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FIXED POINT THEOREMS FOR SOFT $\alpha-\psi$ CONTRACTIVE TYPE MAPPING IN SOFT METRIC SPACES

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Abstract: In this paper, we introduce the notions of soft $\alpha-\psi$-contractive mappings and cyclic soft $(\alpha,\beta)-\psi$-contractive mappings, and the purpose of this paper to prove some fixed point theorems in soft metric space.

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1. Introduction

In 1999, Molodtsov [1] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. He has shown several applications of this theory in solving many practical problems in economics,engineering, social science,medical science, etc. Maji et al [2] introduced several operations on soft sets and applied it to decision making problems.Then the idea of soft topological space was first given by M. Shabir and M. Naz, see [6], S. Das and S.K. Samanta, see [3], introduced the notion of soft metric space and investigated some basic properties of this space.

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Abbas et al [7] introduced the notion of soft contraction mapping based on the theory of soft elements of soft metric spaces. They studied fixed points of soft contraction mappings and obtained among others results, a soft Banach contraction principle. Chen and Lin [8] obtained a soft metric version of the Meir-Keeler fixed point theorem. M.I. Yazar et al [10] introduced soft continuous mappings and investigated properties of soft continuous mappings. In 2012, Samet et al [12] introduced $\alpha - \psi$-contractive mappings and gave some results on a fixed point of the mappings. By using the main idea of Samet et al. we introduce the notions of soft $\alpha - \psi$-contractive mappings and cyclic soft $(\alpha, \beta) - \psi$-contractive mappings, and the purpose of this paper to prove some fixed point theorems in soft metric space.

Definition 1.1. [1] Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if and only if F is a mapping from E into the set of all subsets of the set X , i.e., $F : E \to P(X)$, where $P(X)$ is the power set of X .

Definition 1.2. [11] A soft set (F, E) over X is said to be an absolute soft set denoted by $\tilde{X}$ if for all $e \in E$, $F(e) = X$.

Definition 1.3. [3] A soft set (F, E) over X is said to be a soft point if there is exactly one $e \in E$, such that $F(e) = \{x\}$ for some $x \in X$ and $F(e') = \phi, \forall e' \in E - \{e\}$. It will be denoted by $\tilde{x_e}$.

Definition 1.4. [3] Two soft points $\tilde{x_e}, \tilde{y_{e'}}$ are said to be equal if $e = e'$ and $F(e) = F(e')$ i.e. , $x = y$. Thus $\tilde{x_e} \neq \tilde{y_{e'}}$, $\Leftrightarrow x \neq y$ or $e \neq e'$.

Definition 1.5. [4] Let R be the set of real numbers and $B(R)$ be the collection of all non-empty bounded subsets of R and E be taken as a set of parameters. Then a mapping $F : E \to B(R)$ is called a soft real set. If a real soft set is a singleton soft set, it will be called a soft real number and denoted by $\tilde{r}, \tilde{s}, \tilde{t}$ etc. $\tilde{0}$ and $\tilde{1}$ are the soft real numbers where $\tilde{0}(e) = 0$, $\tilde{1}(e) = 1$for all $e \in E$ respectively.

Definition 1.6. [4] Let $\tilde{r}, \tilde{s}$ be two soft real numbers. Then the following statements hold:

(i) $\tilde{r} \tilde{\leq} \tilde{s}$ if $\tilde{r}(e) \tilde{\leq} \tilde{s}(e)$ for all $e \in E$;

(ii) $\tilde{r} \tilde{\geq} \tilde{s}$ if $\tilde{r}(e) \tilde{\geq} \tilde{s}(e)$ for all $e \in E$;

(iii) $\tilde{r} \tilde{<} \tilde{s}$ if $\tilde{r}(e) \tilde{<} \tilde{s}(e)$ for all $e \in E$;

(iv) $\tilde{r} \tilde{>} \tilde{s}$ if $\tilde{r}(e) \tilde{>} \tilde{s}(e)$ for all $e \in E$.

Let $SP(\tilde{X})$ be the collection of all soft points of $\tilde{X}$ and $R(E)^*$ denote the set of all non-negative soft real numbers.

Definition 1.7. [3] A mapping $\tilde{d} : SP(\tilde{X}) \times SP(\tilde{X}) \to R(E)^*$, is said to be a soft metric on the soft set $\tilde{X}$ if $\tilde{d}$ satisfies the following conditions:

(M_1) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2})\tilde{\geq}\bar{0} \quad \forall\ \tilde{x}_{e_1}, \tilde{y}_{e_2}\tilde{\in}\tilde{X}$;

(M_2) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = \bar{0}$ iff $\tilde{x}_{e_1} = \tilde{y}_{e_2}$;

(M_3) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = \tilde{d}(\tilde{y}_{e_2}, \tilde{x}_{e_1})\ \forall \tilde{x}_{e_1}, \tilde{y}_{e_2}\tilde{\in}\tilde{X}$;

(M_4) For all $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3}\tilde{\in}\tilde{X}$, $\tilde{d}(\tilde{x}_{e_1}, \tilde{z}_{e_3})\tilde{\leq}\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) + \tilde{d}(\tilde{y}_{e_2}, \tilde{z}_{e_3})$.

The soft set $\tilde{X}$ with a soft metric $\tilde{d}$ on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X}, \tilde{d}, E)$.

Definition 1.8. [3] Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space $\tilde{\epsilon}$ be a non negative soft real number. $B(\tilde{x_e}, \tilde{\epsilon}) = \left\{\tilde{y_{e'}}\tilde{\in}\tilde{X} : \tilde{d}(\tilde{x_e}, \tilde{y_{e'}})\tilde{<}\tilde{\epsilon}\right\} \subset SP(\tilde{X})$ is called the soft open ball with center at $\tilde{x_e}$ and radius $\tilde{\epsilon}$ and $B[\tilde{x_e}, \tilde{\epsilon}] = \left\{\tilde{y_{e'}} \in \tilde{X} : \tilde{d}(\tilde{x_e}, \tilde{y_{e'}})\tilde{\leq}\tilde{\epsilon}\right\} \subseteq SP(\tilde{X})$ is called the soft closed ball with center at $\tilde{x_e}$ and radius $\tilde{\epsilon}$.

Definition 1.9. [3] Let $\{\tilde{x}^n_{e_n}\}$ be a sequence of soft points in a soft metric space $(\tilde{X}, \tilde{d}, E)$. The sequence $\{\tilde{x}^n_{e_n}\}$ is said to be convergent in $(\tilde{X}, \tilde{d}, E)$ if there is a soft point $\tilde{y_\mu}\tilde{\in}\tilde{X}$ such that $\tilde{d}(\tilde{x}^n_{e_n}, \tilde{y_\mu}) \to \bar{0}$ as $n \to \infty$. This means for every $\tilde{\epsilon}\tilde{>}\bar{0}$, chosen arbitrarily, $\exists$ a natural number $N = N(\tilde{\epsilon})$ such that $\bar{0} \leq \tilde{d}(\tilde{x}^n_{e_n}, \tilde{y_\mu})\tilde{<}\tilde{\epsilon}$, whenever $n > N$.

