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ANALYSIS OF TWO-PHASE QUEUING SYSTEM WITH IMPATIENT CUSTOMERS, SERVER BREAKDOWNS AND DELAYED REPAIR

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Abstract: We study a two-phase Markovian queueing model where the first phase of service is for all waiting customers in batch mode and second phase of service is in individual mode to each of them. The server may breakdown during any phase of service and may not be repaired immediately due to non-availability of the repair facility. Arriving customers may balk with certain probability and may depart after joining the queue without getting the service due to impatience. Customers reneging is assumed to occur when the server is in individual service or during breakdown rate of individual service. We derive the probability generating functions for the number of customers present in the system when the server is in different states. Further, closed form of expressions for the mean system size for various states of the server, balking rate, reneging rate and loss rate are derived. Numerical examples are presented.

Key Words: two-phase, breakdowns, delayed repair, balking, reneging

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1. Introduction

Many models for customer impatience in queueing systems have been studied in the past, the source of impatience has always been taken to either a long wait already experienced at queue, or a long wait anticipated by a customer upon arrival. Customer impatience leads to loss of potential customers. Queueing systems with impatience customers has special significance for the real world as it has a very negative effect on the revenue generation of the firm.

Queueing systems with impatient customers have been studied by many authors. Altman and Yechiali (2006) presented a comprehensive analysis of the single server M/M/1 and M/G/1 queues, as well as of the multi-server M/M/C queue, for both the multiple and single vacation cases, and obtained various closed form results. They have shown that the customer abandonments under the single-vacation regime is smaller than that under the multiple-vacation discipline. Yue et al. (2011) discussed a two-phase M/M/1 queueing system with impatient customers, multiple vacations, customer balking and reneging. They obtained the closed form expressions for various states of the server, the average rate of balking, the average rate of reneging and the average rate of loss. Kumar and Sarma (2012) studied single as well as multi-server Markovian queueing systems with reneging, balking and retention of reneged customers. They have presented the steady state analysis and obtained measures of performance. Also, derived certain queueing models as special cases. Baruah et al. (2013) studied a batch arrival queue where service is offered in two stages with customer reneging during the system breakdown or vacation periods. They obtained the mean queue length and mean waiting time. Vasanta Kumar and Srinivasa Rao (2013) presented the steady state analysis of the two-phase N-policy$M^x/M/1$ queueing system with server breakdowns and delayed repair. They derived the steady state solution and analyzed the effect of system parameters on the optimal threshold N through numerical illustrations.

In this paper we study the two-phase M/M/1 queueing system with customer balking and reneging for an unreliable server. Customer reneging is considered during individual service, breakdown and delay periods. Rest of the paper is organized as follows:

Mathematical model is presented in Section 2. In Section 3, steady state solution is obtained. Mean system sizes when the server is in different states are derived in Section 4. Some other performance measures are presented in Section 5. Numerical illustrations and conclusions are presented in Section 6 and Section 7, respectively.

2. Mathematical Model

The following assumptions and notations are used to study the steady state behavior of the system.

(a) Customers arrive according to Poisson process with arrival rate λ.

(b) All the customers waiting in the queue are admitted to batch service in the first phase, after completion proceed to individual service in the second phase. The batch service and individual service times are exponential with parameters β and μ respectively. Customers who arrive during the batch service are also admitted into the batch already in service.

(c) While serving in any phase of service, the server may breakdown at any point of time and the service channel will fail for a short interval of time. The breakdowns follow Poisson process with parameter α.

(d) Due to non-availability of the repairman, there may be delay in repair. The delay time and repair time are exponentially distributed with parameters η and γ respectively.

(e) A customer who on arrival finds the server busy (with batch service or individual service) decides to join the queue with probability b_1 and balk with probability $1-b_1$,where as a customer who on arrival finds the server in breakdown or repair state decides to join the queue with probability b_2 and balk with probability $1 - b_2$.

