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RAINBOW VERTEX COLORING FOR CENTRAL AND TOTAL GRAPH OF STAR GRAPH

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Abstract: The conception of rainbow connection was introduced by Chartrand et al [1]. It is gripping and late occurrence, produce lot of papers with good-quality literary works about it. In this paper we endeavour to bring together most of the results and papers that dealt with it. We debut with an introduction and organize the work into categories as rainbow vertex connection number, observation, Results and Conclusion follows.

AMS Subject Classification: 05C15, 05C40
Key Words: rainbow path, rainbow vertex connection number

1. Introduction

The most fundamental graph-theoretic subject is connectivity in dual sense (combinatorial and algorithmic). Graph theory travels with elegant and powerful results on connectivity. There are also many ways to give support to argument about connectivity. Connectivity on behalf of supportive arguments requirement, the rainbow-connection, was instigated by Chartrand et al [1] in 2008. This new concept kneeled, due to fortify the information through communication between agencies of government. While the information needs to protect, since it is concomitant to national security, there must also be procedures that permit access between appropriate parties. Here choosing one or more secure paths between every pair of agencies have no password repeated.

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In order to make minimum passwords needed that allows one or more secure paths between every two agencies so that the passwords along each path are distinct. However, it is matched to Graph-theoretic model. Let G be nontrivial connected Graph on which on which an edge Coloring mapping positive integers is defined, where adjacent edges may be colored the same. A path is rainbow if no two edges of it are colored the same. An edge coloring Graph G is Rainbow connected if every two vertices are connected by rainbow path. An edge - coloring under which G is rainbow connected is called a rainbow coloring. As introduced in [1] the rainbow connection number of a connected Graph, denoted by rc(G), is the smallest number of colors that are needed in order to make G rainbow connected. A rainbow coloring using rc(G) colors is called minimum rainbow coloring. A tree T in G is a rainbow tree if no two edges of T are colored the same This paper is mainly concerned with rainbow vertex coloring and connection number of Central and Total graph of star graph. In addition to regarding it as a natural combinatorial measure and its application to the secure transfer of classified information between agencies, the rainbow connection number can also be motivated by its interesting in the area of networking [2]: Suppose that G represents a network, we wish to route messages between any two vertices in pipeline, and require that each link on the route between the vertices is assigned a distinct channels that we use in our network. This number is precisely rc(G). Let c be rainbow coloring of a connected graph G For any two vertices u and v of G, a rainbow u-v geodesic in G is a Rainbow u-v path of length $d(u, v)$ where $d(u, v)$ is the distance between u and v in G. A Graph G is strongly rainbow connected if there exists a rainbow u-v geodesic for any two vertices u and v in G. In this case, the coloring c is called a strong rainbow coloring of G. Similarly we define the strong rainbow connection number of a connected graph G, denoted $src(G)$, as the smallest number of colors that are needed in order to make G strong rainbow connected [1]. Note this number is also called the rainbow diameter number in [2]. A strong rainbow coloring of G using src(G) colors is called a minimum strong rainbow coloring of G. Clearly, we have $diam(G) \leq rc(G) \leq src(G) \leq m$ where $diam(G)$ denotes the diameter of G and m is the size of $G.rvc(G) = 0$ if G is a clique. In a rainbow coloring, we need only find one rainbow path connecting every two vertices. There is a natural Generalization: the number of rainbow paths between any two vertices is at least an integer k with $k \geq 1$ in some edge coloring. We call an edge coloring a rainbow k- coloring if there are at least k internally disjoint u-v paths connecting any two distinct vertices u and v. A vertex colored graph G is rainbow vertex- connected if any two vertices are connected by a path whose internal vertices have distinct colors. A vertex-coloring under which G is rainbow vertex-connected is called a rainbow vertex-coloring. The Rainbow vertex-connection number of a connected Graph G denoted by $rvc(G)$, is the smallest number of colors that are needed in order to make G is Rainbow vertex-connected. Obviously we have $rvc(G) \leq n-2$ (except for the trivial graph) and $rvc(G) = 0$ iff G is a clique. In rainbow vertex coloring, we assign colors to the vertices as $\{0, 1, 2, \cdots k\}$

Definition 1.1. (Central graph of Star Graph) Let G be a graph, the central graph of star graph (C(G)) is Obtained by subdividing each edge exactly once and joining all the nonadjacent vertices of G.