Theorem 1.10. *[3] Limit of a sequence in a soft metric space, if exist ,is unique.*

Definition 1.11. [3] A sequence $\{\tilde{x}^n_{e_n}\}$ of soft points in $(\tilde{X}, \tilde{d}, E)$ is considered as a Cauchy sequence in $\tilde{X}$ if corresponding to every $\tilde{\epsilon} > \bar{0}$, $\exists\ m \in N$ such that $\tilde{d}(\tilde{x}^i_{e_i}, \tilde{x}^j_{e_j})\tilde{\leq}\tilde{\epsilon}$, $\forall\ i, j \geq m$, i.e., $\tilde{d}(\tilde{x}^i_{e_i}, \tilde{x}^j_{e_j}) \to \bar{0}$ as $i, j \to \infty$.

Definition 1.12. [3] A soft metric space $(\tilde{X}, \tilde{d}, E)$ is called complete if every Cauchy Sequence in $\tilde{X}$converges to some point of $\tilde{X}$.The soft metric space $(\tilde{X}, \tilde{d}, E)$ is called incomplete if it is not complete .

Definition 1.13. Let $(\tilde{X}, \tilde{d}, E)$ and $(\tilde{Y}, \tilde{\rho}, E')$ be two soft metric spaces . The mapping $(f, \phi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{Y}, \tilde{\rho}, E')$ is a soft mapping ,where $f : X \to Y$,$\phi : E \to E'$ are two mappings.

Definition 1.14. [10] Let $(\tilde{X}, \tilde{d}, E)$ and $(\tilde{Y}, \tilde{\rho}, E')$ be two soft metric spaces and $(f, \phi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{Y}, \tilde{\rho}, E')$ be a soft mapping. The mapping (f, ϕ) :

$(\tilde{X}, \tilde{d}, E) \to (\tilde{Y}, \tilde{\rho}, E')$ is a soft continuous mapping at the point $\tilde{x_e} \tilde{\in} SP(\tilde{X})$ if for every soft open ball $B((f,\phi)(\tilde{x_e}), \tilde{\epsilon})$ of $(\tilde{Y}, \tilde{\rho}, E')$, there exists a soft open ball $B(\tilde{x_e}, \tilde{\delta})$ of $(\tilde{X}, \tilde{d}, E)$ such that $f(B(\tilde{x_e}, \tilde{\delta})) \subseteq B((f,\phi)(\tilde{x_e}), \tilde{\epsilon})$.

If (f, ϕ) is soft continuous mapping at every soft point $\tilde{x_e}$ of $(\tilde{X}, \tilde{d}, E)$, then it is said to be soft continuous mapping on $(\tilde{X}, \tilde{d}, E)$.
Now this definition can be expressed using $\tilde{\epsilon} - \tilde{\delta}$ as follows:
The mapping $(f, \phi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{Y}, \tilde{\rho}, E')$ is a soft continuous mapping at the point $\tilde{x_e} \tilde{\in} SP(\tilde{X})$ if for every $\tilde{\epsilon} \tilde{>} 0$ there exists a $\tilde{\delta} \tilde{>} 0$ such that $\tilde{d}(\tilde{x_e}, \tilde{y_{e'}}) \tilde{<} \tilde{\delta}$ implies that $\tilde{\rho}((f,\phi)(\tilde{x_e}), (f,\phi)(\tilde{y_{e'}})) \tilde{<} \tilde{\epsilon}$.

Definition 1.15. [10] The soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{Y}, \tilde{\rho}, E')$ is said to be soft sequentially continuous at $\tilde{x_e} \tilde{\in} SP(\tilde{X})$ iff for every sequence of soft points $\left\{\tilde{x}^n_{e_n}\right\}$ converging to the soft point $\tilde{x_e}$in the soft metric space $(\tilde{X}, \tilde{d}, E)$, the sequence $(f,\phi)(\left\{\tilde{x}^n_{e_n}\right\})$ in $(\tilde{Y}, \tilde{\rho}, E')$ converges to a soft point $(f,\phi)(\tilde{x_e}) \tilde{\in} SP(\tilde{Y})$.

Theorem 1.16. *[10] Soft continuity is equivalent to soft sequential continuity in soft metric spaces.*

Definition 1.17. [12] Let $T : X \to X$ and $\alpha : X \times X \to [0, \infty)$ we say that T is α - admissible if $x, y \in X$, $\alpha(x, y) \geq 1 \Rightarrow \alpha(Tx, Ty) \geq 1$.

Definition 1.18. Let $T : SP(\tilde{X}) \to SP(\tilde{X})$ and $\alpha : SP(\tilde{X}) \times SP(\tilde{X}) \to R(E)^*$ we say that T is soft α - admissible if $\tilde{x_e}, \tilde{y_{e'}} \in \tilde{X}$, $\alpha(x_e, y_{e'}) \tilde{\geq} \bar{1} \Rightarrow \alpha(T\tilde{x_e}, T\tilde{y_{e'}}) \tilde{\geq} \bar{1}$.

Example 1.19. Let $X = E = [0, \infty)$ and $T : SP(\tilde{X}) \to SP(\tilde{X})$ is defined by $T(\tilde{x_e}) = \tilde{x_e}$ $\forall \tilde{x_e} \in \tilde{X}$.Let $\alpha : SP(\tilde{X}) \times SP(\tilde{X}) \to R(E)^*$ be defined by

$$\alpha\left(\tilde{x}_e, \tilde{y}_{e'}\right) = \begin{cases} \bar{2} & if \quad \tilde{x}_e \neq \tilde{y}_{e'} \\ \bar{0} & if \quad \tilde{x}_e = \tilde{y}_{e'} \end{cases}$$

Then T is soft α - admissible.

Definition 1.20. Let Ψ be the family of functions $\psi : R(E)^* \to R(E)^*$ satisfying the following conditions.
(i) ψ is non-decreasing
(ii) $\sum\limits_{n=1}^{+\infty} \psi^n(\tilde{t}) < \infty$ for all $\tilde{t} > \bar{0}$, where ψ^n is the n^{th} iterative of ψ.

Remark: For every function $\psi : R(E)^* \to R(E)^*$ the following holds: if ψ is non decreasing, then for each $\tilde{t} > \bar{0}$, $\lim\limits_{n\to\infty} \psi^n(\tilde{t}) = \bar{0} \Rightarrow \psi(\tilde{t}) \tilde{<} \tilde{t} \Rightarrow \psi(\bar{0}) = \bar{0}$.
There fore if $\psi \in \Psi$ then for each $\tilde{t} \tilde{>} \bar{0}$, $\psi(\tilde{t}) \tilde{<} \tilde{t} \Rightarrow \psi(\bar{0}) = \bar{0}$.