(f) In addition to balking there is reneging (one by one) during the individual service, waiting for repair and repair periods during individual service. Reneging is assumed to follow exponential distribution with parameter ξ and is independent of the number of customers in the system.

(g) All the random variables defined above are statistically independent.

In order to study the steady state behaviour of the system we define the following steady state probabilities.

P_o = The probability that there is no customer in the system.

$P_b(m, 0)$ =The probability that there are m customers in the batch which is in batch service,
m = 1,2 3,...

$P_{bd}(m,0)$ = The probability that there are m customers in the batch which is in batch service, but the server is found to be broken down and waiting for repair, m = 1,2 3,...

$P_{br}(m,0)$ =The probability that there are m customers in the batch which is in batch service, but the server is under repair, m = 1,2 3,...

$P_i(m,n)$ =The probability that there are m customers in the batch queue and n customers in the individual queue while the server is in individual service, m = 1,2 3,... and n = 1, 2, 3,...

$P_{id}(m,n)$ =The probability that there are m customers in the batch queue and n customers in the individual queue while the server is in individual service, but found to be broken down and waiting for repair, m = 0,1,2 3,... and n = 1,2,3, ...

$P_{ir}(m,n)$ = The probability that there are m customers in the batch queue and n customers in the individual queue while the server is in individual service, but is under repair, m = 0,1,2 3,... and n = 1,2,3,...

The balance equations are as follows

$$\begin{aligned}
\lambda b_1 P_0 =& \mu P_i(0,1), && (1)\\
(\lambda b_1+\beta+\alpha)P_b(1,0) =& \mu P_i(1,1)+\gamma P_{br}(1,0)+\lambda b_1 P_0, && (2)\\
(\lambda b_1+\beta+\alpha)P_b(m,0) =& \mu P_i(m,1)+\gamma P_{br}(m,0)\\
&+\lambda b_1 P_b(m-1,0), \quad m>1, && (3)\\
(\lambda b_2+\eta)P_{bd}(1,0) =& \alpha P_b(1,0), && (4)\\
(\lambda b_2+\eta)P_{bd}(m,0) =& \alpha P_b(m,0)+\lambda b_2 P_{bd}(m-1,0), \quad m>1, && (5)\\
(\lambda b_2+\gamma)P_{br}(1,0) =& \eta P_{bd}(1,0), && (6)\\
(\lambda b_2+\gamma)P_{br}(m,0) =& \eta P_{bd}(m,0)+\lambda b_2 P_{br}(m-1,0), \quad m>1, && (7)\\
(\lambda b_1+\mu+\alpha)P_i(0,n) =& \mu P_i(0,n+1)+\beta P_b(n,0)\\
&+\gamma P_{ir}(0,n)+\xi P_i(1,n), \quad n\geq 1, && (8)\\
(\lambda b_1+\mu+\alpha+\xi)P_i(m,n) =& \mu P_i(m,n+1)+\lambda b_1 P_i(m-1,n)\\
&+\gamma P_{ir}(m,n)+\xi P_i(m+1,n),\ m\geq 1,\ n\geq 1, && (9)\\
(\lambda b_2+\eta)P_{id}(0,n) =& \alpha P_i(0,n)+\xi P_{id}(1,n), \quad n\geq 1, && (10)\\
(\lambda b_2+\eta+\xi)P_{id}(m,n) =& \alpha P_i(m,n)+\xi P_{id}(m+1,n)\\
&+\lambda b_2 P_{id}(m-1,n), \quad m\geq 1,\ n\geq 1, && (11)
\end{aligned}$$

$$\begin{aligned}(\lambda b_2+\gamma)P_{ir}(0,n) =&\eta P_{ir}(0,n)+\xi P_{ir}(1,n), \quad n\geq 1, \qquad (12)\\ (\lambda b_2+\gamma+\xi)P_{ir}(m,n) =&\eta P_{ir}(m,n)+\xi P_{ir}(m+1,n)\\ &+\lambda b_2 P_{ir}(m-1,n), \quad m\geq 1,\ n\geq 1. \qquad (13)\end{aligned}$$

