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LEFT FILTERS IN TERNARY SEMIGROUPS

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Abstract: In this paper we consider the left ternary filters in a ternary semigroup. We analyze some relations between the left ternary filters and completely prime ideals of a ternary semigroup T.

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Key Words: left ternary filter, completely prime, c-system, d-system

1. Introduction

Lee S.K. and Lee S.S. in [7], introduced the notion of a left(right) filters in a po-semigroup and gave a characterization of the left(right) filters of T interms of the right(left) prime ideals. Kwon Y.I. [4] and Kostaq H. in [5], characterized filters in ordered semigroups. In [10] Subramanyeswara Rao etc defined some relations between the filters of partially ordered Γ-semigroups S. In this paper, we analyze some relations between the left ternaryfilters and completely primeideals of a ternarysemigroup.

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Definition 2.1. [6] A ternary subsemigroup F of a ternarysemigroup T is known as a left ternaryfilter of T if $a, b, c \in T$; $abc \in F \Rightarrow a \in F$. A ternary subsemigroup F of a ternarysemigroup T is a left ternaryfilter of $T \Leftrightarrow a \in T$, $aTT \subseteq F$ implies $a \in F$.

Example 2.2. Let $T = \{x, y, z\}$ with the multiplication defined by

$$abc = \begin{cases} y & \text{if } a \quad = b \quad = y, \\ z & \text{if } a \quad = b \quad = z, \\ x & \text{if otherwised.} \end{cases},$$

Define a relation on T as $I_T \cup \{(x, y, z); (x, z, y)\}$. Then T is a ternarysemigroup and $\{x, y, z\}$; $\{y\}$ are all ternaryfilters of T.

Theorem 2.3. *The nonempty intersection of two left ternaryfilters of a ternarysemigroup T is also a left ternaryfilter of T.*

Proof. Let A, B be two left ternaryfilters of T. Let $a, b, c \in T$, $abc \in A \cap B$. $abc \in A \cap B \Rightarrow abc \in A$ and $abc \in B$.
$a, b, c \in T$; $abc \in A$; A is a left ternaryfilter of $T \Rightarrow a \in A$.
$a, b, c \in T$; $abc \in B$; B is a left ternaryfilter of $T \Rightarrow a \in B$.
$a \in A$; $a \in B \Rightarrow a \in A \cap B$. $a, b, c \in T$, $abc \in A \cap B \Rightarrow a \in A \cap B$. Therefore $A \cap B$ is a left ternaryfilter of T.

Theorem 2.4. The non empty intersection of a family of left ternaryfilters of a ternarysemigroup T is also a left ternaryfilter.

Proof. Let $\{F_\alpha\}_{\alpha \in \Delta}$ be a family of left ternaryfilters of T and let $F = \bigcap_{\alpha \in \Delta} F_\alpha$. Let $a, b, c \in T$, $abc \in F$. Now $abc \in F \Rightarrow abc \in \bigcap_{\alpha \in \Delta} F_\alpha \Rightarrow abc \in F_\alpha$ for each $\alpha \in \Delta$. $abc \in F_\alpha$; F_α is a left ternaryfilter of $T \Rightarrow a \in F_\alpha$ for each $\alpha \in \Delta \Rightarrow a \in \bigcap_{\alpha \in \Delta} F_\alpha \Rightarrow a \in F$. Therefore F is a left ternaryfilter of T.

Definition 2.5. [9] An ideal A of a ternarysemigroup T is known as a completely primeideal provided $x, y, z \in T$; $xyz \in A \Rightarrow$ either $x \in A$ or $y \in A$ or $z \in A$.

Theorem 2.6. A non empty subset F of a ternary subsemigroup T is a left ternaryfilter $\Leftrightarrow T \setminus F$ is a completely prime right ternaryideal of T or empty.