First of all, we will introduce the concept of a soft $\alpha - \psi$-contraction mapping in the sense of a soft metric distance.

Definition 1.21. Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and $(T, \varphi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{X}, \tilde{d}, E)$ be a given soft mapping we say that (T, φ) is soft $\alpha - \psi$ contractive mapping if there exists two soft functions $(\alpha, \phi) : \tilde{X} \times \tilde{X} \to R(E)^*$ and $\psi \in \Psi$ such that

$$(\alpha, \phi)(\tilde{x}_e, \tilde{y}_{e'})\tilde{d}((T, \varphi)\tilde{x}_e, (T, \varphi)\tilde{y}_{e'})\tilde{\leq}\psi\left(\tilde{d}(\tilde{x}_e, \tilde{y}_{e'})\right), \forall \tilde{x}_e, \tilde{y}_{e'} \in \tilde{X} \quad (1)$$

Now we prove our main result.

2. Main Result

Theorem 2.1. *Let $(\tilde{X}, \tilde{d}, E)$ be a complete soft metric space . Let (T, φ) be a soft $\alpha - \psi$ contractive mapping from $\tilde{X}$ into itself satisfying the following*

(2.1.1) (T, φ) is soft α - admissible.

(2.1.2) there exists $\tilde{x}^0_{e_0} \in \tilde{X}$ such that $(\alpha, \phi)(\tilde{x}^0_{e_0}, (T, \varphi)\tilde{x}^0_{e_0})\tilde{\geq}\bar{1}$,

(2.1.3) (T, φ) is soft continuous.

Then (T, φ) has a fixed point , that is there exists $\tilde{x}_e\tilde{\in}\tilde{X}$ such that $(T, \varphi)\tilde{x}_e = \tilde{x}_e$.

Proof. Let $\tilde{x}^0_{e_0} \in \tilde{X}$ such that $(\alpha, \phi)(x^0_{e_0}, (T, \varphi)x^0_{e_0})\tilde{\geq}\bar{1}$. Define the sequence $\{\tilde{x}^n_{e_n}\}$ in $\tilde{X}$ by

$$\tilde{x}^{n+1}_{e_{n+1}} = (T, \varphi)\tilde{x}^n_{e_n}, \forall n \in N.$$

If $\tilde{x}^n_{e_n} = \tilde{x}^{n+1}_{e_{n+1}}$ for some $n \in N$, then $\tilde{x}_e = \tilde{x}^n_{e_n}$ is a fixed point of (T, φ).
Assume that $\tilde{x}^n_{e_n} \neq \tilde{x}^{n+1}_{e_{n+1}} \forall n \in N$. Since T is soft α - admissible.
we have

$$(\alpha, \phi)(\tilde{x}^0_{e_0}, \tilde{x}^1_{e_1}) = (\alpha, \phi)(\tilde{x}^0_{e_0}, (T, \varphi)\tilde{x}^0_{e_0})\tilde{\geq}\bar{1}.$$

By induction, we get

$$(\alpha, \phi)(\tilde{x}^n_{e_n}, \tilde{x}^{n+1}_{e_{n+1}}) \geq \bar{1}, foralln \in N. \quad (2)$$

Applying the inequality (1) with $\tilde{x}_e = \tilde{x}^{n-1}_{e_{n-1}}$ and $\tilde{y}_{e'} = \tilde{x}^n_{e_n}$, and using (2) , we obtain

$$\begin{aligned} \tilde{d}(\tilde{x}^n_{e_n}, \tilde{x}^{n+1}_{e_{n+1}}) &= \tilde{d}((T, \varphi)\tilde{x}^{n-1}_{e_{n-1}}, (T, \varphi)\tilde{x}^n_{e_n}) \\ &\tilde{\leq}(\alpha, \phi)(\tilde{x}^{n-1}_{e_{n-1}}, \tilde{x}^n_{e_n})\tilde{d}((T, \varphi)\tilde{x}^{n-1}_{e_{n-1}}, (T, \varphi)\tilde{x}^n_{e_n}) \\ &\tilde{\leq}\psi\left(\tilde{d}(\tilde{x}^{n-1}_{e_{n-1}}, \tilde{x}^n_{e_n})\right). \end{aligned}$$

By induction we get

$$\tilde{d}(\tilde{x}^{n}_{e_n}, \tilde{x}^{n+1}_{e_{n+1}}) \tilde{\leq} \psi^n \left(\tilde{d}(\tilde{x}^{0}_{e_0}, \tilde{x}^{1}_{e_1})\right) \forall n \in N. \tag{3}$$

From (3) and using the trinangular inequality and let $n, m \in N$ with $m > n$.

$$\begin{aligned} \tilde{d}(\tilde{x}^{n}_{e_n}, \tilde{x}^{m}_{e_m}) &\tilde{\leq} \tilde{d}(\tilde{x}^{n}_{e_n}, \tilde{x}^{n+1}_{e_{n+1}}) + \tilde{d}(\tilde{x}^{n+1}_{e_{n+1}}, \tilde{x}^{n+2}_{e_{n+2}}) + \cdots + \tilde{d}(\tilde{x}^{m-1}_{e_{m-1}}, \tilde{x}^{m}_{e_m}) \\ &= \sum_{k=n}^{m-1} \tilde{d}(\tilde{x}^{k}_{e_k}, \tilde{x}^{k+1}_{e_{k+1}}) \\ &\tilde{\leq} \sum_{k=n}^{m-1} \psi^k \left(\tilde{d}(\tilde{x}^{0}_{e_0}, \tilde{x}^{1}_{e_1})\right) \\ &\tilde{\leq} \sum_{k=n}^{+\infty} \psi^k \left(\tilde{d}(\tilde{x}^{0}_{e_0}, \tilde{x}^{1}_{e_1})\right). \end{aligned}$$

Letting $k \to \infty$, we obtain $\left\{\tilde{x}^{n}_{e_n}\right\}$ is a Cauchy sequence in soft metric space $(\tilde{X}, \tilde{d}, E)$.Since $(\tilde{X}, \tilde{d}, E)$ is complete, there exists $\tilde{x}_e \tilde{\in} \tilde{X}$ such taht $\tilde{x}^{n}_{e_n} \to \tilde{x}_e$ as $n \to \infty$. From the soft continuity of (T, φ) , it follows that $\tilde{x}^{n+1}_{e_{n+1}} = (T, \varphi)\tilde{x}^{n}_{e_n} \to (T, \varphi)\tilde{x}_e$ as $n \to +\infty$. By the uniqueness of the limit, we get $\tilde{x}_e = (T, \varphi)\tilde{x}_e$. □

Theorem 2.2. *Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space . Let (T, φ) be a soft $\alpha - \psi$ contractive mapping from $\tilde{X}$ into itself satisfying the following*

(2.2.1) (T, φ) is soft α - admissible.