The following probability generating functions are defined to solve the balance equations.

$$\begin{aligned}G_b(z) &= \sum_{m=1}^{\infty} P_b(m,0)z^m, \quad G_{bd}(z)=\sum_{m=1}^{\infty} P_{bd}(m,0)z^m,\\ G_{br}(z) &= \sum_{m=1}^{\infty} P_{br}(m,0)z^m, \quad G_i(z,y)=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty} P_i(m,n)z^m y^n,\\ G_{id}(z,y) &= \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} P_{id}(m,n)z^m y^n, \quad G_{ir}(z,y)=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty} P_{ir}(m,n)z^m y^n,\\ R_n(z) &= \sum_{m=0}^{\infty} P_i(m,n)z^m, \quad S_n(z)=\sum_{m=0}^{\infty} P_{ir}(m,n)z^m,\\ T_n(z) &= \sum_{m=0}^{\infty} P_{ir}(m,n)z^m, \quad \text{where } |z|<1\text{and}|y|<1,\end{aligned}$$

$$G(z,y)=P_0+G_b(z)+G_{bd}(z)+G_{br}(z)+G_i(z,y)+G_{id}(z,y)+G_{ir}(z,y)$$

is the generating function of the queue length distribution.

3. Steady State Solution

We multiply equation (3) by z^m, sum over m from 2 to ∞, and add z times equation (2). Then we have

$$(\lambda b_1(1-z)+\beta+\alpha)G_b(z)=\mu R_1(z)+\gamma G_{br}(z)+\lambda b_1 z P_o-\mu P_i(0,1), \qquad (14)$$

Using equation (1), equation (14) becomes

$$(\lambda b_1(1-z)+\beta+\alpha)G_b(z)=\mu R_1(z)+\gamma G_{br}(z)+\lambda b_1(z-1)P_0. \qquad (15)$$

Multiply equation (5) by z^m, sum over m from 2 to ∞, and add z times equation (4). Then we have

$$(\lambda b_2(1-z)+\eta)G_{bd}(z)=\alpha G_b(z). \qquad (16)$$

Multiply equation (7) z^m, sum over m from 2 to ∞, and add z times equation (6). Then we have

$$(\lambda b_2(1-z)+\gamma)G_{br}(z)=\eta G_{bd}(z). \tag{17}$$

Multiply equation (9) by z^m, sum over m from 1 to ∞, and add z times equation (8). Then we have

$$\Big(\lambda b_1(1-z)+\xi(1-\frac{1}{z})\Big)R_n(z)-\mu R_{n+1}(z)=\gamma T_n(z)+\beta P_b(n,0).$$

Multiply this equation by y^n, sum over n from 1 to ∞. Then we have

$$\begin{aligned}(\lambda b_1yz(1-z)-\mu z(1-y)-\xi y(1-z)+\alpha yz)G_i(z,y)\\=-\mu yzR_1(z,y)+\gamma yzG_{ir}(z,y)+\beta yzG_b(y).\end{aligned} \tag{18}$$

Multiply equation (11) by z^m, sum over m from 1 to ∞, and add equation (10). Then we have

$$(\lambda b_2(1-z)+\xi(1-\frac{1}{z})+\eta)S_n(z)=\alpha R_n(z).$$

Multiply this equation by y^n,sum over n from 1 to ∞. Then we have

$$(\lambda b_2(1-z)+\xi(1-\frac{1}{z})+\eta)G_{id}(z,y)=\alpha G_i(z,y). \tag{19}$$

Multiply equation (13)by z^m,sum over m from 1 to ∞,and add equation (11). Then we have

$$(\lambda b_2(1-z)+\xi(1-\frac{1}{z})+\gamma)T_n(z)=\eta S_n(z)$$

Multiply this equation by y^n,sum over n from 1 to ∞.Then we have

$$(\lambda b_2(1-z)+\xi(1-\frac{1}{z})+\gamma)G_{ir}(z,y)=\eta G_{id}(z,y). \tag{20}$$