(Total Graph Of Star Graph) Let G be a graph with vertex set V (G) and edge set E (G). The total graph of G is denoted by T(G) and defined as follows. The vertex set of T (G) is V(G)U E(G). Two vertices x, y in the vertex set of T(G) are adjacent in T(G) in case one of the following holds:

1. x, y are in V(G) and x is adjacent to y in G
2. x, y are in E(G) and x, y are adjacent in G.
3. x is in V(G), y is in E (G), and x, y are incident in G.

2. Preliminaries

Proposition 2.1. [1] Let G be a nontrivial connected graph of size m. Then

(a) $rc(G) = src(G) = 1$ if and only if G is complete,

(b) $rc(G) = src(G) = m$ if and only if G is a Tree,

(c) $rc(G) = 2$ if and only if $src(G) = 2$.

Proposition 2.2. [1] For each integer $n \geq 4, rc(C_n) = src(C_n) = \lceil \frac{n}{2} \rceil$

Proposition 2.3. [1] For each integer $n \geq 3$, we have $rc(W_n) = 1$ if $n = 3$ $rc(W_n) = 2$ if $4 \leq n \leq 6$ and $rc(W_n) = 3$ if $n \geq 7$

Proposition 2.4. [1] For integers s and t with $2 \leq s \leq t$, $rc(K_{s,t}) = \min\{\lfloor \sqrt[s]{t} \rfloor, 4\}$ and for integers s and t with $1 \leq s \leq t$, $src(K_{s,t}) = \lfloor \sqrt[s]{t} \rfloor$

Theorem 2.1. *[3] A connected Graph G with n vertices has* $rvc(G) < \frac{11n}{\delta(G)}$

Theorem 2.2. *[4] A connected Graph G of order n with minimum degree* δ *has* $rvc(G) \leq \frac{3n}{\delta+1} + 5$ *for* $\delta \geq \sqrt{n-1} - 1$, $n \geq 290$ *while* $rvc(G) \leq \frac{4n}{\delta+1} + 5$ *for* $16 \leq \delta \leq \sqrt{n-1} - 2$, $rvc(G) \leq \frac{4n}{\delta+1} + c(\delta)$ *for* $6 \leq \delta \leq 16$ *where* $C(\delta) = e^{\frac{3\log(\delta^3+2\delta^2+3)-3(\log 3-1)}{\delta-3}} - 2$
$rvc(G) \leq \frac{n}{2} - 2$ *for* $\delta = 5$, $rvc(G) \leq \frac{3n}{5} - \frac{8}{5}$ *for* $\delta = 4$, $rvc(G) \leq \frac{3n}{4} - 2$ *for* $\delta = 3$

Theorem 2.3. *[5] Let G be a connected Graph of order n with k independent vertices, then* $rvc(G) \leq \frac{(4k+2k^2)n}{\sigma_k+k} + 5k$ *if* $\sigma_k \leq 7k$ *or* $\sigma_k \geq 8k$ *whereas* $rvc(G)$ *whereas* $rvc(G) \leq \frac{\left(\frac{38k}{9}+2k^2\right)n}{\sigma_k+k} + 5k$ *if* $7k < \sigma_k < 8k$

Theorem 2.4. *[6] Let G be a 2-connected graph of order* $n(n \geq 3)$. *Then* $rvc(G) = 0$ *if* $n = 3$; *1 if* $n = 4, 5$ *and* $rvc(G) = 3$ *if* $n = 9$;
if $\left[\frac{n}{2} - 1\right]$, $n = 6, 7, 8, 10, 12, 13$ *or* 15 : $\left[\frac{n}{2}\right]$ *if* $n \geq 16$ *or* $n = 14$ *and the upper bound can be achieved by the cycle* C_n.

Lemma 2.1. *[7] For every Sunlet Graph* $(S_n), n \geq 3$, $rvc(G)$ *is atleast 3.*

Lemma 2.2. *[7] Let* (S_n) *be Sunlet Graph where* $n \geq 3$ *and n is even then* $rvc(G) = rc(G) + \chi(G)$.

Lemma 2.3. *[7] Let* (S_n) *be Sunlet Graph where* $n \geq 3$ *and n is odd then* $rvc(G) = rc(G) + \chi(G) - 1$.