(2.2.2) there exists $\tilde{x}^{0}_{e_0} \in \tilde{X}$ such that $(\alpha, \phi)(\tilde{x}^{0}_{e_0}, (T, \varphi)\tilde{x}^{0}_{e_0}) \tilde{\geq} \bar{1}$,

(2.2.3) if $\left\{\tilde{x}^{n}_{e_n}\right\}$ is a sequence in $\tilde{X}$ such that $(\alpha, \phi)(\tilde{x}^{n}_{e_n}, \tilde{x}^{n+1}_{e_{n+1}}) \tilde{\geq} \bar{1}$ for all n and $\tilde{x}^{n}_{e_n} \to \tilde{x}_e \in \tilde{X}$ as $n \to +\infty$, then $(\alpha, \phi)(\tilde{x}^{n}_{e_n}, \tilde{x}_e) \tilde{\geq} \bar{1}$ for all n. Then, (T, φ) has a fixed point.

Proof. Following the proof of Theorem (2.1), we know that $\left\{\tilde{x}^{n}_{e_n}\right\}$ is a Cauchy sequence in complete soft metric space $(\tilde{X}, \tilde{d}, E)$.Then , there exists $\tilde{x}_e \in \tilde{X}$ such taht $\tilde{x}^{n}_{e_n} \to \tilde{x}_e$ as $n \to \infty$. On the other hand from (2) and the hypothesis (2.2.3) we have

$$(\alpha, \phi)(\tilde{x}^{n}_{e_n}, \tilde{x}_e) \tilde{\geq} \bar{1} \forall n \in N \tag{4}$$

Now, using the triangular inequality , (1) and (4)

$$\begin{aligned} \tilde{d}((T, \varphi)\tilde{x}_e, \tilde{x}_e) &\tilde{\leq} \tilde{d}((T, \varphi)\tilde{x}^{n}_{e_n}, (T, \varphi)\tilde{x}_e) + \tilde{d}(\tilde{x}^{n+1}_{e_{n+1}}, \tilde{x}_e) \\ &\tilde{\leq} (\alpha, \phi)(\tilde{x}^{n}_{e_n}, \tilde{x}_e)\tilde{d}((T, \varphi)\tilde{x}^{n}_{e_n}, (T, \varphi)\tilde{x}_e) + \tilde{d}(\tilde{x}^{n+1}_{e_{n+1}}, \tilde{x}_e) \\ &\tilde{\leq} \psi(\tilde{d}(\tilde{x}^{n}_{e_n}, \tilde{x}_e)) + \tilde{d}(\tilde{x}^{n+1}_{e_{n+1}}, \tilde{x}_e). \end{aligned}$$

Letting $n \to \infty$, since ψ is continuous at $\bar{0}$ we obtain $\tilde{d}((T, \varphi)\tilde{x}_e, \tilde{x}_e) = \bar{0}$, that is , $(T, \varphi)\tilde{x}_e = \tilde{x}_e$. □

To assure the uniqueness of the fixed point, we will consider the following hypothesis.
(H) : For all $\tilde{x_e}, \tilde{y}_{e'} \tilde{\in} \tilde{X}$, there exists $\tilde{z}_{e^*} \in \tilde{X}$ such that $(\alpha,\phi)(\tilde{x_e},\tilde{z}_{e^*})\tilde{\geq}\bar{1}$ and $(\alpha,\phi)(\tilde{y}_{e'},\tilde{z}_{e^*})\tilde{\geq}\bar{1}$.

Theorem 2.3. *Adding condition (H) to the hypotheses of Theorem* (2.1)*(resp. Theorem* (2.2)) *we obtain uniqueness of the fixed point of* (T,φ).

Proof. Suppose $\tilde{x_e}$ and $\tilde{y}_{e'}$ are two fixed points of (T,φ). From (H) , there exists $\tilde{z}_{e^*}\tilde{\in}\tilde{X}$ such that

$$(\alpha,\phi)(\tilde{x_e},\tilde{z}_{e^*})\tilde{\geq}\bar{1} \quad \text{and } (\alpha,\phi)(\tilde{y}_{e'},\tilde{z}_{e^*})\tilde{\geq}\bar{1}. \tag{5}$$

Since (T,φ) is soft α - admissible, from (5), we get

$$(\alpha,\phi)(\tilde{x_e},(T,\varphi)^n\tilde{z}_{e^*})\tilde{\geq}\bar{1} \quad \text{and } (\alpha,\phi)(\tilde{y}_{e'},(T,\varphi)^n\tilde{z}_{e^*})\tilde{\geq}\bar{1}, \forall n\in N. \tag{6}$$

Define the sequence $\left\{\tilde{z}^n_{e^*_n}\right\}$ in $\tilde{X}$ by $\tilde{z}^{n+1}_{e^*_{n+1}} = (T,\varphi)\tilde{z}^n_{e^*_n}$ for all $n\geq 0$ and $\tilde{z}^0_{e^*_0} = \tilde{z}_{e^*}$. Using (6) and (1) , we have

$$\begin{aligned}\tilde{d}(\tilde{x}_e,\tilde{z}^{n+1}_{e^*_{n+1}}) &= \tilde{d}((T,\varphi)\tilde{x}_e,(T,\varphi)\tilde{z}^n_{e^*_n})\\ &\tilde{\leq}(\alpha,\phi)(\tilde{x}_e,\tilde{z}^n_{e^*_n})\tilde{d}((T,\varphi)\tilde{x}_e,(T,\varphi)\tilde{z}^n_{e^*_n})\\ &\tilde{\leq}\psi(\tilde{d}(\tilde{x}_e,\tilde{z}^n_{e^*_n})).\end{aligned}$$

This implies that

$$\tilde{d}(\tilde{x}_e,\tilde{z}^{n+1}_{e^*_{n+1}})\tilde{\leq}\psi(\tilde{d}(\tilde{x}_e,\tilde{z}_{e^*})), \forall n\in N.$$

This implies that

$$\tilde{d}(\tilde{x}_e,\tilde{z}^n_{e^*_n})\tilde{\leq}\psi^n(\tilde{d}(\tilde{x}_e,\tilde{z}_{e^*})), \forall n\geq 1.$$

Then , letting $n\to\infty$, we have

$$\tilde{z}^n_{e^*_n}\to\tilde{x}_e. \tag{7}$$

Similarly, using (6) and (1), we get

$$\tilde{z}^n_{e^*_n}\to\tilde{y}_{e'}. \tag{8}$$

Using (7) and (8) , the uniqueness of limit gives us $\tilde{x}_e = \tilde{y}_{e'}$.Thus we proved that $\tilde{x}_e$ is the unique fixed point of (T,φ). □