Substitute the values of $G_b(z)$and $G_{ir}(z)$from equations (14)and (20)in equation (18), and put y=z to get

$$\begin{aligned}\Big[(\lambda b_1(1-z)z^2-\mu z(1-z)+\alpha z^2-\xi z(1-z))(\lambda b_2(1-z)+\xi(1-\frac{1}{z})+\eta)\\(\lambda b_2(1-z)+\xi(1-\frac{1}{z})+\gamma)-\alpha\gamma\eta z^2\Big]G_i(z,z)=\end{aligned}$$

$$(\beta z^2 G_b(z) - \mu z^2 R_1(z))(\lambda b_2(1-z) + \xi(1-\frac{1}{z}) + \eta)(\lambda b_2(1-z) + \xi(1-\frac{1}{z}) + \gamma)). \tag{21}$$

Let z=1 and y=1 in equations (14), (15),(16), (17),(19), (20), and (21) to obtain

$$G_b(z) = \frac{\mu}{\beta} R_1(1), \tag{22}$$

$$G_{bd} = \frac{\alpha}{\eta} G_b(1), \tag{23}$$

$$G_{br}(1) = \frac{\alpha}{\gamma}, \tag{24}$$

$$G_i(1,1) = \frac{\left(\frac{\lambda b_1}{\mu} + \frac{\lambda b_2}{\mu}\left(\frac{\alpha}{\gamma} + \frac{\alpha}{\eta}\right)\right) G_b(1) + \lambda b_1 P_0}{\left(1 - \frac{\lambda b_1}{\mu} - \frac{\lambda b_2}{\mu}\left(\frac{\alpha}{\gamma} + \frac{\alpha}{\eta}\right)\right) + \frac{\xi}{\mu}\left(1 + \frac{\alpha}{\gamma} + \frac{\alpha}{\eta}\right)}, \tag{25}$$

$$G_{id}(1,1) = \frac{\alpha}{\eta} G_i(1,1), \tag{26}$$

and

$$G_{ir} = \frac{\alpha}{\gamma} G_i(1,1). \tag{27}$$

Probability that the server is neither doing batch service nor individual service is given by

$$\begin{aligned} P_0 &= 1 - \left(\frac{\lambda b_1}{\beta} + \frac{\lambda b_2}{\beta}\left(\frac{\alpha}{\gamma} + \frac{\alpha}{\eta}\right)\right) - \left(\frac{\lambda b_1 - \xi}{\mu} + \frac{\lambda b_2 - \xi}{\mu}\left(\frac{\alpha}{\gamma} + \frac{\alpha}{\eta}\right)\right) \\ &= 1 - \rho_1 - \rho_2, \end{aligned} \tag{28}$$

where

$$\rho_1 = \left(\frac{\lambda b_1}{\beta} + \frac{\lambda b_2}{\beta}\left(\frac{\alpha}{\gamma} + \frac{\alpha}{\eta}\right)\right)$$

and

$$\rho_2 = \left(\frac{\lambda b_2 - \xi}{\mu} + \frac{(\lambda b_2 - \xi)}{\mu}\left(\frac{\alpha}{\gamma} + \frac{\alpha}{\eta}\right)\right).$$

To find $R_1(1)$, we use the normalizing condition

$$P_0 + G_b(1) + G_{bd}(1) + G_{br}(1) + G_i(1,1) + G_{id}(1,1) + G_{ir}(1,1) = 1. \tag{29}$$

Substitute equations (22) to (27) in equation (29) to get

$$\left(1+\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\right)\left(1+\frac{\xi}{\mu}\Big(1+\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\Big)\right)\frac{\mu}{\beta}R_1(1)$$
$$=1-\rho_2-P_0\left[1+\frac{\lambda}{\mu}(b_1-b_2)\Big(\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\Big)+\frac{\xi}{\mu}\Big(1+\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\Big)\right] \quad (30)$$