Proof. Suppose that $T \setminus F \neq \phi$. Let $x \in T \setminus F$; $y, z \in T$. Suppose if possible $xyz \notin T \setminus F$. Then $xyz \in F$. Since F is a left ternaryfilter, $x \in F$. It is a contradiction. Thus $xyz \in T \setminus F$ and so $(T \setminus F)TT \subseteq T \setminus F$. Therefore $T \setminus F$ is a right ternaryideal. Now shall we prove that $T \setminus F$ is a completelyprime. Let $x, y, z \in T$ and $xyz \in T \setminus F$. Suppose if possible $x \notin T \setminus F$; $y \notin T \setminus F$ and $z \notin T \setminus F$. Then $x, y, z \in F$. Since F is a ternary subsemigroup of T, $xyz \in F$. It is a contradiction. Thus $x \in T \setminus F$ or $y \in T \setminus F$ or $z \in T \setminus F$. Hence $T \setminus F$ is completely prime. Therefore $T \setminus F$ is a completely prime rightideal of T.

Contrary assume that $T \setminus F$ is a completely prime right ternaryideal of T or empty. If $T \setminus F = \phi$ then $F = T$. Thus F is a left ternaryfilter of T. Suppose that $T \setminus F$ is a completely prime right ternaryideal of T. Let $x, y, z \in F$. Suppose if possible $xyz \notin F$. Then $xyz \in T \setminus F$. Since $T \setminus F$ is a completely prime, $x \in$
$T \setminus F$ or $y \in T \setminus F$ or $z \in T \setminus F$. It is a contradiction. Thus $xyz \in F$ and hence F is a ternary subsemigroup of T. Let $x, y, z \in T$; $xyz \in F$. If $x \notin F$ then
$x \in T \setminus F$. Since $T \setminus F$ is a completely prime rightideal of T, $xyz \in$
$(T \setminus F)TT \subseteq T \setminus F$. It is a contradiction. Thus $x \in F$. Therefore F is a left ternaryfilter of T.

Definition 2.7. [10] A ternaryideal P of a ternarysemigroup T is known as primeideal provided A, B and C are ideals of T and $ABC \subseteq P \Rightarrow$ either $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$.

Corollary 2.8. Let T be a ternarysemigroup and F is a left ternaryfilter of T. Then $T \setminus F$ is a prime right ternaryideal of T or empty.

Proof. Since F is a left ternaryfilter; by theorem 2.6, $T \setminus F$ is a completely prime right ternaryideal of T or empty.

We now introduce the notion of a c-system of a ternarysemigroup.

Definition 2.9. Let T be a ternarysemigroup. A non empty subset A of T is called a c-system of T if for each $a, b \in A$ and $c \in T$ there exist an element $x \in A$ such that $x = acb$.

A non empty subset A of a ternarysemigroup T is a c-system of T if for each $a, b \in A$ there exist an element $c \in A$ such that $c \in aTb$.

Theorem 2.10. Every ternary subsemigroup of a ternarysemigroup T is a c-system.

Proof. Let S be a ternary subsemigroup of T and $a, b, c \in T$. Since S is a ternary subsemigroup of T, $abc \in T$. Let $x = abc$. Therefore there exist an element $x \in S$ such that $x = abc$. Therefore S is a c-system.

Theorem 2.11. A ternaryideal P of a ternarysemigroup T. If $T \setminus P$ is either a c-system of T or empty then P is completely prime.

Proof. Assume that $T \setminus P$ is a c-system of T or $T \setminus P$ is empty. If $T \setminus P$ is empty then $P = T$ and hence P is a completely prime. Suppose that $T \setminus P$ is a c-system of T. Let $a, b, c \in T$ and $abc \in P$. Suppose if possible $a \notin P$; $b \notin P$ and $c \notin P$. Then $a \in T \setminus P$; $b \in T \setminus P$ and $c \in T \setminus P$.
Since $T \setminus P$ is a c-system, there exists $x \in T \setminus P$ such that $x = abc$. $x = abc \in P$. Since P is a completely primeideal of T, we have $x \in P$. It is a contradiction. Hence either $a \in P$ or $b \in P$ or $c \in P$. Therefore P is a completely prime ternaryideal of T.

Definition 2.12. A ternaryideal A of a ternarysemigroup T is called a completely semiprime ideal provided $x^3 \in A$; $x \in T \Rightarrow x \in A$.

Theorem 2.13. Every completely primeideal of a ternarysemigroup T is a completely semiprimeideal of T.

Proof. Let A be a completely primeideal of a ternarysemigroup T. Suppose that $x \in T$ and $x^3 \in A$. Since A is a completely primeideal of T, $x \in A$. Therefore T is a completely semiprime ideal.

