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A TRIANGULAR FUZZY DEA MODEL FOR EFFICIENCY EVALUATION

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Abstract: Data envelopment analysis is a widely used non-parametric technique to measure and evaluate the relative efficiency of similar decision making units. Classical DEA models evaluate the efficiency from input and output values which are precise or crisp in nature. But when it is applied in real life situations input and output values vary even over small intervals of time. Hence mostly the data will be imprecise or fluctuating, which can very well be modelled by fuzzy set theory. So in this paper a DEA model is developed which can handle input output values which are fuzzy in nature. The fuzzy DEA model is developed as a fully fuzzy fractional programming problem and a methodology is suggested for solving it.

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1. Introduction

Data envelopment analysis (DEA) is a linear programming based non-parametric

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technique used to measure the relative efficiency of similar decision making units (DMUs) that uses homogeneous multiple inputs to produce homogeneous multiple outputs. Following the seminal work by Charnes et al [5], a wide variety of DEA models are introduced by various researchers (see [4], [11]). The main objective of the DEA technique is the ranking of the DMUs based on their relative efficiency.

The basic DEA models are developed based on the assumption that the input-output values are known exactly. However, in reality input output values are usually fluctuating so that they are found to be within a certain interval. Since DEA is an envelopment model, a slight variation in data may cause a drastic change in the efficiency value of DMUs. Cooper et al were the first to address the problem of imprecise data in DEA (IDEA) [12]. Later Despotis and Smirlis [15] considered the problem of IDEA in a different way. Their approach transforms a non linear DEA model to an equivalent LP problem by applying transformations on the variables. The resulting efficiency score will lie in an interval. Entani et al [17] also proposed a DEA model with interval efficiency measured from both optimistic and pessimistic view point. This model was first developed for crisp data and then extended to interval and fuzzy data. Later Wang et al [25] developed a model to handle interval DEA. A minimum regret based approach is suggested to compare and rank DMUs based on their interval efficiency. Rather than using interval valued data for inputs and outputs which give equal importance to all values in that interval, fuzzy numbers allow modelling of data with varying levels of preference to the values in the interval. Among such fuzzy numbers modelling input output values by triangular fuzzy numbers seems more appropriate in the case of DEA as it allows one particular value to be more preferable than other values in the interval.

Basic DEA model is formulated as a linear fractional programming problem. Charnes and Cooper [3] proposed a method for solving linear fractional problem by transforming it into an equivalent linear problem. Later on, several authors developed different methods to solve generalized fractional programming problems [13]. Several methods are available for solving fuzzy fractional programming problems. Saraj and Safei [24] suggested the use of a Taylor series for fuzzy linear fractional bi level multi objective programming problems. Pop et al. [22] proposed a method to solve the fully fuzzified linear fractional programming problem, where all the variables and parameters are represented by triangular fuzzy numbers. Safei [23] proposed a new method for solving a fully fuzzy linear fractional programming problem with triangular fuzzy numbers. Das et al. [14] pointed out some drawbacks of Safei and suggested a modified algorithm for the same. In this paper imprecise/ambiguous input output values are represented by triangular fuzzy numbers. The efficiency of each DMU is evaluated by solving a fully fuzzy linear fractional programming problem with triangular fuzzy numbers. The method suggested by Akyar et [1] al for ranking of triangular fuzzy numbers is used to rank the DMUs. The rest of the paper is organized as follows. In Section 2 basic definitions and operations in fuzzy set theory and basic model of DEA are explained. Fully fuzzy linear fractional programming is given in Section 3. Proposed model and solution methodology for the efficiency evaluation is given in Section 4. Section 5 illustrates the model using a numerical example. Conclusion is explained in Section 6.