Definition 2.4. [9] Let X be a non empty set, $T : X \to X$ and $\alpha, \beta : X \to [0, \infty)$ be two mappings .We say that T is a cyclic (α, β) - admissible mapping if

$$x \in X, \alpha(x) \geq 1 \Rightarrow \beta(Tx) \geq 1$$

and

$$x \in X, \beta(x) \geq 1 \Rightarrow \alpha(Tx) \geq 1.$$

Definition 2.5. Let $SP(\tilde{X})$ be collection of all soft points of $\tilde{X}$, $T : SP(\tilde{X}) \to SP(\tilde{X})$ and $\alpha, \beta : SP(\tilde{X}) \to R(E)^*$ be two mappings . We say that T is a cyclic soft (α, β) - admissible mapping if

$$\tilde{x}_e \in \tilde{X}, \alpha(\tilde{x}_e) \tilde{\geq} \bar{1} \Rightarrow \beta(T\tilde{x}_e) \geq \bar{1}$$

and

$$\tilde{x}_e \in \tilde{X}, \beta(\tilde{x}_e) \tilde{\geq} \bar{1} \Rightarrow \alpha(T\tilde{x}_e) \geq \bar{1}.$$

Definition 2.6. Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and $(T, \varphi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{X}, \tilde{d}, E)$ be a given soft mapping we say that (T, φ) is soft (α, β) - Banach contractive mapping if there exists two soft functions $(\alpha, \psi), (\beta, \phi) : \tilde{X} \to R(E)^*$ and $\bar{0} \leq \tilde{r} < \bar{1}$such that

$$(\alpha, \psi)(\tilde{x}_e)(\beta, \phi)(\tilde{y}_{e'})\tilde{d}((T, \varphi)\tilde{x}_e, (T, \varphi)\tilde{y}_{e'}) \tilde{\leq} \tilde{r}\tilde{d}(\tilde{x}_e, \tilde{y}_{e'}), \forall \tilde{x}_e, \tilde{y}_{e'} \in \tilde{X} \quad (9)$$

Next, we give some fixed point result for soft (α, β)-Banach-contraction mappings in complete soft metric space.

Theorem 2.7. *Let $(\tilde{X}, \tilde{d}, E)$ be a complete soft metric space . Let (T, φ) be a soft (α, β) - Banach contractive mapping from $\tilde{X}$ into itself satisfying the following*

(2.7.1) there exists $\tilde{x}^0_{e_0} \in \tilde{X}$ such that $(\alpha, \psi)(\tilde{x}^0_{e_0}) \tilde{\geq} \bar{1}$ and ,$(\beta, \phi)(\tilde{x}^0_{e_0}) \tilde{\geq} \bar{1}$,

(2.7.2) (T, φ) is a cyclic soft (α, β) - admissible.

(2.7.3) one of the following conditions holds;

- *(2.7.3.1) (T, φ) is soft continuous,*
- *(2.7.3.2) if $\{\tilde{x}^n_{e_n}\}$ is a sequence in $\tilde{X}$ such that $\{\tilde{x}^n_{e_n}\} \to \tilde{x}_e \tilde{\in} \tilde{X}$ as $n \to \infty$ and $(\beta, \phi)(\tilde{x}^n_{e_n}) \tilde{\geq} \bar{1}$ $\forall n \in N$, then $(\beta, \phi)(\tilde{x}_e) \tilde{\geq} \bar{1}$.*

Then (T, φ) has a fixed point. Futhermore, if $(\alpha, \psi)(\tilde{x}_e) \tilde{\geq} \bar{1}$ and $(\beta, \phi)(\tilde{x}_e) \tilde{\geq} \bar{1}$ for all fixed point $\tilde{x}_e \tilde{\in} \tilde{X}$, then (T, φ) has a unique fixed point.

Proof. Let $\tilde{x}^0_{e_0} \in \tilde{X}$ such that $(\alpha,\psi)(\tilde{x}^0_{e_0})\tilde{\succeq}\bar{1}$ and ,$(\beta,\phi)(\tilde{x}^0_{e_0})\tilde{\succeq}\bar{1}$. We will construct the iterative sequence $\left\{\tilde{x}^n_{e_n}\right\}$, where $\tilde{x}^n_{e_n} = (T,\varphi)\tilde{x}^{n-1}_{e_{n-1}}$ for all $n \in N$. Since (T,φ) is a cyclic soft (α,β) - admissible mapping, we have

$$(\alpha,\psi)(\tilde{x}^0_{e_0})\tilde{\succeq}\bar{1} \Rightarrow (\beta,\phi)\tilde{x}^1_{e_1} = (\beta,\phi)((T,\varphi)\tilde{x}^0_{e_0})\tilde{\succeq}\bar{1} \tag{10}$$

and

$$(\beta,\phi)(\tilde{x}^0_{e_0})\tilde{\succeq}\bar{1} \Rightarrow (\alpha,\psi)\tilde{x}^1_{e_1} = (\alpha,\psi)((T,\varphi)\tilde{x}^0_{e_0})\tilde{\succeq}\bar{1} \tag{11}$$

by similar method , we get

$$(\alpha,\psi)(\tilde{x}^n_{e_n})\tilde{\succeq}\bar{1} \qquad and \qquad (\beta,\phi)(\tilde{x}^n_{e_n})\tilde{\succeq}\bar{1} \qquad \forall \quad n \in N.$$

This implies that

$$(\alpha,\psi)(\tilde{x}^{n-1}_{e_{n-1}})(\beta,\phi)(\tilde{x}^n_{e_n})\tilde{\succeq}\bar{1} \qquad \forall \quad n \in N.$$

From the soft (α,β)-Banach contractive condition of (T,φ) , we have

$$\begin{aligned}
\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n+1}_{e_{n+1}}) &= \tilde{d}((T,\varphi)\tilde{x}^{n-1}_{e_{n-1}},(T,\varphi)\tilde{x}^n_{e_n}) \\
&\tilde{\preceq}(\alpha,\psi)(\tilde{x}^{n-1}_{e_{n-1}})(\beta,\phi)(\tilde{x}^n_{e_n}).\tilde{d}((T,\varphi)\tilde{x}^{n-1}_{e_{n-1}},(T,\varphi)\tilde{x}^n_{e_n}) \\
&\tilde{\preceq}\tilde{r}\tilde{d}(\tilde{x}^{n-1}_{e_{n-1}},\tilde{x}^n_{e_n}) \\
&\vdots \\
&\tilde{\preceq}\tilde{r}^n\tilde{d}(\tilde{x}^0_{e_0},\tilde{x}^1_{e_1})
\end{aligned}$$