Lemma 2.4. *[7] For every Helm Graph* H_n *of order n,* $n \geq 3$ *then* $rvc(G)$ *is at least n.*

Lemma 2.5. *[8] Let* $G = CG_n$ *(Comb Graph)of even order where* $n \geq 2$, *then* $rvc(G) = 0$ *if* $n = 2$, $rvc(G) = \frac{n}{2}$ *if* $n > 2$.

Lemma 2.6. *[8] Let* $G = CG_n$ *of even order where* $n \geq 2$, *then for every graph G with q vertices of degree atleast 2 then* $rvc(G) = \lceil \frac{qn}{m} \rceil - 1$.

Lemma 2.7. *[8] Let* $G = CG_n$ *of even order where* $n \geq 2$, *then* $rvc(G) < \frac{n^2}{n^2-2m}$ *if* $n = 2$, $rvc(G) > \frac{n^2}{n^2-2m}$ *if* $n > 2$

Proposition 2.5. [9] Let $G = M[K_{1,n}]$ then $rvc(G) = \chi(G) - 1 = \alpha(G) - 1 = \omega(G) - 1$

Proposition 2.6. [9] Let $G = M[K_{1,n}]$ without isolated vertices then $rvc(G) = \gamma(G)$

Proposition 2.7. [9] Let $G = M[K_{1,n}]$ on n vertices with q vertices of degree atleast 2 then $rvc(G) < n - 1 + q$

Proposition 2.8. [9] Let $G = M[K_{1,n}]$ then $rvc(G) < \lfloor \frac{n+\omega(G)}{2} \rfloor$

Proposition 2.9. [9] Let $G = M[K_{1,n}]$ then $rvc(G) = \lfloor \frac{n}{2} \rfloor - \lceil \frac{n}{4} \rceil - \lfloor \frac{n}{4} \rfloor - 1$

Theorem 2.5. *[9] Let $G = M[K_{1,n}]$ then $rvc(G) = 0$ if G is a trivial graph, $rvc(G) = \lfloor \frac{n}{2} \rfloor$ if $n = 3, 5, 7, \cdots$*

3. Main Results

Theorem 3.1. *Let $G = C[K_{1,n}]$ of order $2n+1$ where $n \in N$, then $rvc(G) = 1$.*

Proof. Let us write G for $C[K_{1,n}]$, where

$$V(G) = \{v, e_1, e_2, \cdots e_n, u_1, u_2, \cdots u_n\},$$

$$E(G) = \{ve_i/1 \leq i \leq n\} \cup \{vu_i/1 \leq i \leq n\} \cup \{e_iu_j/1 \leq i, j \leq n \text{ifi} = j\} \cup \{u_iu_j/1 \leq i, j \leq n\}.$$

Here $V(G) = 2n+1$, $U = \{u_i/1 \leq i \leq n\}$ forms a clique of order n. We show that $rvc(G) = 1$ in the following cases:

Case 1: When $n = 8k-3, k \in N$. Define a coloring $c : V(G) \to \{0\}$ as

$$c(v) = 0,$$
$$c(e_i) = 0, \text{for } 1 \leq i \leq n,$$
$$c(u_i) = 0, \text{for } 1 \leq i \leq n.$$

Every path is rainbow vertex connected with this coloring. Therefore $rvc(G) = 1$.

Suppose we define a coloring $c : V(G) \to \{0, 1\}$ as

$$c(v) = 0,$$
$$c(e_{2s-1}) = 0, \text{ where } s \in N,$$
$$c(e_{2s}) = 1, \text{ where } s \in N,$$
$$c(u_{2s-1}) = c(u_{2s}) = 1, \text{ where } s \in N.$$

Every path is rainbow vertex connected with two colors Chosen. It is possible to make rainbow vertex connected with only one color chosen.

Similarly if we define a coloring $c : V(G) \to \{0, 1, \cdots k\}$ as $c(v) = 0$,

$$c(e_{2s-1}) = c(u_{2s-1}) = 2s-1, \text{ where } s \in N,$$

$$c(e_{2s}) = c(u_{2s}) = 2s, \text{ where } s \in N.$$

Every path is rainbow vertex connected with maximum number of colors chosen which contradicts the definition of rainbow vertex connection number.