2. Basics of Fuzzy set Theory and Data Envelopment Analysis

2.1. Triangular Fuzzy Numbers and its Operations

The characteristic function $\mu_{\widetilde{A}}(x)$ indicate the membership grade of each element x in the fuzzy set $\widetilde{A}$. The set $\widetilde{A} = \{(x, \mu_{\widetilde{A}}); x \in X\}$ defined by $\mu_{\widetilde{A}}$ for each $x \in X$ is called a fuzzy set. A fuzzy number $\widetilde{A} = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)}, & a \leq x \leq b, \\ \frac{(x-c)}{(b-c)}, & b \leq x \leq c, \\ 0, & \\ quadhboxotherwise. & \end{cases} \tag{1}$$

A triangular fuzzy number $\widetilde{A} = (a, b, c)$ is said to be non negative if and only if $a \geq 0$. Two fuzzy numbers $\widetilde{A} = (a, b, c)$ and $\widetilde{B} = (e, f, g)$ are said to be equal if and only if $a = e, b = f, c = g$. Also we say that $\widetilde{A} \preceq \widetilde{B}$if and only if $a \leq e$, $b \leq f$, $c \leq g$.

Let $\widetilde{A} = (a, b, c)$ and $\widetilde{B} = (e, f, g)$ be two triangular fuzzy numbers, then:

$$\begin{aligned} \widetilde{A} \oplus \widetilde{B} =& (a, b, c) \oplus (e, f, g) = (a + e, b + f, c + g), \\ \ominus \widetilde{A} =& -(a, b, c) = (-c, -b, -a), \\ \widetilde{A} \ominus \widetilde{B} =& (a, b, c) \ominus (e, f, g) = (a - g, b - f, c - e). \end{aligned}$$

Let $\widetilde{A} = (a, b, c)$ be any triangular fuzzy number and $\widetilde{B} = (e, f, g)$ be a non negative triangular fuzzy number, then

$$\widetilde{A} \otimes \widetilde{B} = (a,b,c) \otimes (e,f,g) = \begin{cases} (ae, bf, cg), & a \geq 0, \\ (ag, bf, cg), & \text{a<0, c}\geq \text{ 0}, \\ (ag, bf, cg), & c < 0, \end{cases}$$

$$\widetilde{A} \div \widetilde{B} = \frac{(a,b,c)}{(e,f,g)} = (\frac{a}{g}, \frac{b}{f}, \frac{c}{e}).$$

2.2. Basic Model of DEA

Assume that there are n DMUs to be evaluated. Each DMU uses m homogeneous inputs to produce s homogeneous outputs. In particular, DMUj consumes amounts $x_j = (x_{ij})$ of inputs $(i = 1, 2, ...m)$ and produces amounts $y_j = (y_{rj})$ of outputs $(r = 1, 2, ...s)$. The efficiency of each DMU for $j = 1, 2, ...n$ can be evaluated by solving the linear fractional programming problem.

$$\begin{aligned} \text{Maximize } \theta &= \frac{uy_o}{vx_o}, \\ \text{subject to } \frac{uY}{vX} &\leq 1, \\ u, v &\geq 0. \end{aligned}$$

Here X is the input matrix and Y is the output matrix. We can transform this fractional programming problem as a linear programming problem in the form

$$\begin{aligned} \text{Maximize } \theta &= uy_o, \\ \text{subject to } vx_o &= 1, \\ uY &\leq vX, \\ u, v &\geq 0. \end{aligned}$$

DMU_o is said to be efficient if $\theta^{\square} = 1$ and there exist at least one optimal $(u^{\square}, v^{\square})$ with $u^{\square} > 0$ and $v^{\square} > 0$. Otherwise it is inefficient.

We can also determine the relative efficiency by solving its dual envelopment problem

$$\begin{aligned} &\text{Maximize } \theta, \\ &\text{subject to } \theta x_0 - X\lambda \geq 0, \\ &Y\lambda \geq y_0, \\ &\lambda \geq 0. \end{aligned}$$

2.3. Triangular Fuzzy DEA Model

If we assume the input-output values as triangular fuzzy numbers, then the basic DEA model will become a fully fuzzy linear fractional programming problem (FFLFP) of the form

$$\begin{aligned} \text{Maximize } \widetilde{\theta} &= \frac{\widetilde{u}\widetilde{y}_o}{\widetilde{v}\widetilde{x}_o}, \\ \text{subject to } &\frac{\widetilde{u}\widetilde{Y}}{\widetilde{v}\widetilde{X}} \preceq 1, \\ &\widetilde{u}, \widetilde{v} \succeq 0, \end{aligned}$$

In this problem the parameters are all triangular fuzzy numbers.