$\forall n \in N$. Let $m,n \in N$ such that $m < n$, then we get

$$\begin{aligned}
\tilde{d}(\tilde{x}^m_{e_m},\tilde{x}^n_{e_n}) &\tilde{\preceq}\tilde{d}(\tilde{x}^m_{e_m},\tilde{x}^{m+1}_{e_{m+1}}) + \tilde{d}(\tilde{x}^{m+1}_{e_{m+1}},\tilde{x}^{m+2}_{e_{m+2}}) + \cdots + \tilde{d}(\tilde{x}^{n-1}_{e_{n-1}},\tilde{x}^n_{e_n}) \\
&\tilde{\preceq}(\tilde{r}^m + \tilde{r}^{m+1} + \cdots + \tilde{r}^{n-1})\tilde{d}(\tilde{x}^0_{e_0},\tilde{x}^1_{e_1}) \\
&\tilde{\preceq}\tfrac{\tilde{r}^m}{1-\tilde{r}}\tilde{d}(\tilde{x}^0_{e_0},\tilde{x}^1_{e_1}).
\end{aligned}$$

Letting $m,n \to \infty$, we get $\tilde{d}(\tilde{x}^m_{e_m},\tilde{x}^n_{e_n}) \to \bar{0}$ and so the sequence $\left\{\tilde{x}^n_{e_n}\right\}$ is Cauchy. From the completeness of $\tilde{X}$, there exists $\tilde{x}_e\tilde{\in}\tilde{X}$ such that $\tilde{x}^n_{e_n} \to \tilde{x}_e$ as $n \to \infty$.

Now we assume that (T,φ) is soft continuous.

Hence, we obtain

$$\tilde{x}_e = \lim_{n\to\infty} \tilde{x}^{n+1}_{e_{n+1}} = \lim_{n\to\infty}(T,\varphi)\tilde{x}^n_{e_n} = (T,\varphi)(\lim_{n\to\infty}\tilde{x}^n_{e_n}) = (T,\varphi)\tilde{x}_e.$$

Now we will assume that the condition (2.7.3.2) holds.

Hence $(\beta,\phi)(\tilde{x}_e)\tilde{\succeq}\bar{1}$. Then we have , for each $n \in N$,

$$\begin{aligned}
\tilde{d}((T,\varphi)\tilde{x}_e,\tilde{x}_e) &\tilde{\preceq}\tilde{d}((T,\varphi)\tilde{x}_e,(T,\varphi)\tilde{x}^n_{e_n}) + \tilde{d}((T,\varphi)\tilde{x}^n_{e_n},\tilde{x}_e) \\
&\tilde{\preceq}(\alpha,\psi)(\tilde{x}^n_{e_n})(\beta,\phi)(\tilde{x}_e).\tilde{d}((T,\varphi)\tilde{x}_e,(T,\varphi)\tilde{x}^n_{e_n}) + \tilde{d}((T,\varphi)\tilde{x}^n_{e_n},\tilde{x}_e) \\
&\tilde{\preceq}\tilde{r}\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}_e) + \tilde{d}(\tilde{x}^{n+1}_{e_{n+1}},\tilde{x}_e).
\end{aligned}$$

Letting $n \to \infty$, we get $\tilde{d}((T,\varphi)\tilde{x}_e, \tilde{x}_e) = \bar{0}$, that is , $(T,\varphi)\tilde{x}_e = \tilde{x}_e$. This shows that $\tilde{x}_e$ is a fixed point of (T,φ).
Now , we show that $\tilde{x}_e$ is the unique fixed point of (T,φ). Assume that $\tilde{y}_{e'}$ is another fixed point of (T,φ) . From hypothesis, we find that $(\alpha,\psi)\tilde{x}_e \tilde{\geq} \bar{1}$ and $(\beta,\phi)\tilde{y}_{e'} \tilde{\geq} \bar{1}$, and hence

$$\begin{aligned}\tilde{d}(\tilde{x}_e, \tilde{y}_{e'}) &= \tilde{d}((T,\varphi)\tilde{x}_e, (T,\varphi)\tilde{y}_{e'}) \\ &\tilde{\leq}(\alpha,\psi)\tilde{x}_e(\beta,\phi)\tilde{y}_{e'}.\tilde{d}((T,\varphi)\tilde{x}_e, (T,\varphi)\tilde{y}_{e'}) \\ &\tilde{\leq}\tilde{r}\tilde{d}(\tilde{x}_e, \tilde{y}_{e'}).\end{aligned}$$

This shows that $\tilde{d}(\tilde{x}_e, \tilde{y}_{e'}) = \bar{0}$ and then $\tilde{x}_e = \tilde{y}_{e'}$. Therefore , $\tilde{x}_e$ is the unique fixed point of (T,φ). □

Now generalized cyclic soft (α,β) - contractive type mappings as follows

Definition 2.8. Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and $(T,\varphi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{X}, \tilde{d}, E)$ be a given soft mapping we say that (T,φ) is generalized soft (α,β) - contraction if there exists two soft functions $(\alpha,\psi), (\beta,\phi) : \tilde{X} \to R(E)^*$ and $\bar{0} \leq \tilde{r} < \bar{1}$such that

$$(\alpha,\psi)(\tilde{x}_e)(\beta,\phi)(\tilde{y}_{e'})\tilde{d}((T,\varphi)\tilde{x}_e, (T,\varphi)\tilde{y}_{e'}) \tilde{\leq} \tilde{r} M(\tilde{x}_e, \tilde{y}_{e'}), \tag{12}$$

where

$$M(\tilde{x}_e, \tilde{y}_{e'}) = \max\left\{ \begin{array}{c} \tilde{d}(\tilde{x}_e, \tilde{y}_{e'}), \quad \tilde{d}((T,\varphi)\tilde{x}_e, \tilde{x}_e), \quad \tilde{d}((T,\varphi)\tilde{y}_{e'}, \tilde{y}_{e'}), \\ \frac{1}{2}\left[\tilde{d}((T,\varphi)\tilde{x}_e, \tilde{y}_{e'}) + \tilde{d}(\tilde{x}_e, (T,\varphi)\tilde{y}_{e'})\right] \end{array} \right\},$$

$\forall \tilde{x}_e, \tilde{y}_{e'} \in \tilde{X}$

Theorem 2.9. *Let $(\tilde{X}, \tilde{d}, E)$ be a complete soft metric space . Let (T,φ) be a generalized soft (α,β) - contraction mapping from $\tilde{X}$ into itself satisfying the following*

(2.9.1) there exists $\tilde{x}^0_{e_0} \in \tilde{X}$ such that $(\alpha,\psi)(\tilde{x}^0_{e_0}) \tilde{\geq} \bar{1}$ and ,$(\beta,\phi)(\tilde{x}^0_{e_0}) \tilde{\geq} \bar{1}$,

(2.9.2) (T,φ) is a cyclic soft (α,β) - admissible.