Theorem 2.14. A ternaryideal P of a ternarysemigroup T. If $T \setminus P$ is either a c-system of T or empty then P is completely semiprime ideal.

Proof. By Theorem 2.11; P is completely prime.
By Theorem 2.13; P is a completely semiprimeideal.

We now introduce the notion of a d-system of a ternarysemigroup.

Definition 2.15. Let T be a ternarysemigroup. A non empty subset A of T is called a d-system of T if for each $a \in A$ and $b \in T$, there exist an element $x \in A$ such that $x = aba$.

A non empty subset A of a ternarysemigroup T is a d-system of $T \Leftrightarrow$ for each $a \in A$ there exist $x \in A$ such that $x \in aTa$.

Theorem 2.16. An ideal P of a ternarysemigroup T is a completely semiprime $\Leftrightarrow T \setminus P$ is a d-system of T or empty.

Proof. Assume that P is completely semiprime ideal of T and $T \setminus P \neq \phi$. Let $a \in T \setminus P$. Then $a \notin P$. Suppose if possible $x \notin aTa$ for every $x \in T \setminus P$ and $aTa \subseteq P$. $aTa \subseteq P$, Since P is a completely semiprime, $a \in P$. It is a contradiction. Therefore there exist an element $x \in T \setminus P$ such that $x = aba$; $b \in T$. Therefore
$T \setminus P$ is a d-system of T.

Contrary suppose that $T \setminus P$ is a d-system of T or $T \setminus P$ is empty. If $T \setminus P$ is empty then $P = T$ and hence P is completely semiprime. Suppose that $T \setminus P$ is a d-system of T. Let $a \in T$ and $a^3 \in P$. Suppose if possible $a \notin P$. Then $a \in T \setminus P$. Since $T \setminus P$ is a d-system, there exists an element $x \in T \setminus P$ such that $x = aba$ for $b \in T$. $x = aba \in aTa \subseteq P$. Therefore $x \in P$. It is a contradiction. Hence $a \in P$. Thus P is completely semiprime ideal of T.

We now introduce the notion of a left ternaryfilter of T generated by A.

Definition 2.17. Let T be a ternarysemigroup and A be a non empty subset of T. The smallest leftfilter of $T \subseteq A$ is known as a left ternaryfilter of T generated by A and is symbolized by $F_l(A)$.

Theorem 2.18. The left ternaryfilter of a ternarysemigroup T generated by a non empty subset A of T is the intersection of all left ternaryfilters of $T \subseteq A$.

Proof. Let Δ be the set of all left ternaryfilter of $T \subseteq A$. Since T itself is a left ternaryfilter of $T \subseteq A$, $T \in \Delta$. So $\Delta \neq \phi$.

Let $F^* = \bigcap_{\alpha\in\Delta} F$. Since $A \subseteq F$ for all $F \in \Delta$, $A \subseteq F^*$. So $F^* \neq \phi$. By theorem 2.3, F^* is a left ternaryfilter of T. Let K be a left ternaryfilter of $T \subseteq A$. Clearly $A \subseteq K$ and K is a left ternaryfilter of T. Therefore $K \in \Delta \Rightarrow F^* \subseteq K$. Therefore F^* is the smallest left ternaryfilter of $T \subseteq A$ and hence F^* is the left ternaryfilter of T generated by A.

Definition 2.19. A left ternaryfilter F of a ternarysemigroup T is known as a principal leftfilter provided F is a left ternaryfilter generated by $\{a\}$ for some $a \in T$. It is symbolized by $F_l(a)$.

Example 2.20. As in the example 2.2, T is a ternarysemigroup and $F_l(a) = \{a, b, c\}$, $F_l(b) = \{b\}$ and $F_l(c) = \{c\}$ are all the principal left ternaryfilters of the ternarysemigroup T.

Corollary 2.21. Let T be a ternarysemigroup and $a \in T$. Then $F_l(a)$ is the least left ternaryfilter of T containing $\{a\}$

For every $a \in T$, the intersection of all left ternaryfilters containing $\{a\}$ is again a left ternaryfilter and thus the least left ternaryfilter containing $\{a\}$.

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