3. Fully Fuzzy Linear Fractional Programming Problems

Let us consider a fully fuzzy linear fractional programming problem (FFLFP) in the form

$$\begin{aligned} \text{Maximize } \widetilde{L}(\widetilde{X}) &= \frac{(\widetilde{C} \otimes \widetilde{X}) \oplus \widetilde{\alpha}}{(\widetilde{D} \otimes \widetilde{X}) \oplus \widetilde{\beta}}, \\ \text{subject to } &\widetilde{A} \otimes \widetilde{X} \preceq \widetilde{B}. \end{aligned}$$

Here $\widetilde{X}$is a non negative triangular fuzzy number, and all components of the parameters used are triangular fuzzy numbers. N Safei [23] developed a method to solve such a FFLFP problem. Das et al [14] pointed out some drawbacks of the method and developed their own method to solve such a problem.

By using the arithmetic operations defined above we can express the FFLFP in the form

$$\begin{aligned} \text{Maximize } \widetilde{L} &= \sum_j \frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)}, \\ \text{subject to } &\sum_j (m_i, n_i, t_i) \preceq (b_i, g_i, h_i) \text{ for all i}, \end{aligned}$$

and all decision variables are non negative.

Here (r_j^1, r_j^2, r_j^3) and (s_j^1, s_j^2, s_j^3) denote the lower middle and upper values of $(\widetilde{C} \otimes \widetilde{X}) \oplus \widetilde{\alpha}$ and $(\widetilde{D} \otimes \widetilde{X}) \oplus \widetilde{\beta}$ respectively with membership values as given by equation (1). similarly (m_i, n_i, t_i) and (b_i, g_i, h_i) denote the lower middle and upper values of $\widetilde{A} \otimes \widetilde{X}$ and $\widetilde{B}$ respectively.

Now considering the objective function as three crisp objective functions, we can consider the problem in the form,

$$\text{Maximize } L_1 = \sum_j \left(\frac{r_j^1}{s_j^3}\right), \quad \left(\text{lower value of } \sum_j \frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)}\right),$$

$$\text{Maximize } L_2 = \sum_j \left(\frac{r_j^2}{s_j^2}\right), \quad \left(\text{middle value of } \sum_j \frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)}\right),$$

$$\text{Maximize } L_3 = \sum_j \left(\frac{r_j^3}{s_j^1}\right), \quad \left(\text{upper value of } \sum_j \frac{(r_j^1, r_j^2, r_j^3)}{(s_j^1, s_j^2, s_j^3)}\right),$$

$$\text{subject to } \sum_j m_i \leq b_i, \text{ for all } i,$$

$$\sum_j n_i \leq g_i, \text{ for all } i,$$

$$\sum_j t_i \leq h_i, \text{ for all } i.$$

With all decision variables are non negative.

Here three different objective functions on the same set of constraints are to be maximized.

4. Proposed Method for the Triangular Fuzzy DEA

4.1. Evaluation of Efficiency as Triangular Fuzzy Numbers

Clearly the triangular fuzzy DEA model defined in Section 2.3 has fractional functions in the objective function and constraints. Since the parameters involved in the model are fuzzy numbers, it is not desirable to consider the associated linear programming problem as given in Section 2.2. Using the definitions given in Section 3 we will consider the model as a FFLFP and therefore we will consider a set of three crisp LP problems associated with the model. On solving these crisp problems it is possible to define the efficiency of each DMU as a triangular fuzzy number. For this we will proceed as follows.

Let us consider DMUo whose efficiency is to be evaluated. Assume the input-output values associated with each DMU are non-negative triangular fuzzy numbers. The efficiency $\widetilde{\theta}$ of DMUo is given by solving

$$\begin{aligned} \text{Maximize } \widetilde{\theta} &\approx \frac{\sum_r \widetilde{u}_r \otimes \widetilde{y}_{ro}}{\sum_i \widetilde{v}_i \otimes \widetilde{x}_{io}}, \\ \text{subject to } &\frac{\sum_r \widetilde{u}_r \otimes \widetilde{y}_{rj}}{\sum_i \widetilde{v}_i \otimes \widetilde{x}_{ij}} \preceq 1,\ j = 1, 2, \ldots, n, \end{aligned}$$

$\widetilde{u}_r, \widetilde{v}_i$ are non-negative triangular fuzzy numbers in the form (u_r^L, u_r^M, u_r^U) and (v_i^L, v_i^M, v_i^U). Here $\sum_r$ and $\sum_i$ denote the summation of triangular fuzzy numbers.

