases} 1, & \text{if } x = y, \\ 0, & \text{if } x \neq y. \end{cases}$$

The pair $(X,\mathcal{U})$ is called an L-fuzzy (K,E)-soft pre-uniform space.

An L- fuzzy (K,E)-soft pre-uniformity is an L- fuzzy (K,E)-soft quasi-uniformity if

(UQ) $\mathcal{U}_k(u) \leq \bigvee\{\mathcal{U}_k(v)\odot\mathcal{U}_k(w) \mid v\circ w \sqsubseteq u\}$, where

$$v_e \circ w_e(x,z) = \bigvee_{y\in X} v_e(x,y)\odot w_e(y,z),$$

An L-fuzzy (K,E)-soft quasi-uniform space $(X,\mathcal{U})$ is said to be an L-fuzzy (K,E)-soft uniform space if

(U) $\mathcal{U}_k(u) \leq \mathcal{U}_k(u^{-1})$, where $(u^{-1})_e(x,y) = u_e(y,x)$ for each $k\in K$ and $u \in (L^{X\times X})^E$.

Let $(X,\mathcal{U}^1)$ be an L-fuzzy (K_1,E_1)-soft pre-uniform space and $(Y,\mathcal{U}^2)$ be an L-fuzzy (K_2,E_2)-soft pre-uniform space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from $(X,\mathcal{U}^1)$ into $(Y,\mathcal{U}^2)$ is called L-fuzzy soft uniformly continuous if

$$\mathcal{U}^2_{\eta(k)}(v) \leq \mathcal{U}^1_k((\varphi\times\varphi)^{-1}_{\psi}(v)) \ \ \forall\, v \in (L^{Y\times Y})^{E_2}, k\in K_1.$$

Remark 2.7. Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft uniform space.

(1) By (SU1) and (SU2), we have $\mathcal{U}_k(1_{X\times X}) = 1$ because $u \sqsubseteq 1_{X\times X}$ for all $u \in (L^{X\times X})^E$.

(2) Since $\mathcal{U}_k(u) \leq \mathcal{U}_k(u^{-1}) \leq \mathcal{U}_k((u^{-1})^{-1}) = \mathcal{U}_k(u)$, then $\mathcal{U}_k(u) = \mathcal{U}_k(u^{-1})$.

3. L-fuzzy (K,E)-soft pre-proximities induced by L-fuzzy (K,E)-soft pre-uniformities

Definition 3.1. A mapping $\delta : K \to L^{(L^X)^E\times(L^X)^E}$ ($\delta_k = \delta(k) : (L^X)^E \times (L^X)^E \to L$) is called an L-fuzzy (K,E)-soft pre-proximity on X if it satisfies the following axioms.

(SP1) $\delta_k(0_X,1_X) = \delta_k(1_X,0_X) = 0$.

(SP2) If $\delta_k(f,g) \neq 1$, then $f \sqsubseteq g^*$.

(SP3) If $f_1 \sqsubseteq f_2$ and $g_1 \sqsubseteq g_2$, then $\delta_k(f_1,g_1) \leq \delta_k(f_2,g_2)$.

(SP4) $\delta_k(f_1\odot f_2, g_1\oplus g_2) \leq \delta_k(f_1,g_1)\oplus\delta_k(f_2,g_2)$.

The pair (X,δ) is called an L-fuzzy (K,E)-soft pre-proximity space.

An L-fuzzy (K,E)-soft pre-proximity is called an L-fuzzy (K,E)-soft quasi-proximity on X if

(PQ) $\delta_k(f,g) \geq \bigwedge_h\{\delta_k(f,h)\oplus\delta_k(h^*,g)\}$.

An L-fuzzy (K,E)-soft quasi-proximity is called an L-fuzzy (K,E)-soft proximity on X if

(SP) $\delta_k(f,g)=\delta_k(g,f)$.

Let (X,δ^1) be an L-fuzzy (K_1,E_1)-soft quasi proximity space and (Y,δ^2) be an L-fuzzy (K_2,E_2)-soft pre-proximity space. Let $\varphi: X\to Y$, $\psi: E_1\to E_2$ and $\eta: K_1\to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from (X,δ^1) into (Y,δ^2) is called L-fuzzy soft proximally continuous if

$$\delta^1_k(f,g)\le\delta^2_{\eta(k)}(\varphi_\psi(f),\varphi_\psi(g))\;\;\forall\, f,g\in(L^X)^{E_1},k\in K_1.$$

or equivalently, $\delta^1_k(\varphi^{-1}_\psi(f),\varphi^{-1}_\psi(g))\le\delta^2_{\eta(k)}(f,g)\;\;\forall\, f,g\in(L^Y)^{E_2},k\in K_1.$