Case 2: When $n = 8k - 1, k \in N$. Define a coloring $c : V(G) \to \{0\}$ as
$c(v) = 0$
$c(e_i) = 0$ for $1 \le i \le n$
$c(u_i) = 0$ for $1 \le i \le n$
Every path is rainbow vertex connected with this coloring.

Therefore $rvc(G) = 1$.

Case 3: When $n = 8k+1, k \in N$. Define a coloring as in case 2 of theorem 1. 5.

Every path is rainbow vertex connected with this coloring.

Therefore $rvc(G) = 1$. □

Theorem 3.2. *Let $G = C[K_{1,n}]$ of order $2n+1$, where $n \in N$. If a graph G is k-critical graph has (k-1) edge - connected then $rvc(G) = \chi(G) - \omega(G)$ without affecting the definition of $C[k_{1,n}]$.*

Proof. Write G for $C[K_{1,n}]$ since graph G is k-critical graph it has $(k-1)$ edge-connected

$$V(G) = \{v, e_1, e_2, \cdots e_n, u_1, u_2, \cdots u_n\},$$

$$E(G) = \{ve_i/1 \le i \le n\} \cup \{vu_i/1 \le i \le n\} \cup \{e_iu_j/1 \le i, j \le n \text{ if } i = j\} \cup \{u_iu_j/1 \le i, j \le n\}.$$

Here $V(G) = 2n + 1$.

Define a coloring $c : V(G) \to \{0\}$ as: $c(v) = 0$
$c(e_i) = 0$ for $1 \le i \le n$
$c(u_i) = 0$ for $1 \le i \le n$
Every path is rainbow vertex connected with this coloring.

Therefore $rvc(G) = 1$.

Case 1: When $\chi(G) = 3k, k \in N$, $\omega(G) = k - 1, k \in N$. Obviously for every graph G we have $\chi(G) - \omega(G) = 1$. Hence $rvc(G) = \chi(G) - \omega(G)$.

Case 2: When $\chi(G) = 3k + 1, k \in N$, $\omega(G) = 3k, k \in N$. Obviously for every graph G we have $\chi(G) - \omega(G) = 1$. Hence $rvc(G) = \chi(G) - \omega(G)$.

Case 3: When $\chi(G) = 3k + 2, k \in N$, $\omega(G) = 3k + 1, k \in N$. Obviously for every graph G we have $\chi(G) - \omega(G) = 1$. Hence $rvc(G) = \chi(G) - \omega(G)$.

[10] Chartrand proved that every k-critical graph has $(k-1)$ edge-connected. To show that if a graph G is k-critical graph has $(k-1)$ edge - connected then $rvc(G) = \chi(G) - \omega(G)$ without affecting the definition of $C[k_{1,n}]$. When $n = 4s+1, s \in N$. $\chi(G) = 2k+1, k \in N$ and $\omega(G) = 2k, k \in N$.
Obviously for every graph G we have $\chi(G) - \omega(G) = 1$
Let H be any subgraph of G, it can easily verify that $\chi(H) \leq 2k, k \in N$, clearly the graph G is k-critical. Obviously whenever the graph G is k-critical then $rvc(G) = \chi(G) - \omega(G)$
Assume on the contrary, if the graph G is not k-critical then $\chi(H) = \chi(G) = 2k+1, k \in N$ and $\omega(G) = 2k, k \in N$ but $rvc(G) = 1$ and $\chi(G) - \omega(G) = 1$ but it contradicts the definition of $C[k_{1,n}]$

□

Theorem 3.3. *(i) Let $G = T[K_{1,n}]$. If $\omega(G) = 2n+1$ where $n \in N$ then $rvc(G) = 0$*
(ii) Let G be graph of order $2n+1$ then $rvc(G) = \omega(G) - \alpha(G)$