Using the condition $G(1,1)=1-P_0$ in the forms

$$\lim_{y\to 1} G(1,y)=1-P_0 \text{and} \lim_{z\to 1} G(z,1)=1-P_0, \quad (31)$$

we obtain

$$R_1'(1)=\frac{\rho_2}{1-\rho_2}\left[\frac{\lambda b_1}{\mu}+\frac{\lambda b_2}{\mu}\Big(\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\Big)G_b(1)+\frac{\lambda b_1}{\mu}-P_0\right]. \quad (32)$$

4. Mean System Sizes

Mean system sizes for various states of the server are presented in this section. Let $L_b, L_{bd}, L_{br}, L_i, L_{id}$,and L_{ir} be the mean system sizes when the server is in batch service, waiting for repair during batch service, under repair during batch service, in individual service, waiting for repair during individual service and under repair during individual service states, respectively.

Then

$$L_b=\sum_{m=0}^{\infty} mP_b(m,0)=G_b'(1)$$
$$=\left(\frac{\lambda b_1}{\beta}+\frac{\lambda b_2}{\beta}\Big(\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\Big)\right)G_b(1)+\frac{\lambda b_1}{\beta}P_0+\frac{\mu}{\beta}R_1'(1), \quad (33)$$

$$L_{bd}=\sum_{m=1}^{\infty} mP_{bd}(m,0)=G_{bd}'(1)=\frac{\lambda b_2\alpha}{\eta^2}G_b(1)+\frac{\alpha}{\eta}G_b'(1), \quad (34)$$

$$L_{br}=\sum_{m=1}^{\infty} mP_{br}(m,0)=G_{br}'(1)$$
$$=\frac{\lambda b_2}{\gamma}\left(\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\right)G_b(1)+\frac{\alpha}{\gamma}G_b'(1), \quad (35)$$

$$
\begin{aligned}
L_i = &\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(m+n)P_i(m,n) = G_i'(1,1)\\
=&\frac{1}{(1-\rho_2)}\Bigg[\frac{(2\lambda b_1-\xi)-1}{\mu}+\frac{(\lambda b_2-\xi)}{\mu}(\mu\\
&+2\alpha+\xi-\lambda b_1)\left(\frac{1}{\gamma}+\frac{1}{\eta}\right)+\frac{\xi}{\mu}\left(\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\right)\\
&-\frac{\alpha}{\mu\gamma\eta}(\lambda b_2-\xi)^2\Bigg]G_i(1,1)\\
&+\frac{1}{(1-\rho_2)}\Bigg[2\left(\frac{\lambda b_1}{\mu}+\frac{\lambda b_2}{\mu}\Big(\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\Big)\right)G_b(1)\\
&+\left(\frac{\lambda b_1}{\mu}+\frac{\lambda b_2}{\mu}\Big(\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\Big)\right)G_b'(1)\\
&+\frac{(\lambda b_2)^2}{\mu}\left(\frac{\alpha}{\gamma^2}+\frac{\alpha}{\gamma\eta}+\frac{\alpha}{\eta^2}\right)G_b(1)+\frac{2\lambda b_1}{\mu}P_0\Bigg]\\
&-\frac{1}{(1-\rho_2)}\left[\left(\frac{\lambda b_1}{\mu}+\frac{\lambda b_2}{\mu}\left(\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\right)\right)G_b(1)+\frac{\lambda b_1}{\mu}P_0\right]\\
&\left(\frac{1}{\gamma}+\frac{1}{\eta}\right)(\lambda b_2-\xi), \qquad (36)\\
L_{id} = &\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(m+n)P_{id}(m,n) = G_{id}'(1,1)\\
=&\frac{\alpha}{\eta}G_i'(1,1)+\frac{(\lambda b_2-\xi)\alpha}{\eta^2}G_i(1,1), \qquad (37)\\
L_{ir} = &\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(m+n)P_{ir}(m,n) = G_{ir}'(1,1)\\
=&\frac{\alpha}{\gamma}G_i'(1,1)+\frac{(\lambda b_2-\xi)}{\gamma}\left(\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\right)G_i(1,1). \qquad (38)
\end{aligned}
$$