(2.9.3) one of the following conditions holds;

(2.9.3.1) (T,φ) is soft continuous,

(2.9.3.2) if $\{\tilde{x}^n_{e_n}\}$ is a sequence in $\tilde{X}$ such that $\{\tilde{x}^n_{e_n}\} \to \tilde{x}_e \tilde{\in} \tilde{X}$ as $n \to \infty$ and $(\beta,\phi)(\tilde{x}^n_{e_n}) \tilde{\geq} \bar{1}$ $\forall n \in N$, then $(\beta,\phi)(\tilde{x}_e) \tilde{\geq} \bar{1}$.

Then (T,φ) has a fixed point. Futhermore, if $(\alpha,\psi)(\tilde{x}_e)\tilde{\geq}\bar{1}$ and $(\beta,\phi)(\tilde{x}_e)\tilde{\geq}\bar{1}$ for all fixed point $\tilde{x}_e\tilde{\in}\tilde{X}$, then (T,φ) has a unique fixed point.

Proof. Let $\tilde{x}^0_{e_0}\in\tilde{X}$ such that $(\alpha,\psi)(\tilde{x}^0_{e_0})\tilde{\geq}\bar{1}$ and ,$(\beta,\phi)(\tilde{x}^0_{e_0})\tilde{\geq}\bar{1}$. We will construct the iterative sequence $\left\{\tilde{x}^n_{e_n}\right\}$, where $\tilde{x}^n_{e_n}=(T,\varphi)\tilde{x}^{n-1}_{e_{n-1}}$ for all $n\in N$. Since (T,φ) is a cyclic soft (α,β) - admissible mapping, we have

$$(\alpha,\psi)(\tilde{x}^0_{e_0})\tilde{\geq}\bar{1}\Rightarrow(\beta,\phi)\tilde{x}^1_{e_1}=(\beta,\phi)((T,\varphi)\tilde{x}^0_{e_0})\tilde{\geq}\bar{1} \tag{13}$$

and

$$(\beta,\phi)(\tilde{x}^0_{e_0})\tilde{\geq}\bar{1}\Rightarrow(\alpha,\psi)\tilde{x}^1_{e_1}=(\alpha,\psi)((T,\varphi)\tilde{x}^0_{e_0})\tilde{\geq}\bar{1} \tag{14}$$

by similar method , we get

$$(\alpha,\psi)(\tilde{x}^n_{e_n})\tilde{\geq}\bar{1}\qquad and\qquad(\beta,\phi)(\tilde{x}^n_{e_n})\tilde{\geq}\bar{1}\qquad\forall\quad n\in N.$$

This implies that

$$(\alpha,\psi)(\tilde{x}^{n-1}_{e_{n-1}})(\beta,\phi)(\tilde{x}^n_{e_n})\tilde{\geq}\bar{1}\qquad\forall\quad n\in N.$$

From the generalized soft (α,β)-contractive condition of (T,φ) , we have

$$\begin{aligned}\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n+1}_{e_{n+1}})\ &=\tilde{d}((T,\varphi)\tilde{x}^{n-1}_{e_{n-1}},(T,\varphi)\tilde{x}^n_{e_n})\\ &\tilde{\leq}(\alpha,\psi)(\tilde{x}^{n-1}_{e_{n-1}})(\beta,\phi)(\tilde{x}^n_{e_n}).\tilde{d}((T,\varphi)\tilde{x}^{n-1}_{e_{n-1}},(T,\varphi)\tilde{x}^n_{e_n})\\ &\tilde{\leq}\tilde{r}M(\tilde{x}^{n-1}_{e_{n-1}},\tilde{x}^n_{e_n}),\end{aligned}$$

where

$$\begin{aligned}&M(\tilde{x}^{n-1}_{e_{n-1}},\tilde{x}^n_{e_n})\\ &=\max\left\{\begin{array}{c}\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n-1}_{e_{n-1}}),\qquad\tilde{d}((T,\varphi)\tilde{x}^{n-1}_{e_{n-1}},\tilde{x}^{n-1}_{e_{n-1}}),\\ \tilde{d}((T,\varphi)\tilde{x}^n_{e_n},\tilde{x}^n_{e_n}),\\ \frac{1}{2}\left[\tilde{d}((T,\varphi)\tilde{x}^{n-1}_{e_{n-1}},\tilde{x}^n_{e_n})+\tilde{d}(\tilde{x}^{n-1}_{e_{n-1}},(T,\varphi)\tilde{x}^n_{e_n})\right]\end{array}\right\}\\ &=\max\left\{\begin{array}{c}\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n-1}_{e_{n-1}}),\quad\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n-1}_{e_{n-1}}),\quad\tilde{d}(\tilde{x}^{n+1}_{e_{n+1}},\tilde{x}^n_{e_n}),\\ \frac{1}{2}\left[\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^n_{e_n})+\tilde{d}(\tilde{x}^{n-1}_{e_{n-1}},\tilde{x}^{n+1}_{e_{n+1}})\right]\end{array}\right\}\\ &\tilde{\leq}\max\left\{\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n-1}_{e_{n-1}}),\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n+1}_{e_{n+1}})\right\}.\end{aligned}$$

Thus

$$\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n+1}_{e_{n+1}})\quad\tilde{\leq}\tilde{r}\max\left\{\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n-1}_{e_{n-1}}),\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n+1}_{e_{n+1}})\right\}.$$

Suppose $\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n+1}_{e_{n+1}})$ is maximum.
Then

$$\begin{aligned}\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n+1}_{e_{n+1}})\ &\tilde{\leq}\tilde{r}\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n+1}_{e_{n+1}})\\ &<\tilde{d}(\tilde{x}^n_{e_n},\tilde{x}^{n+1}_{e_{n+1}})\end{aligned}$$

is a contradiction.
Hence

$$\tilde{d}(\tilde{x}^n_{e_n}, \tilde{x}^{n+1}_{e_{n+1}})\tilde{\leq}\tilde{r}\tilde{d}(\tilde{x}^n_{e_n}, \tilde{x}^{n-1}_{e_{n-1}}), \forall n \in N.$$

Continuing this way , we get

$$\tilde{d}(\tilde{x}^n_{e_n}, \tilde{x}^{n+1}_{e_{n+1}})\tilde{\leq}\tilde{r}^n\tilde{d}(\tilde{x}^0_{e_0}, \tilde{x}^1_{e_1}), \forall n \in N.$$