Theorem 3.2. Let $(X,\mathcal{U})$ be an L-fuzzy (K,E)-soft pre-uniform space. Define a mapping $\delta^{\mathcal{U}}: K\to L^{(L^X)^E\times(L^X)^E}$ by

$$\delta^{\mathcal{U}}_k(f,g)=\bigwedge\{\mathcal{U}^*_k(u)\mid u[f]\sqsubseteq g^*\},$$

where $u_e[f_e](x)=\bigvee_{y\in X}(f_e(y)\odot u_e(y,x)),\ \forall x\in X,\forall e\in E,\ \forall u\in(L^{X\times X})^E$ and $f\in(L^X)^E$. Then we have the following properties.

(1) $\delta^{\mathcal{U}}$ is an L-fuzzy (K,E)-soft pre-proximity space.

(2) If $(X,\mathcal{U})$ is an L-fuzzy (K,E)-soft quasi-uniform space, then $\delta^{\mathcal{U}}$ is L-fuzzy (K,E)-soft quasi-proximity space.

Proof. (1) (SP1) Since $u[0_X]=0_X$ and $u[1_X]=1_X$ for $\mathcal{U}_k(u)=1$, we have $\delta^{\mathcal{U}}_k(0_X,1_X)=0$ and $\delta^{\mathcal{U}}_k(0_X,1_X)=0$.

(SP2) Let $f\not\sqsubseteq g^*$ be given. Since $f\sqsubseteq u[f]$ for all $\mathcal{U}_k(u)>0$, we have $u[f]\not\sqsubseteq g^*$. By the definition of $\delta^{\mathcal{U}}_k$, we have $\delta^{\mathcal{U}}_k(f,g)=1$.

(SP3) Easily proved.

(SP4) First we show that $(u\odot v)[f\odot g]\sqsubseteq u[f]\odot v[g]$, from:

$$\begin{aligned}
&u_e[f_e](x)\odot v_e[g_e](x)\\
&=\Big(\bigvee_{y\in X}(f_e(y)\odot u_e(y,x))\Big)\odot\Big(\bigvee_{z\in X}(g_e(z)\odot v_e(z,x))\Big)\\
&\ge\bigvee_{y\in X}\Big(f_e(y)\odot u_e(y,x))\Big)\odot\Big(g_e(y)\odot v_e(y,x)\Big)\\
&=\bigvee_{y\in X}\Big((f_e\odot g_e)(y)\odot(u_e\odot v_e)(y,x)\Big)\\
&=(u_e\odot v_e)[f_e\odot g_e](x).
\end{aligned}$$

$$\begin{aligned}
&\delta_k^{\mathcal{U}}(f_1, g_1) \oplus \delta_k^{\mathcal{U}}(f_2, g_2)\\
&= \bigwedge\{\mathcal{U}_k^*(u) \mid u[f_1] \sqsubseteq g_1^*\} \oplus \bigwedge\{\mathcal{U}_k^*(v) \mid v[f_2] \sqsubseteq g_2^*\}\\
&= \bigwedge\{\mathcal{U}_k^*(u) \oplus \mathcal{U}_k^*(v) \mid u[f_1] \sqsubseteq g_1^*,\ v[f_2] \sqsubseteq g_2^*\}\\
&\geq \bigwedge\{\mathcal{U}_k^*(u) \oplus \mathcal{U}_k^*(v) \mid u[f_1] \odot v[f_2] \sqsubseteq g_1^* \odot g_2^*\}\\
&\geq \bigwedge\{\mathcal{U}_k^*(u \odot v) \mid (u \odot v)[f_1 \odot f_2] \sqsubseteq (g_1 \oplus g_2)^*\}\\
&\geq \bigwedge\{\mathcal{U}_k^*(w) \mid w[f_1 \odot f_2] \sqsubseteq (g_1 \oplus g_2)^*\}\\
&= \delta_k^{\mathcal{U}}(f_1 \odot f_2, g_1 \oplus g_2).
\end{aligned}$$

Hence $\delta^{\mathcal{U}}$ is an L-fuzzy (K, E)-soft pre-proximity space.
(2) (PQ) For each $u \in (L^{X\times X})^E$ such that $u[f] \sqsubseteq g^*$, by (UQ), we have

$$\mathcal{U}_k(u) \leq \bigvee\{\mathcal{U}_k(v) \odot \mathcal{U}_k(w) \mid v \circ w \sqsubseteq u\}.$$