The mean system size can be calculated by

$$
\begin{aligned}
L(N) =&G_b'(1)+G_{bd}'(1)+G_r'(1)+G_i'(1,1)+G_{id}'(1,1)+G_{ir}'(1,1)\\
=&\left(1+\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\right)G_b'(1,1)+\left(1+\frac{\alpha}{\gamma}+\frac{\alpha}{\eta}\right)G_i'(1,1)
\end{aligned}
$$

$$
\begin{aligned}
&+ \lambda b_2 \left(\frac{\alpha}{\eta^2} + \frac{\alpha}{\gamma\eta} + \frac{\alpha}{\gamma^2} \right) G_b(1) \\
&+ (\lambda b_2 - \xi) \left(\frac{\alpha}{\eta^2} + \frac{\alpha}{\gamma\eta} + \frac{\alpha}{\gamma^2} \right) G_i(1,1). \qquad (39)
\end{aligned}
$$

5. Some Performance Measures

In this section performance measures like the average rate of balking(RB), the average rate of reneging(RR) and the average rate of loss(RL) are presented.

The average rate of balking is given by

$$
\begin{aligned}
RB = &\sum_{m=1}^{\infty} \lambda(1-b_1)P_b(m,0) + \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \lambda(1-b_1)P_i(m,n) \\
&+ \sum_{m=1}^{\infty} \lambda(1-b_2)(P_{bd}(m,0) + P_{br}(m,0)) \\
&+ \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \lambda(1-b_2)(P_{id}(m,n) + P_{ir}(m,n)) \qquad (40) \\
=&\lambda(1-b_1)\Big(G_b(1) + G_i(1,1)\Big) \\
&+ \lambda(1-b_2)\Big(G_{bd}(1) + G_{br}(1) + G_{id}(1,1) + G_{ir}(1,1)\Big).
\end{aligned}
$$

The average rate of reneging is given by

$$
\begin{aligned}
RR = &\sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \xi[P_i(m,n) + P_{id}(m,n) + P_{ir}(m,n)] \\
&= \xi[G_i(m,n) + G_{id}(m,n) + G_{ir}(m,n)]. \qquad (41)
\end{aligned}
$$

The average rate of loss due to balking and reneging is given by

$$RL = RB + RR.$$

$\mu = 5, \beta = 8$			$\lambda = 2.2, \beta = 8$			$\lambda = 2.2, \mu = 5$		
λ	$L(N)$	RL	μ	$L(N)$	RL	β	$L(N)$	RL
2.0	0.62	0.91	3	2.12	0.81	5	1.08	0.82
2.6	1.36	1.19	4	1.30	0.96	6	0.98	0.91
2.9	1.75	1.24	5	0.85	1.02	8	0.85	1.02
3.5	2.22	1.00	6	0.57	1.06	10	0.78	1.09
3.8	1.98	0.50	7	0.38	1.08	11	0.75	1.11
3.9	1.76	0.35	8	0.24	1.09	12	0.73	1.13

Table 1: The mean system size and the mean rate of loss by varying (λ, μ, β) $(\alpha = 0.1, \gamma = 2, \eta = 3, \xi = 0.2, b_1 = 0.8, b_2 = 0.3)$