Proof. Write $G = T[K_{1,n}]$. Let $V(G) = \{v, e_1, e_2, \cdots e_n, u_1, u_2, \cdots u_n\}$, $E(G) = \{ve_i/1 \leq i \leq n\} \cup \{vu_i/1 \leq i \leq n\} \cup \{e_iu_j/1 \leq i,j \leq n \text{ if } i = j\} \cup \{e_ie_j/1 \leq i,j \leq n\}$
Here $V(G) = 2n+1$
If $\omega(G) = 2n+1$, then the vertex v is adjacent to 2n vertices, which implies that the graph G is complete.
Obviously $rvc(G) = 0$ iff G is complete.
(ii) Let $X_1 = \{e_1, u_2, e_3, \cdots e_{n-1}, u_n\}, X_2 = \{u_1, e_2, \cdots u_{n-1}, e_n\}$ and $X_3 = \{u_1, u_2, \cdots u_n\}$ are maximum independent sets.
Then $|X_1| = |X_2| = |X_3| = \alpha(G)$.
Define coloring $c : V(G) \to \{0\}$ as
$c(v) = 0$
$c(e_i) = 0$ for $1 \leq i \leq n$
$c(u_i) = 0$ for $1 \leq i \leq n$
Every path is rainbow vertex connected with this coloring.
Therefore $rvc(G) = 1$
On the other hand, Clearly $\omega(G) = k+2, k \in N$ and $\alpha(G) = k+1, k \in N$.
Hence the result. □

Theorem 3.4. *Let $G = T[K_{1,n}]$ of order $2n+1$ where $n > 1$. Then $rvc(G) = \min\{\lfloor \sqrt[m]{k} \rfloor, \triangle(G)\}$ where m is number of vertices of degree exactly two and k is an independence number.*

Proof. Write $G = T[K_{1,n}]$. Let $V(G) = \{v, e_1, e_2, \cdots e_n, u_1, u_2, \cdots u_n\}$, $E(G) = \{ve_i/1 \leq i \leq n\} \cup \{vu_i/1 \leq i \leq n\} \cup \{e_iu_j/1 \leq i, j \leq n \text{ if } i = j\} \cup \{e_ie_j/1 \leq i, j \leq n\}$
Here $V(G) = 2n + 1$
Define coloring $c : V(G) \to \{0\}$ as
$c(v) = 0$
$c(e_i) = 0$ for $1 \leq i \leq n$
$c(u_i) = 0$ for $1 \leq i \leq n$
Every path is rainbow vertex connected with this Coloring.
Therefore $rvc(G) = 1$
For every graph G we have $\lfloor \sqrt[m]{k} \rfloor = 1$, where m and k is already defined in the statement. On the other hand $\triangle(G) > \lfloor \sqrt[m]{k} \rfloor$ for every graph G.
To show that $\lfloor \sqrt[m]{k} \rfloor = 1$ for the following two cases:
Case 1:
When $n = 4s + 1, s \in N$
Clearly $k = \alpha(G) = 2t$, where $t \in N$.
Here $m = 2t$ where $t \in N$. This implies that m=k.
It can be easily verify that $\lfloor \sqrt[m]{k} \rfloor = 1$, for every graph G where $m = k = 2t$.
Therefore $rvc(G) = \min\{\lfloor \sqrt[m]{k} \rfloor, \triangle(G)\} = 1$
Case 2:
When $n = 4s + 3, s \in N$
Clearly $k = \alpha(G) = 2t + 1$, where $t \in N$.
m is number of vertices of degree exactly two then m is clearly $2t + 1$ where $t \in N$. It can be shown clearly that $\lfloor \sqrt[m]{k} \rfloor = 1$, for every graph G of order $4s + 3$ where $m = k = 2t + 1, t \in N$.
Therefore $rvc(G) = \min\{\lfloor \sqrt[m]{k} \rfloor, \triangle(G)\} = 1$
Hence in both cases, the result is true. □

Theorem 3.5. *Let $G = T[K_{1,n}]$ then $rvc(G) = 0$ if $n = 3$, $rvc(G) = 1$ if $n = 5, 7, \cdots 2n + 3$*

Proof. Write $G = T[K_{1,n}]$. Let $V(G) = \{v, e_1, e_2, \cdots e_n, u_1, u_2, \cdots u_n\}$,
Here $V(G) = 2n + 1$
Assume that $n = 3$, the vertex is adjacent to remaining 2 vertices e_1 and u_1 forms a clique with clique number 3, but $rvc(G) = 0$ iff G is clique. Hence $rvc(G) = 0$
Case 1:
When $n = 6k - 1, k \in N$
Define coloring $c : V(G) \to \{0\}$ as
$c(v) = 0$