Since $\widetilde{x}_{ij}, \widetilde{y}_{rj}, \widetilde{u}_r, \widetilde{v}_i$ are all non-negative triangular fuzzy numbers, using the arithmetic operations on triangular fuzzy numbers, we can express the problem in the form

$$\begin{aligned} \text{Maximize } (\theta_1, \theta_2, \theta_3) &= \frac{(\alpha_1, \alpha_2, \alpha_3)}{(\beta_1, \beta_2, \beta_3)}, \\ \text{subject to } &\frac{(\alpha_1, \alpha_2, \alpha_3)}{(\beta_1, \beta_2, \beta_3)} \preceq (1, 1, 1),\ j = 1, 2, \ldots, n. \end{aligned}$$

Using $(a, b, c) \preceq (e, f, g)$ if and only if $a \le e, b \le f, c \le g$ and $\frac{(a,b,c)}{(e,f,g)} = (\frac{a}{g}, \frac{b}{f}, \frac{c}{e})$ we can consider three crisp fractional programming problems associated with the model having same set of constraints together with the hierarchy conditions on $\widetilde{u}_r = (u_r^L, u_r^M, u_r^U)$ and $\widetilde{v}_i = (v_i^L, v_i^M, v_i^U)$ in the form $u_r^L \le u_r^M \le u_r^U$ and $v_i^L \le v_i^M \le v_i^U$ *for all i&r.*

The crisp fractional programming problem to determine the lower bound efficiency θ_o^L of DMUo is given by

$$\begin{aligned} \text{Maximize } \theta_o^L &= \frac{\sum_{r=1}^s u_r^L y_{ro}^L}{\sum_{i=1}^m v_i^U x_{io}^U}, \\ \text{subject to } &\sum_{r=1}^s u_r^L y_{rj}^L \le \sum_{i=1}^m v_i^U x_{ij}^U, \\ &\sum_{r=1}^s u_r^M y_{rj}^M \le \sum_{i=1}^m v_i^M x_{ij}^M, \\ &\sum_{r=1}^s u_r^U y_{rj}^U \le \sum_{i=1}^m v_i^L x_{ij}^L, \text{ for } j = 1, 2, \ldots, n, \\ &0 \le u_r^L \le u_r^M \le u_r^U, \text{ for } r = 1, 2, \ldots, s, \end{aligned}$$

$$0 \leq v_i^L \leq v_i^M \leq v_i^U, \text{ for } i = 1, 2, \ldots, m.$$

Similarly to determine middle value of the efficiency θ_o^M of DMUo, we have to replace the objective function by

$$\text{Maximize } \theta_o^M = \frac{\sum_{r=1}^{s} u_r^M y_{ro}^M}{\sum_{i=1}^{m} v_i^M x_{io}^M}$$

and to determine the upper bound of efficiency we will consider the objective function

$$\text{Maximize } \theta_o^U = \frac{\sum_{r=1}^{s} u_r^U y_{ro}^U}{\sum_{i=1}^{m} v_i^L x_{io}^L}.$$

We can make these crisp fractional problems as LP problems by normalizing the denominator of the objective function. On solving these LP problems we can obtain the triangular fuzzy number efficiency of DMUo. By considering DMUs one by one we can measure the relative efficiency of each DMU as a triangular fuzzy number. Using any of the ranking method for triangular fuzzy numbers we can rank the DMUs for the relative performance [1].

4.2. Ranking of DMUs

Numerous methods on ranking triangular fuzzy numbers have been proposed by many researchers (see [1], [6], [7], [8], [9], [10], [20], [21], [26]. Among these methods, the method suggested by Akyar et al used the incircle of the triangle for the ordering of triangular fuzzy numbers.