$\eta = 3, \gamma = 2$			$\alpha = 0.1, \gamma = 2$			$\alpha = 0.1, \eta = 3$		
α	L(N)	RL	η	L(N)	RL	γ	L(N)	RL
0.05	0.80	0.94	2.0	0.95	1.05	1.0	1.18	1.12
0.15	0.91	1.10	2.5	0.89	1.03	1.5	0.95	1.05
0.25	1.02	1.26	3.0	0.85	1.02	2.0	0.85	1.02
0.35	1.13	1.41	3.5	0.82	1.01	2.5	0.79	1.00
0.40	1.19	1.48	4.5	0.79	1.00	3.0	0.75	0.99
0.45	1.25	1.54	5.5	0.76	0.99	3.5	0.73	0.98

Table 2: The mean system size and the mean rate of loss by varying (α, η, γ) $(\lambda = 2.2, \mu = 5, \beta = 8, \xi = 0.8, b_1 = 0.8, b_2 = 0.3)$

6. Numerical Illustrations

In order to study the effect of different parameters on the mean system size and the mean rate of loss due to balking and reneging, we compute some numerical results.

From Table 1, we observe that with increase in the values of λ, both L(N) and mean rate of loss are convex functions of λ , while with increase in the values of μ and β, L(N) decreases and mean rate of loss increases.

It is clear from Table 2 that (i) with increase in the values of breakdown rate α, L(N) and mean rate of loss increase, and (ii) with decrease in the values of waiting time for repair $\frac{1}{\eta}$ and repair time $\dfrac{1}{\gamma}$ both L(N) and mean rate of loss decrease.

One can observe from Table 3 (i) with decrease in the values of the balking

$b_2 = 0.3, \xi = 2$			$b_1 = 0.8, \xi = 0.2$			$b_1 = 0.8, b_2 = 0.3$		
α	L(N)	RL	η	L(N)	RL	γ	L(N)	RL
0.4	0.13	1.47	0.1	0.43	1.08	0.10	0.98	0.89
0.5	0.25	1.43	0.2	0.64	1.05	0.20	0.85	1.02
0.6	0.41	1.38	0.4	1.06	0.99	0.30	0.72	1.16
0.7	0.61	1.24	0.5	1.26	0.96	0.35	0.65	1.23
0.8	0.85	1.02	0.7	1.66	0.90	0.40	0.59	1.30
0.9	1.12	0.74	0.8	1.86	0.88	0.45	0.52	1.37

Table 3: The mean system size and the mean rate of loss by varying (b_1, b_2, ξ) $(\lambda = 2.2, \mu = 0.5, \beta = 8, \alpha = 0.1\gamma = 2, \eta = 3)$

probabilities $(1-b_1)$ and $(1-b_2)$, L(N) increases and mean rate of loss decreases, and (ii) with increase in the values of reneging probability ξ , L(N) decreases and mean rate of loss increases.

7. Conclusions

We have studied the impatient behaviour of customers in a two-phase M/M/1 queuing system with server breakdowns and delayed repair. We obtained the probability generating function of the queue length distribution in steady state. The mean system size and the average rate of loss are obtained. Numerical illustration of the effect of system parameters on the mean system size and the average rate of loss is presented.

References

[1] E. Altman, U. Yechaili, Analysis of customers impatience with server vacations, *Queueing Systems*, **52** (2006), 261-279.

[2] M. Baruah, K.C. Madan, T. Eldabi, A two stage batch arrival queue with reneging during vacation and breakdown periods, *American Journal of Operations Research*, **3** (2013), 570-580.

[3] R. Kumar, S.K. Sharma, Queueing with reneging, balking and retention of reneged customers, *International Journal of Mathematical Models and Methods in Applied Sciences*, **6**, No. 7 (2012), 819-827.

[4] Kumar Vasanta, V. Srinivasa Rao, T. Optimal control of an N-policy queueing system with server startup subject to the server breakdowns and delayed repair, *American International Journal of Research in Science, Technology, Engineering and Mathematics*, **1**, No. 3 (2013), 93-102.

[5] D. Yue, W. Yue, X. Li, Analysis of a two-phase queueing system with impatient customers and multiple vacations, In: *The Tenth International Symposium on Operations Research and Its Applications* (2011), 292-298.