Let $m, n \in N$ such that $m < n$, then we get

$$\begin{aligned}\tilde{d}(\tilde{x}^m_{e_m}, \tilde{x}^n_{e_n}) &\tilde{\leq}\tilde{d}(\tilde{x}^m_{e_m}, \tilde{x}^{m+1}_{e_{m+1}}) + \tilde{d}(\tilde{x}^{m+1}_{e_{m+1}}, \tilde{x}^{m+2}_{e_{m+2}}) + \cdots + \tilde{d}(\tilde{x}^{n-1}_{e_{n-1}}, \tilde{x}^n_{e_n})\\ &\tilde{\leq}(\tilde{r}^m + \tilde{r}^{m+1} + \cdots + \tilde{r}^{n-1})\tilde{d}(\tilde{x}^0_{e_0}, \tilde{x}^1_{e_1})\\ &\tilde{\leq}\tfrac{\tilde{r}^m}{1-\tilde{r}}\tilde{d}(\tilde{x}^0_{e_0}, \tilde{x}^1_{e_1}).\end{aligned}$$

Letting $m, n \to \infty$, we get $\tilde{d}(\tilde{x}^m_{e_m}, \tilde{x}^n_{e_n}) \to \bar{0}$ and so the sequence $\{\tilde{x}^n_{e_n}\}$ is Cauchy. From the completeness of $\tilde{X}$, there exists $\tilde{x}_e\tilde{\in}\tilde{X}$ such that $\tilde{x}^n_{e_n} \to \tilde{x}_e$ as $n \to \infty$.
Now we assume that (T, φ) is soft continuous.
Hence, we obtain

$$\tilde{x}_e = \lim_{n\to\infty} \tilde{x}^{n+1}_{e_{n+1}} = \lim_{n\to\infty}(T, \varphi)\tilde{x}^n_{e_n} = (T, \varphi)(\lim_{n\to\infty} \tilde{x}^n_{e_n}) = (T, \varphi)\tilde{x}_e.$$

Now we will assume that the condition (2.9.3.2) holds.
Hence $(\beta, \phi)(\tilde{x}_e)\tilde{\geq}\bar{1}$. Then we have , for each $n \in N$,

$$\begin{aligned}\tilde{d}((T, \varphi)\tilde{x}_e, \tilde{x}_e) &\tilde{\leq}\tilde{d}((T, \varphi)\tilde{x}_e, (T, \varphi)\tilde{x}^n_{e_n}) + \tilde{d}((T, \varphi)\tilde{x}^n_{e_n}, \tilde{x}_e)\\ &\tilde{\leq}(\alpha, \psi)(\tilde{x}^n_{e_n})(\beta, \phi)(\tilde{x}_e).\tilde{d}((T, \varphi)\tilde{x}_e, (T, \varphi)\tilde{x}^n_{e_n}) + \tilde{d}((T, \varphi)\tilde{x}^n_{e_n}, \tilde{x}_e)\\ &\tilde{\leq}\tilde{r}M(\tilde{x}^n_{e_n}, \tilde{x}_e) + \tilde{d}(\tilde{x}^{n+1}_{e_{n+1}}, \tilde{x}_e)\\ &= \tilde{r}\max\left\{\begin{array}{c}\tilde{d}(\tilde{x}^n_{e_n}, \tilde{x}_e), \quad \tilde{d}((T, \varphi)\tilde{x}^n_{e_n}, \tilde{x}^n_{e_n}), \quad \tilde{d}((T, \varphi)\tilde{x}_e, \tilde{x}_e),\\ \frac{1}{2}\left[\tilde{d}((T, \varphi)\tilde{x}^n_{e_n}, \tilde{x}_e) + \tilde{d}(\tilde{x}^n_{e_n}, (T, \varphi)\tilde{x}_e)\right]\end{array}\right\}\\ &+\tilde{d}(\tilde{x}^{n+1}_{e_{n+1}}, \tilde{x}_e)\end{aligned}$$

letting $n \to \infty$, we get

$$\begin{aligned}\tilde{d}((T, \varphi)\tilde{x}_e, \tilde{x}_e) &\tilde{\leq}\tilde{r}\tilde{d}((T, \varphi)\tilde{x}_e, \tilde{x}_e)\\ &< \tilde{d}((T, \varphi)\tilde{x}_e, \tilde{x}_e)\end{aligned}$$

is a contradiction.
Hence $(T, \varphi)\tilde{x}_e = \tilde{x}_e$. This shows that $\tilde{x}_e$ is a fixed point of (T, φ).
Now , we show that $\tilde{x}_e$ is the unique fixed point of (T, φ). Assume that $\tilde{y}_{e'}$ is

another fixed point of (T,φ) . From hypothesis, we find that $(\alpha,\psi)\tilde{x}_e\tilde{\geq}\bar{0}$ and $(\beta,\phi)\tilde{y}_{e'}\tilde{\geq}\bar{0}$, and hence

$$\begin{aligned}
\tilde{d}(\tilde{x}_e,\tilde{y}_{e'}) &= \tilde{d}((T,\varphi)\tilde{x}_e,(T,\varphi)\tilde{y}_{e'}) \\
&\tilde{\leq}(\alpha,\psi)\tilde{x}_e(\beta,\phi)\tilde{y}_{e'}.\tilde{d}((T,\varphi)\tilde{x}_e,(T,\varphi)\tilde{y}_{e'}) \\
&\tilde{\leq}\tilde{r}M(\tilde{x}_e,\tilde{y}_{e'}) \\
&= \tilde{r}\max\left\{\begin{array}{c} \tilde{d}(\tilde{x}_e,\tilde{y}_{e'}),\quad \tilde{d}((T,\varphi)\tilde{x}_e,\tilde{x}_e),\quad \tilde{d}((T,\varphi)\tilde{y}_{e'},\tilde{y}_{e'}), \\ \frac{1}{2}\left[\tilde{d}((T,\varphi)\tilde{x}_e,\tilde{y}_{e'})+\tilde{d}(\tilde{x}_e,(T,\varphi)\tilde{y}_{e'})\right] \end{array}\right\} \\
&= \tilde{r}\max\left\{\begin{array}{c} \tilde{d}(\tilde{x}_e,\tilde{y}_{e'}),\quad \tilde{d}(\tilde{x}_e,\tilde{x}_e),\quad \tilde{d}(\tilde{y}_{e'},\tilde{y}_{e'}), \\ \frac{1}{2}\left[\tilde{d}(\tilde{x}_e,\tilde{y}_{e'})+\tilde{d}(\tilde{x}_e,\tilde{y}_{e'})\right] \end{array}\right\} \\
&\tilde{\leq}\tilde{r}\tilde{d}(\tilde{x}_e,\tilde{y}_{e'}) \\
&< \tilde{d}(\tilde{x}_e,\tilde{y}_{e'})
\end{aligned}$$

is a contradiction.
Hence $\tilde{x}_e=\tilde{y}_{e'}$. Therefore , $\tilde{x}_e$ is the unique fixed point of (T,φ). □

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