Thus,

$$\begin{aligned}
\delta_k^{\mathcal{U}}(f, g) &= \bigwedge\{\mathcal{U}_k^*(u) \mid u[f] \sqsubseteq g^*\}\\
&\geq \bigwedge\{\mathcal{U}_k^*(v) \oplus \mathcal{U}_k^*(w) \mid v[w[f]] \sqsubseteq g^*\}\\
&= \bigwedge\{\mathcal{U}_k^*(v) \oplus \mathcal{U}_k^*(w) \mid w[f] = (w[f]^*)^*, v[w[f]] \sqsubseteq g^*\}\\
&\geq \bigwedge_{h\in L^X}\{\bigwedge\{\mathcal{U}_k^*(v) \oplus \mathcal{U}_k^*(w) \mid w[f] \sqsubseteq h^*,\ v[h^*] \sqsubseteq g^*\}\}\\
&\geq \bigwedge_{h\in L^X}\{\bigwedge\{\mathcal{U}_k^*(w) \mid w[f] \sqsubseteq h^*\} \oplus \bigwedge\{\mathcal{U}_k^*(v) \mid v[h^*] \sqsubseteq g^*\}\}\\
&= \bigwedge_{h\in L^X}(\delta_k^{\mathcal{U}}(f, h) \oplus \delta_k^{\mathcal{U}}(h^*, g)).
\end{aligned}$$

Hence, $\delta_k^{\mathcal{U}}(f, g) = \bigwedge\{\mathcal{U}_k^*(u) \mid u[f] \sqsubseteq g^*\} \geq \bigwedge_{h\in L^X}(\delta_k^{\mathcal{U}}(f, h) \oplus \delta_k^{\mathcal{U}}(h^*, g))$.

Theorem 3.3. Let $(X, \mathcal{U})$ be an L-fuzzy (K_1, E_1)-soft quasi- uniform space and $(Y, \mathcal{V})$ be an L-fuzzy (K_2, E_2)-soft quasi- uniform space, $\phi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ are functions. If $\phi_{\psi,\eta}$ from $(X, \mathcal{U})$ into $(X, \mathcal{V})$ is L-fuzzy soft uniformly continuous, then $\phi_{\psi,\eta} : (X, \delta^{\mathcal{U}}) \to (Y, \delta^{\mathcal{V}})$ is L-fuzzy soft proximally continuous.

Proof. Since

$$\begin{aligned}
&(\phi \times \phi)_\psi^{-1}(v)_e[(\phi_\psi^{-1}(f))_e](x)\\
&= \bigvee_{z\in X} \phi_\psi^{-1}(f)_e(z) \odot (\phi \times \phi)_\psi^{-1}(v_e)(z, x)\\
&= \bigvee_{z\in X} f_{\psi(e)}(\phi(z)) \odot v_{\psi(e)}(\phi(z), \phi(x))\\
&\leq \bigvee_{y\in Y} f_{\psi(e)}(y) \odot v_{\psi(e)}(y, \phi(x))\\
&= \phi_\psi^{-1}(v_{\psi(e)}[f_{\psi(e)}])(x),
\end{aligned}$$

we have

$$\begin{aligned}
&\delta^{\mathcal{V}}_{\eta(k)}(f,h) = \bigwedge\{\mathcal{V}^*_{\eta(k)}(v) \mid v[f] \leq h^*\}\\
&\geq \bigwedge\{\mathcal{U}^*_k((\phi\times\phi)^{-1}_{\psi}(v)) \mid \phi^{-1}_{\psi}(v)[f] \leq \phi^{-1}_{\psi}(h)^*\}\\
&\geq \bigwedge\{\mathcal{U}^*_k((\phi\times\phi)^{-1}_{\psi}(v)) \mid (\phi\times\phi)^{-1}_{\psi}(v)[\phi^{-1}_{\psi}(f)] \leq \phi^{-1}_{\psi}(h)^*\}\\
&\geq \bigwedge\{\mathcal{U}^*_k(u) \mid u[\phi^{-1}_{\psi}(f)] \leq \phi^{-1}_{\psi}(h)^*\}\\
&= \delta^{\mathcal{U}}_k(\phi^{-1}_{\psi}(f), \phi^{-1}_{\psi}(h)).
\end{aligned}$$

Hence, $\phi_{\psi,\eta} : (X, \delta^{\mathcal{U}}) \to (Y, \delta^{\mathcal{V}})$ is L-fuzzy soft proximally continuous.