$c(e_i) = 0$ for $1 \leq i \leq n$
$c(u_i) = 0$ for $1 \leq i \leq n$
Every path is rainbow vertex connected with this Coloring.
Therefore $rvc(G) = 1$
Suppose if we define coloring $c : V(G) \rightarrow \{0, 1\}$ as $c(v) = 0$
$c(u_i) = c(e_i) = 0$ for $i = 2s + 1$ where $s = \{0, 1, 2, \cdots\}$
$c(u_i) = c(e_i) = 1$ for $i = 2s$ where $s \in N$
then every path is rainbow vertex connected with this coloring in which two colors are chosen, but it is possible to choose only on color in order to make rainbow vertex connected.
Similarly if we, define coloring $c : V(G) \rightarrow \{0, 1, 2, \cdots\}$, every path is rainbow vertex connected with maximum colors chosen which contradicts the definition of rainbow vertex connection number.
Hence $rvc(G) = 1$
Case 2:
When $n = 6k + 1, k \in N$
Define coloring $c : V(G) \rightarrow \{0\}$ as
$c(v) = 0$
$c(e_i) = 0$ for $1 \leq i \leq n$
$c(u_i) = 0$ for $1 \leq i \leq n$
Every path is rainbow vertex connected with this Coloring.
Hence $rvc(G) = 1$
Case 3:
When $n = 6k + 3, k \in N$
Define coloring $c : V(G) \rightarrow \{0\}$ as
$c(v) = 0$
$c(e_i) = 0$ for $1 \leq i \leq n$
$c(u_i) = 0$ for $1 \leq i \leq n$
Every path is rainbow vertex connected with this Coloring.
Hence $rvc(G) = 1$ □

4. Conclusion

i The rainbow vertex connection number for central graph of star graph is 1

ii The rainbow vertex connection number for central graph of star graph is the difference between chromatic number and clique number without affecting the definition of central graph of star graph

iii The rainbow vertex connection number for total graph of star graph is zero if the clique number is order of the central graph of star graph

iv The rainbow vertex connection number of central graph of star graph is 0 if $n = 3, rvc(G)$ is 1 if $n = 5, 7, ..., 2n + 3$

5. Applications

Its application is to secure transfer of classified information between agencies and it also be stimulated by its interesting elucidation in the area of networking. The information needs to be protected since it relates to national security; there must also be procedures that permit access between adequate parties. This twofold issue can be addressed by assigning information transfer paths between agencies which have many other agencies as intermediaries while requiring large enough number of passwords and firewalls that is prohibitive to intruders. The minimum number of passwords or firewalls needed that allows one or more paths between every two agencies so that the passwords along each path are distinct. Suppose that G is represents a network. We wish to route messages between any two vertices in a pipeline and require that each link on the route between the vertices. Clearly, we want to minimize the number of distinct channels that we use in our network.

References

[1] G. Chartrand, G. L. Johns, K. A. Mckeon, P. Zhang, Rainbow connection in graphs, *Math Bohem.* **133** (2008), 85-98

[2] S. Chakraborty, E. Fischer, A. Matslaih, R. Yuster, Hardness and algorithms for rainbow connectivity, *26th international symposium on Theoretical Aspects of Computer Science STACS 2009* (2009), 243-254.

[3] M. Krivelevich, R. Yuster, The rainbow connection of a graph is (atmost) reciprocal to its minimum degree, *J. Graph Theory* **63** 3 (2009) 185-191.

[4] X. Li, Y. Shi, *On the rainbow vertex-connection, Discuss. Math. Graph theory* (2012).

[5] J. Dong, X. Li, Rainbow connection numbers and the minimum degree sum of a Graph, *Sci. China Ser, A* (2012).

[6] X. Li, S. Liu, Rainbow vertex-connection number of 2-connected graphs, arXiv:1110. 5770[math. CO](2011)

[7] Hariram Kumar, N. Parvathi, Rainbow Vertex Coloring for Sunlet and Helm Graph, *National Conference NCAGL 2015* (2015), 35-36.

[8] Hariram Kumar, N. Parvathi, Rainbow Vertex Coloring for Comb Graph, *International Conference ICEME* **1** (2015), 86-88.

[9] Hariram Kumar, N. Parvathi, Rainbow vertex coloring for middle graph of star graph Global, *Journal of Pure and Applied Mathematics* **12** 1 (2016), 166-169.

[10] Chartrand and P. Zhaung, *Chromatic Graph Theory*, *CRC PRESS* (2008), 176.