Let $\widetilde{a} = (a^L, a^M, a^U)$ be a triangular fuzzy number. Then we define, the radius of the incircle of the triangle with vertices $(a^L, 0), (a^U, 0)$ and $(a^M, 1)$ as

$$r_{\widetilde{a}} = \frac{a^U - a^L}{a^U - a^L + \sqrt{1 + (a^U - a^M)^2} + \sqrt{1 + (a^M - a^L)^2}}$$

and the incentre as

$$I_{\widetilde{a}}(x_{\widetilde{a}}, y_{\widetilde{a}}) = \frac{(a^U - a^L)(a^M, 1) + \sqrt{1 + (a^U - a^M)^2}(a^L, 0) + \sqrt{1 + (a^M - a^L)^2}(a^U, 0)}{a^U - a^L + \sqrt{1 + (a^U - a^M)^2} + \sqrt{1 + (a^M - a^L)^2}}.$$

Using the incircle, the rank of the triangular fuzzy number $\widetilde{a} = (a^L, a^M, a^U)$ is given by

$$\text{Rank}(\widetilde{a}) = (x_{\widetilde{a}} - \frac{1}{2}y_{\widetilde{a}}, 1 - y_{\widetilde{a}}, a^M).$$

That is, rank of a triangular fuzzy number is defined as a triplet.

Now let, $\widetilde{a} = (a^L, a^M, a^U)$ and $\widetilde{b} = (b^L, b^M, b^U)$ be two triangular fuzzy numbers. Then using the above definition, the ordering of these fuzzy numbers can be defined as

$$\widetilde{a} \preceq \widetilde{b} \text{ if and only if } \text{Rank}(\widetilde{a}) <_L \text{Rank}(\widetilde{b}).$$

Here the symbol $<_L$ denotes the lexicographical order and is defined by

$$(x_1, x_2, x_3) <_L (y_1, y_2, y_3) \Leftrightarrow (\exists m = 1, 2, 3)\forall i < m, (x_i = y_i) \wedge (x_m < y_m).$$

5. Example

As an example consider 5 DMUs each use 2 inputs to produce 2 outputs. Each input and output is given as a triangular fuzzy number. Table 1 gives the data.

DMU	$\widetilde{X}_1$	$\widetilde{X}_2$	$\widetilde{Y}_1$	$\widetilde{Y}_2$
A	$(4, 8, 12)$	$(16, 27, 35)$	$(57, 70, 98)$	$(21, 25, 29)$
B	$(10, 14, 17)$	$(10, 40, 70)$	$(43, 50, 59)$	$(28, 32, 35)$
C	$(13, 14, 15)$	$(6, 8, 9)$	$(57, 59, 61)$	$(28, 35, 41)$
D	$(12, 13, 15)$	$(21, 32, 48)$	$(38, 40, 44)$	$(20, 21, 22)$
E	$(19, 21, 22)$	$(12, 16, 19)$	$(58, 70, 81)$	$(21, 23, 25)$

Table 1: Input output values for five DMUs

Using the proposed model the efficiency of each DMU can be evaluated as a triangular fuzzy number. Using the ranking method of Akyar et al for triangular fuzzy numbers we can evaluate the relative ranking of the considered DMUs. This relative ranking along with the efficiency is given in table 2.

Using the proposed model, the efficiency of each DMU can be evaluated and is given in table 2. Using any ranking method for triangular fuzzy numbers we can consider the relative ranking of the DMUs. From table 2 we can see that the DMU C is most efficient followed by E, A, B and D in order.

DMU	Efficiency	Rank
A	$(0.29, 0.48, 1.00)$	3
B	$(0.23, 0.39, 0.93)$	4
C	$(0.70, 0.79, 1.00)$	1
D	$(0.20, 0.30, 0.39)$	5
E	$(0.38, 0.52, 0.76)$	2

Table 2: Efficiency and rank of each DMU using the proposed method

6. Conclusions

The proposed model evaluates the relative efficiency of the DMUs whose input output values are in the form of triangular fuzzy numbers. A FFLFP model is used here which gives the efficiency of each DMU also as a triangular fuzzy number. Comparison of these fuzzy numbers are done according to a method suggested by Akyar et al. This method is discriminative as it can rank correctly even fuzzy numbers having the same centroid. It also works well in the case of crisp numbers unlike certain other methods used for comparison of fuzzy numbers. By ranking the fuzzy efficiency values, DMUs are also rated in terms of their performance.

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