Example 3.4. Let $X = \{h_i \mid i = \{1, 2, 3\}\}$ with h_i=house and $E = \{e, b\}$ with e=expensive, b= beautiful. Define a binary operation $\odot$ on $[0, 1]$ by

$$x \odot y = \max\{0, x + y - 1\},\ x \to y = \min\{1 - x + y, 1\}$$

$$x \oplus y = \min\{1, x + y\},\ x^* = 1 - x$$

Then $([0, 1], \wedge, \to, 0, 1)$ is a stsc-quantle.

(1) Put $v, v \odot v, w \in ([0, 1]^{X\times X})^E$ as

$$v_e = \begin{pmatrix} 1 & 0.6 & 0.5\\ 0.3 & 1 & 0.5\\ 0.4 & 0.6 & 1 \end{pmatrix}, v_b = \begin{pmatrix} 1 & 0.5 & 0.3\\ 0.7 & 1 & 0.5\\ 0.6 & 0.6 & 1 \end{pmatrix}$$

$$(v\odot v)_e = \begin{pmatrix} 1 & 0.2 & 0\\ 0 & 1 & 0\\ 0 & 0.2 & 1 \end{pmatrix}, (v\odot v)_b = \begin{pmatrix} 1 & 0 & 0\\ 0.4 & 1 & 0\\ 0.2 & 0.2 & 1 \end{pmatrix}$$

$$w_e = \begin{pmatrix} 1 & 0.4 & 0.5\\ 0.4 & 1 & 0.5\\ 0.4 & 0.6 & 1 \end{pmatrix}, v_b = \begin{pmatrix} 1 & 0.5 & 0.3\\ 0.3 & 1 & 0.5\\ 0.2 & 0.3 & 1 \end{pmatrix}$$

We define $\mathcal{U} : E \to [0, 1]^{([0,1]^{X\times X})^E}$ as follows:

$$\mathcal{U}_e(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y}\\ 0.6, & \text{if } v \sqsubseteq u \neq 1_{Y\times Y},\\ 0.3, & \text{if } v\odot v \sqsubseteq u \not\sqsupseteq v,\\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{U}_b(u) = \begin{cases} 1, & \text{if } u = 1_{Y\times Y}\\ 0.5, & \text{if } w \sqsubseteq u \neq 1_{Y\times Y},\\ 0, & \text{otherwise.} \end{cases}$$

Since $v \circ v = v$, $w \circ w = w$ and $(v \odot v) \circ (v \odot v) = (v \odot v)$, $\mathcal{U}$ is not a $[0,1]$-fuzzy (E, E)-soft quasi-uniformity on X because

$$0.6 = \mathcal{U}_k(v) \not\leq \bigvee \{\mathcal{U}_k(u) \odot \mathcal{U}_k(w) \mid u \circ w \sqsubseteq v\} = 0.2.$$

From Theorem 3.2, we obtain a $[0,1]$-fuzzy (E, E)-soft pre-proximity $\delta^{\mathcal{U}} : E \to [0,1]^{([0,1]^X)^E \times ([0,1]^X)^E}$ as follows

$$\delta^{\mathcal{U}}_e(f, g) = \begin{cases} 0, & \text{if } [1_{X\times X}](f) \sqsubseteq g^* \not\sqsupseteq v[f], \\ 0.4, & \text{if } v[f] \sqsubseteq g^* \not\sqsupseteq (v \odot v)[f], \\ 0.7, & \text{if } \ (v \odot v)[f] \sqsubseteq g^* \\ 1, & \text{otherwise,} \end{cases}$$

$$\delta^{\mathcal{U}}_b(f, g) = \begin{cases} 0, & \text{if } [1_{X\times X}](f) \sqsubseteq g^* \not\sqsupseteq w[f], \\ 0.5, & \text{if } w[f] \sqsubseteq g^*, \\ 1, & \text{otherwise,} \end{cases}$$

But it is not a $[0,1]$-fuzzy (E, E) soft quasi-proximity because, for each $0_X \neq f \in ([0,1]^X)^E$ with $w[f] \leq g^* \neq 1_X$, $\delta^{\mathcal{U}}_b(f, h) \neq 0$ and $\delta^{\mathcal{U}}_b(h^*, g) \neq 0$ imply

$$0.5 = \delta^{\mathcal{U}}_b(f, g) \not\geq \bigwedge_{h \in ([0,1]^X)^E} (\delta^{\mathcal{U}}_b(f, h) \oplus \delta^{\mathcal{U}}_b(h^*, g)).$$

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